## Molecular Dynamics and Accelerated Molecular Dynamics

Arthur F. Voter Theoretical Division Los Alamos National Laboratory



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# **Overall Outline**

- Day 1: Molecular dynamics methods
- Day 2: Molecular dynamics methods
- Day 3: Infrequent events, transition state theory, saddle finding, etc.
- Day 4: Accelerated molecular dynamics methods

# **Overall Outline**

- Day 1: Molecular dynamics methods
- Day 2: Molecular dynamics methods
- Day 3: Infrequent events, transition state theory, saddle finding, etc.
- Day 4: Accelerated molecular dynamics methods
  - Quick review of key concepts
  - Parallel-replica dynamics
  - Hyperdynamics
  - Temperature accelerated dynamics (TAD)

## Infrequent Event System



# Review of some key points

For an infrequent-event system, the probability of escaping to any particular final state is proportional to the rate constant for that escape path.

The bottleneck for escape may be energetic (a barrier) or entropic (a free-energy barrier), or some of both.

In transition state theory (TST), we approximate this rate constant as the flux through the dividing surface that separates the two states.

The TST rate is an equilibrium property of the system. We could run a very long MD trajectory and count the crossings per time, or (for the canonical ensemble) we can simply determine the Boltzmann probability of being in the dividing surface subspace and multiply it by the average velocity normal to that dividing surface [ $(2k_BT/\pi m)^{1/2}$ ].

Successive crossings may be correlated; so  $k^{exact}$  is less than or equal to  $k^{TST}$ .

## Review of some key points (cont.)

We can correct for these recrossings, to compute k<sup>exact</sup>, by initiating "half trajectories" at the dividing surface (the same one we used to compute the TST rate), and tallying up where they come to rest. This assumes there is a separation of time scales -- that the correlation time ( $\tau_{corr}$ ) is much shorter than the average reaction time ( $\tau_{rxn}=1/k$ ).

We can actually pick almost any dividing surface we want, and the TST + dynamical corrections formalism will still give the same rate constant.

If there is not a separation of time scales, even the rate itself is ill defined, since there is no longer a first-order (exponential) population decay.

For a true infrequent-event system, after a correlation time has passed since entering the state, the probability distribution function for the time to the next escape event is exponentially distributed (i.e., a poisson process).

#### **Exponential escape-time distribution**

Infrequent events give an exponential distribution of first-escape times (after correlation time has passed since entering state).



We will exploit this in some of the accelerated MD methods.

#### Accelerated molecular dynamics concept



The trajectory finds an appropriate way out (i.e., proportional to the rate constant) without knowing about any of the escape paths except the first one it sees. In accelerated dynamics we try to maintain this key characteristic, while making the first escape happen sooner.

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## **Parallel Replica Dynamics**

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## **Parallel Replica Dynamics**

Parallelizes time evolution

**Assumptions:** 

- infrequent events
- transitions can be detected
- exponential distribution of first-escape times



- correlation time known

AFV, Phys. Rev. B, 57, R13985 (1998) A.F. Voter, September, 2005 LAUR-05-8125

#### Parallel Replica Dynamics - derivation

Consider an infrequent-event system with a set of possible escape events



Total escape rate:

$$k_{tot} = \sum_{i} k_{i}$$

#### Probability distribution function for time to first event:

$$p(t) = k_{tot} \exp(-k_{tot}t)$$

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This behaves like a physical supersystem that is M times larger, with the simulation time on processor 1 representing the time (t) for the whole system.



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At some point, a transition will occur on one of the processors. What can we say about it?

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Define  $t_{\mbox{\tiny sum}}$  as the sum of times on all processors

 $t_{sum} = Mt$ 

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Define  $t_{\mbox{\tiny sum}}$  as the sum of times on all processors

 $t_{sum} = Mt$ Then  $p(t) = k_{super} \exp(-k_{super}t)$   $p(t_{sum}/M) = Mk_{tot} \exp(-Mk_{tot}t)$   $(1/M)p(t_{sum}/M) = k_{tot} \exp(-k_{tot}Mt)$   $p(t_{sum}) = k_{tot} \exp(-k_{tot}t_{sum})$ 

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Notice

$$p(t_{sum}) = k_{tot} \exp(-k_{tot}t_{sum}) \quad <=> \quad p(t) = k_{tot} \exp(-k_{tot}t)$$

Expressed in this summed time, the probability distribution function for the first escape time from this state is exactly that for the original system.

We have done nothing to corrupt the relative escape rates for the different pathways.

The time  $(t_{sum})$  for the supersystem accumulates M times faster.

We have parallelized the time leading up to the first escape event.

#### <u>Comment</u>

Processors need not run at the same speed, nor at constant speed

Replicate entire system on each of M processors.



Randomize momenta independently on each processor.



Run MD for short time ( $\tau_{dephase}$ ) to dephase the replicas.



Start clock and run thermostatted MD on each processor. Watch for transition...



Stop all trajectories when first transition occurs on any processor.



Sum the trajectory times over all M processors. Advance simulation clock by this  $\rm t_{sum}$ 



On the processor where a transition occurred, continue trajectory for a time  $\tau_{corr}$  to allow correlated dynamical events.



Advance simulation clock by  $\tau_{\text{corr}}$ 



Replicate the new state and begin procedure again.



#### **Parallel Replica Dynamics**



The summed time  $(t_{sum})$  obeys the correct exponential distribution, and the system escapes to an appropriate state.

State-to-state dynamics are thus correct;  $\tau_{corr}$  stage even releases the TST assumption [AFV, Phys. Rev. B, 57, R13985 (1998)].

Good parallel efficiency if  $\tau_{rxn}$  / M >>  $\tau_{dephase} + \tau_{corr}$ 

Applicable to any system with exponential first-event statistics

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- best method depends on the system
- simple method for EAM metal systems: periodically perform steepest-descent quench; see if geometry at basin minimum has changed



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# Parallel-replica dynamics example

Ag(111) island-on-island decay

Embedded atom (EAM) potential

Temperature = 400K

5 days on 32 processors (1 GHz Pentium-IIIs)

Upper island decays into lower island via step-edge exchange events.

For this case, parallel replica is even faster than temperatureaccelerated dynamics, since barriers are low relative to T.

AFV, F. Montalenti and T.C. Germann, Ann. Rev. Mater. Res. 32, 321 (2002). A.F. Voter, September, 2005 LAUR-05-8125













t=0.45 μs



t=1.00 µs



#### Must p(t) be exponential?

What if p(t) looks like this?

p(t)

#### Must p(t) be exponential?



If  $p(t)=\delta(t-t_0)$ , parallel replica simulation will detect an event on all M processors at the same time  $(t_0)$ , giving no boost. Worse, the summed time will predict a transition time too large by exactly a factor of M.

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### General comments about non-exponential p(t)



A non-exponential p(t) is almost certainly caused by "hidden" transitions, or, similarly, a diffusive character to some coordinate(s).

Thus, applying parallel-replica dynamics when p(t) is nonexponential will give incorrect times, and incorrect dynamics.

If every transition is detected, each p(t) will be exponential for a (non-diffusive) infrequent-event system, and parallel-replica dynamics will be valid.

# Parallel Replica Dynamics Can we overlook fast transitions?

The maximum boost is limited by the fastest processes in the system. If the rapid transitions seem to be unimportant, can we ignore them? Yes - but be careful...

### Safe case



Trajectory will go back and forth many times before escaping



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#### Dangerous case



Parallel replica on floppy systems (ignoring fast conformational transitions)

Small "fast folding" proteins (Pande et al, 2000, 2001,...)

- folding transitions detected based on energy fluctuations
- screen-saver based (SETI-at-home-like) parallelization
- microsecond time scales, folding seen

Hydrocarbon pyrolysis (Kum, Stuart et al, Clemson, 2002)

- bond-breaking transitions detected
- microseconds at T=2000K (usually T=3000K with MD)
- unexpected ring-formation observed

#### Pyrolysis of n-hexadecane (C<sub>16</sub>H<sub>34</sub>)

3-carbon-ring formation (not in standard models)

Subsequent ring opening leads to branched structure



Cyclic intermediate (11 carbons left) A.F. Voter, September, 2005 LAUR-05-8125



Branched radical after ring reopens.

# **REBO** potential

**Reactive Bond-Order potential** 

Brenner et al, J. Phys.: Condens. Matter 14, 783 (2002)

- fit to lots of C and C-H data
- improved property predictions relative to "Tersoff Brenner" form Brenner, Phys. Rev. B 42, 9458 (1990).
- dihedral angle term for C=C bonds
- good description of graphite

## Summary - Parallel replica dynamics

Most exact of the accelerated dynamics methods

- no harmonic approximation
- goes beyond TST to include correlated dynamical events
- no assumption that barrier is energetic can be entropic

Easy to implement; Requirements:

- transition detection
- good estimate of correlation time

Very general applicability

- any system with exponentially distributed events

Good match to increasing availability of parallel processing power, distributed computing, etc.

#### Limitations - Parallel replica dynamics

Only get parallel boost

Boost drops off when events are frequent (or become frequent because so many processors)

Boost especially limited if correlation time is long

For complex systems (e.g., proteins) even transition detection can be tricky.

Hyperdynamics

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# Hyperdynamics

Builds on umbrella-sampling techniques (e.g., Valleau 1970's) Assumptions:

- infrequent events
- transition state theory (no recrossings)



Procedure:

- design bias potential  $\Delta V$  (zero at dividing surfaces; causes no recrossings)
- run thermostatted trajectory on the biased surface (V+ $\Delta$ V)
- accumulate hypertime as

 $t_{hyper} = \Sigma \Delta t_{MD} exp[\Delta V(R(t))/k_BT]$ 

- time converges on correct value in long-time limit (vanishing relative error)

AFV, J. Chem. Phys. 106, 4665 (1997)

# The hypertime clock



Boost = hypertime/(MD clock time)

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# The hypertime clock



Boost = hypertime/(MD clock time)

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#### Hyperdynamics - derivation

Viewgraphs here

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# Hyperdynamics

Key challenge is designing a bias potential that meets the requirements of the derivation and is computationally efficient.

A few forms have been proposed and tested. Still a subject of ongoing research...

# Flat bias potential

An extremely simple form: flat bias potential



M. M. Steiner, P.-A. Genilloud, and J. W. Wilkins, Phys. Rev. B 57, 10236 (1998).

- no more expensive than normal MD (negative overhead(!))
- very effective for low-dimensional systems
- diminishing boost factor for more than a few atoms.

## Hessian-based bias potential

Detect ridgetop using local approximation of Sevick, Bell and Theodorou (1993),

 $\epsilon_1 < 0$  and  $C_1g = 0$ 

( $\epsilon_1$ ,  $C_1$  = lowest eigenvalue, eigenvector of Hessian; g = gradient)

Design bias potential that turns off smoothly in proximity of ridgetop

Iterative method for finding  $\epsilon_1$  using only first derivatives of potential

Iterative method for finding  $C_1g$  and its derivative using only first derivatives of potential

Good boost, but very tight convergence required for accurate forces

AFV, Phys. Rev. Lett., **78**, 3908 (1997) A.F. Voter, September, 2005 LAUR-05-8125

### Bond-boost bias potential

R.A. Miron and K.A. Fichthorn J. Chem. Phys. **119**, 6210 (2003)

Assumes any transition will signal itself by significant changes in bond lengths

 $\Delta V =$  sum of contributions from every bond

Total  $\Delta V \rightarrow 0$  when any bond is stretched beyond some threshold value

Looks promising:

- fairly general
- very low overhead
- for metal surface diffusion, boost factors up to 10<sup>6</sup>

### Summary - Hyperdynamics

- Powerful if an effective bias potential can be constructed
- Need not detect transitions
- Exponential boost factors possible
- Especially effective if barriers high relative to T
- Possibilities for powerful system-specific bias potentials
- Lots of possibilities for future development of advanced bias potential forms

### **Limitations - Hyperdynamics**

Must design bias potential

Assumes TST holds (though Langevin jiggle recrossings may be OK)

Boost drops off when events are frequent

Harder to implement properly if bottlenecks are entropic (but possible)



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# Temperature Accelerated Dynamics (TAD)

#### Concept:

Raise temperature of system to make events occur more frequently. Filter out the events that should not have occurred at the lower temperature.

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Raise temperature of system to make events occur more frequently. Filter out the events that should not have occurred at the lower temperature.

#### Assumptions:

- infrequent-event system
- transition state theory (no correlated events)
- harmonic transition state theory (gives Arrhenius behavior)

 $k = v_0 \exp[-\Delta E/k_BT]$ 

- all preexponentials ( $\mathrm{v}_0$ ) are greater than  $\mathrm{v}_{min}$ 

#### [Sørensen and Voter, J. Chem. Phys. 112, 9599 (2000)]

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- Run MD at elevated temperature  $(T_{high})$  in state A.
- Intercept each attempted escape from basin A
  - find saddle point (and hence barrier height)
    - (e.g., using nudged elastic band method of Jonsson et al).
  - extrapolate to predict event time at T<sub>low</sub>.
- Reflect system back into basin A and continue.
- When safe, accept transition with shortest time at  $T_{low}$ .
- Go to new state and repeat.



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#### TAD temperature-extrapolated time

Because each rate is assumed to be Arrhenius,

 $k = v_0 \exp[-\Delta E/k_BT]$ ,

the time for each particular event at high T can be extrapolated to low T:

$$t_{low} = t_{high} \exp[\Delta E(1/k_B T_{low} - 1/k_B T_{high})].$$

This time is sampled correctly from the exponential distribution at low T, mapped from the high T sample:


#### The Arrhenius view



#### The confidence line

For a given rate  $k_i$ , the time required to be f-confident to see the first event (e.g., f=0.99 for 99% confidence) is

$$\tau_i f = (1/k_i) \ln[1/(1-f)]$$
 (or  $\delta = 1-f$ )

If rate is Arrhenius (k =  $v_0 exp[-\Delta E/k_BT]$ ), then fraction f of first attempts will occur *above* the line with slope  $-\Delta E/k_B$  and intercept  $v_0^{f} = v_0/ln[1/(1-f)]$ 



TAD - when can we stop the MD and accept an event?



After time  $t_{stop}$ , with confidence 1- $\delta$ , no event can replace shortest-time event seen at low T.

Move system to this state and start again.

Exact dynamics, assuming harmonic TST,  $v_{min}$ , uncertainty  $\delta$ .

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# MD+TAD metal deposition simulation

- MD for each deposition event (2 ps)
- TAD for intervening time (~1 s)
- Embedded atom method (EAM) for fcc metals (e.g., Cu, Ag, ...; LANL fit)



# MD+TAD deposition of Cu/Cu(100)

T=77K, flux= 0.04 ML/s, matching deposition conditions of Egelhoff and Jacob (1989).



Voter, Montalenti, and Germann, Ann. Rev. Mater. Res. 32, 321 (2002) A.F. Voter, September, 2005 LAUR-05-8125

# MD+TAD deposition of Cu/Cu(100) Concerted events observed at T=77K and T=100K:









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# Slice of a nanotube evolved with TAD

High Temperature: 3000K Low Temperature: 1500K States Visited: 100 shown Boost: 129



**0 - 2** μ**s** 

Blas Uberuaga Steve Stuart

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High Temperature: 3000K Low Temperature: 1500K States Visited: 100 shown Boost: 129



after 14 µS Blas Uberuaga Steve Stuart

## TAD: More 60-atom Nanotube Fragments

T=1500K (High T = 3000K), after first 1000 transitions

5x0	6x0	9x0 (63 atoms)	10x0	5x5	6x6	
		То	tal Simulatio	n Time (μs)		
59	20	2.7	0.2 (133 states)	4	3	
			Boos	st		
1445	796	80	7	108	100	

Blas Uberuaga Los Alamos

## TAD: More 60-atom Nanotube Fragments

T=1500K (High T = 3000K), after first 1000 transitions

1445	796	80	7	108	100	29
			Boos	st		
			(133 states)			
59	20	2.7	0.2	4	3	1
		То	tal Simulatio	n Time (μs)		
520	DXU	(63 atoms)	1020	525	020	1 X I
5v0	6 <b>v</b> 0	9x0	10-20	5×5	6×6	777

Blas Uberuaga Los Alamos

# Slice of a 7x7 nanotube evolved with TAD

High Temperature: 3000K

Low Temperature: 1500K



first few transitions

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# Slice of a 7x7 nanotube evolved with TAD





t=0

### 8.5 μs (~25 eV above buckyball)

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### Using minimum barrier to stop TAD sooner

Assume we know the minimum barrier ( $\Delta E_{min}$ ) for escape from this state



F. Montalenti and AFV, J. Chem. Phys. 116, 4819 (2002). A.F. Voter, September, 2005 LAUR-05-8125

### Combining dimer method with TAD

#### **Dimer-TAD**

- Use dimer method (Henkelman and Jonsson, 1999) to find a number of saddles and assume the lowest barrier ( $\Delta E_{min}$ ) is among them
- Supply this  $\Delta E_{min}$  to ETAD for this state
- ----> accuracy of TAD (unless lowest barrier missed), with roughly the speed of the dimer method

# MgO Radiation Damage Annealing T=300K



Coulombic Buckingham potential

Interesting picture emerges for annealing after 400 eV cascade.

Uberuaga, Smith, Cleave, Montalenti, Henkelman, Grimes, Voter, and Sickafus, Phys. Rev. Lett., **92**, 115505 (2004); Phys. Rev. **71**, 104102 (2005).

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### Growth of interstitial clusters in MgO, T=300K

Typical 400 eV collision event forms a few vacancies and interstitials Diffusing interstitials coalesce into clusters (vacancies are immobile) Mono-interstitial - diffuses on ns-µs time scale Di-interstitial - diffuses on s time scale

Tetra-interstitial - immobile (years)

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Is the tetramer a sink for all larger clusters?

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Is the tetramer a sink for all larger clusters? No!

# TAD Simulation: dimer + tetramer interstitial clusters



TAD simulation, Uberuaga et al, 2003

# **Interstitial Cluster Mobility**



- Diffusion barrier follows no clear pattern versus cluster size
- Metastable clusters can be surprisingly mobile
- Metastable clusters can be very stable:
  - metastable  $I_6$  lifetime is years at room temperature

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# Issues for future

- Boost limited by lowest barrier general problem for many realistic systems (e.g., interstitials in metals)
- Roughness in potentials lowers effectiveness/efficiency of saddle searches (e.g., needed in TAD) and transition detection
  - shallow local minima
  - cusps at cutoffs and bond-switching points
- Currently limited to small systems
  - Trying to achieve N scaling
  - Spatial parallelization (with Jacques Amar at U. Toledo)
- Feeding information about atomistic behavior to higher-level models

# Summary

- Accelerated molecular dynamics concept:
  - Let the trajectory find an appropriate way out or state, but coax it into doing so more quickly
- Significant speedup over standard MD when barriers are high relative to temperature
- Often encounter unexpected behavior
- Ongoing challenges
  - low barriers and pesky local minima
  - cuspy potentials
  - scaling with system size

Review: Voter, Montalenti, and Germann, Ann. Rev. Mater. Res. 32, 321 (2002)