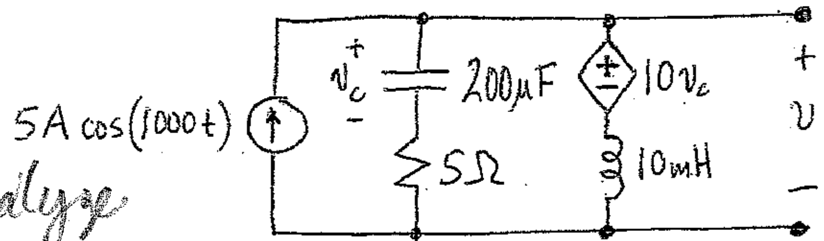
Name Solutions UW ID#: ____________ Section: ___ Version A

Final Exam — EE 233

Spring 2002

The test is closed book, with two sheets of 8.5 by 11 inch notes and standard calculators allowed. Show all work. Be sure to state all assumptions made and **check** them when possible. The number of points per problem are indicated in parentheses. Total of 150 points in 6 problems on 6 pages. A table of Laplace transform pairs are attached as page 7.

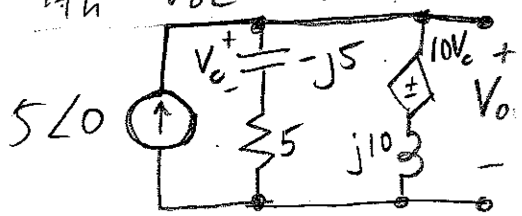
1. Determine the Thevenin equivalent of the circuit shown to the right. (20)



Sinusoidal source, analyze as steady-state (phasor)

$$Z_C = \frac{1}{j\omega C} = \frac{-j}{(1000)(2\times10^{-4})} = -j5$$

$$V_{Th} = V_{oc} = V_0$$



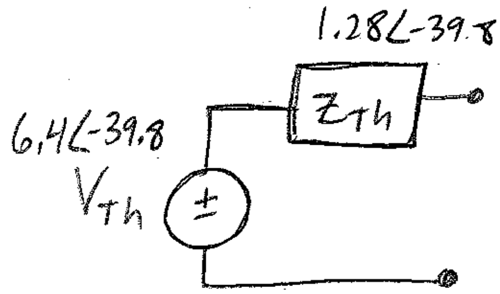
$$Z_L = j\omega L = j(1000)(10^{-2}) = j10$$

$$V_c = V_{oc}\left(\frac{-j5}{5-j5}\right) = V_{oc}\left(\frac{-j}{1-j}\right) = V_{oc}\left(\frac{1-j}{2}\right)$$

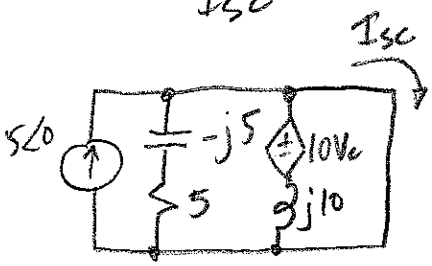
$$5\angle 0 = \frac{V_0}{5(1-j)} + \frac{V_0 - 10V_c}{j10} = \frac{V_0(1+j)}{10} + \frac{V_0\left[1-5(1-j)\right]}{j10}$$

$$= \frac{V_0}{10}\left[1+j-j(1-5+5j)\right] = \frac{V_0}{10}\left[1+j+4j+5\right] = V_0\,\frac{6+5j}{10}$$

$$V_{oc} = \frac{50}{6+5j} = 6.4\angle -39.8$$



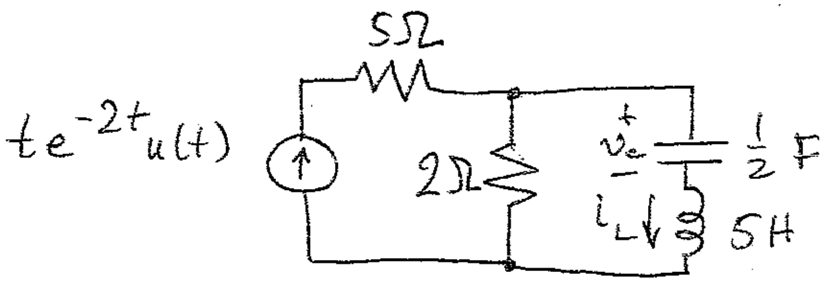
$$Z_{Th} = \frac{V_{oc}}{I_{sc}} \qquad V_0 = 0,\ \text{so } V_c = 0$$



$$I_{sc} = 5\angle 0$$

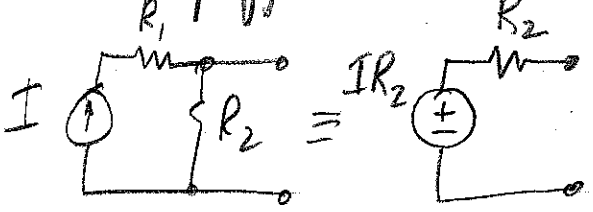
$$Z_{Th} = \frac{10}{6+5j} = 1.28\angle -39.8$$

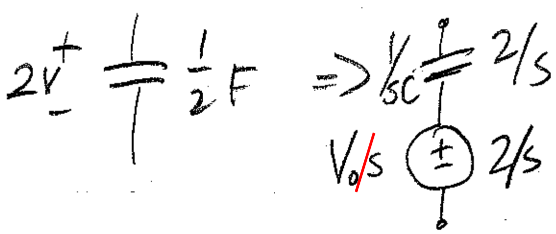
2. In the circuit to the right, $v_C = 2\,\mathrm{V}$ and $i_L = 0.2\,\mathrm{A}$ at $t = 0^-$.
Draw the s-domain circuit valid for $t > 0$ and determine $I_L(s)$. (25)

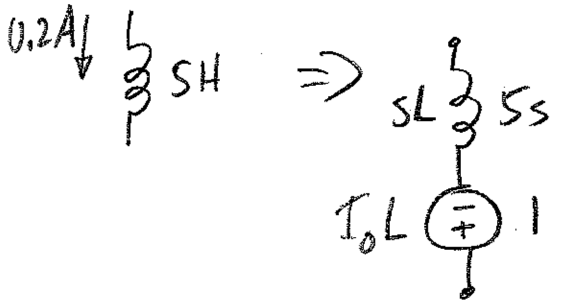


$$\mathcal{L}\{te^{-2t}u(t)\} = \frac{1}{(s+2)^2}$$

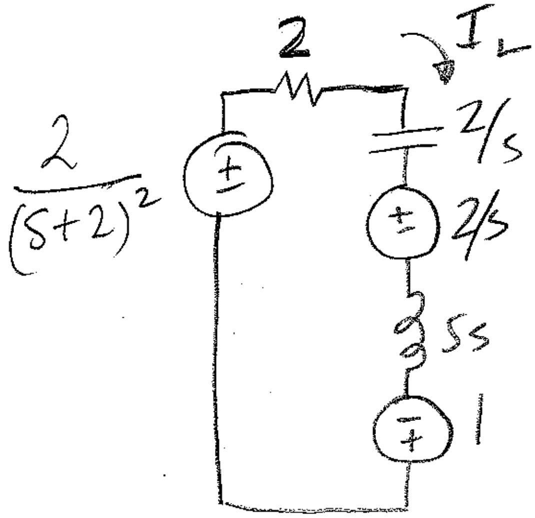
Can simplify source







Combine to series circuit:



$$I_L = \frac{\frac{2}{(s+2)^2} - 2/s + 1}{2 + 2/s + 5s}$$

$$I_L = \frac{\left[2 - (2/s)(s+2)^2 + (s+2)^2\right]s}{(s+2)^2(2s+2+5s^2)}$$

Test w/ IVT $\lim_{s\to\infty} sI_L = \lim_{s\to\infty} \frac{s^4 + \cdots}{5s^4 + \cdots} = \frac{1}{5} = 0.2$ ✓

3. The output of a circuit in the s-domain is:

$$V(s) = \frac{10s}{(s+3)^2} + \frac{s^2}{(s^2+4s+5)}$$

Find $v(t)$. (25)

$$\frac{10s}{(s+3)^2} = \frac{K_1}{(s+3)^2} + \frac{K_2}{(s+3)}$$

$$K_1 = \frac{10s}{\cancel{(s+3)^2}}\cancel{(s+3)^2}\Big|_{s=-3} = -30$$

$$K_2 = \frac{d}{ds}\Big[10s\Big]_{s=-3} = 10$$

Check: $\frac{-30}{(s+3)^2} + \frac{10}{s+3}$

$$= \frac{\cancel{-30} + 10(s+\cancel{3})}{(s+3)^2}$$

$$\frac{s^2}{s^2+4s+5} = 1 - \frac{4s+5}{s^2+4s+5}$$

$$= 1 + \frac{K}{s+2-j} + \frac{K^*}{s+2+j}$$

$$K = \frac{-4s-5}{s+2+j}\Big|_{s=-2+j} = \frac{8-4j-5}{-2+j+2+j}$$

$$= \frac{3-4j}{2j} = -2-\frac{3}{2}j = \frac{5}{2}\angle{-143.1^\circ}$$

Check: For $s=-2$

$$\frac{-2-\frac{3}{2}j}{-2+2-j} + \frac{-2+\frac{3}{2}j}{-2+2+j}$$

$$= \frac{+2+\frac{3}{2}j}{+j} + \frac{-2+\frac{3}{2}j}{+j}$$

$$= 3$$

$$\frac{-4s-5}{s^2+4s+5} = \frac{8-5}{4-8+5} = 3$$

$$\mathcal{L}^{-1}\left\{\frac{-30}{(s+3)^2} + \frac{10}{(s+3)}\right\} = \left[-30te^{-3t} + 10e^{-3t}\right]u(t)$$

$$\mathcal{L}^{-1}\left\{\frac{\frac{5}{2}\angle{-143.1^\circ}}{s+2-j} + \frac{5/2\angle 143.1^\circ}{s+2+j}\right\} = 5e^{-2t}\cos(t-143.1^\circ)$$

$$\mathcal{L}^{-1}\{1\} = \delta(t)$$

$$v(t) = \delta(t) + \left[10e^{-3t} - 30te^{-3t} + 5e^{-2t}\cos(t-143.1^\circ)\right]u(t)$$

$143.1^\circ = 2.5$ rad

4. For the transfer function

$$H(s) = \frac{s(s+10^5)}{(s^2+2000s+10^8)}:$$

Sketch the *asymptotic* Bode plot (gain in dB and phase in degrees versus $\log_{10}\omega$). (25)

- On both plots, indicate the slopes of the asymptotic behavior within each frequency range.
- At the corner frequencies of the amplitude, calculate the amplitude and phase for the asymptotic approximation.
- On the amplitude plot, also indicate the actual gain at each corner frequency.

$$s^2 + 2000s + 10^8 = s^2 + \frac{\omega_0}{Q}s + \omega_0^2 \Rightarrow \omega_0 = 10^4,\ Q = 5$$

At low frequencies $H(s) \Rightarrow \frac{10^5 s}{10^8} \Rightarrow H(j\omega) \Rightarrow \frac{j\omega}{10^3}$

Zeroes at $\omega = 0, 10^5$

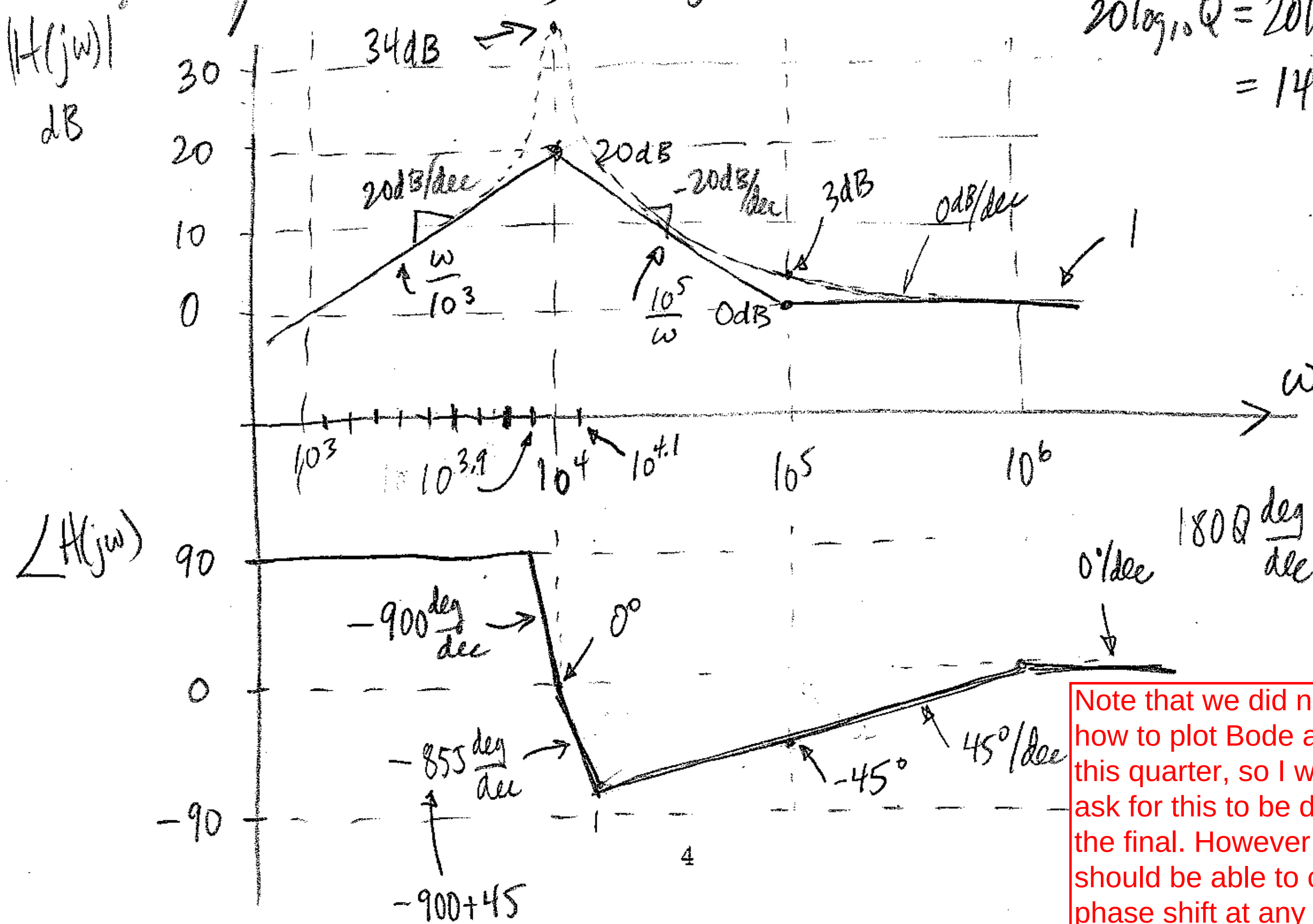
2 poles at $\omega = 10^4$

For $10^4 \ll \omega \ll 10^5$ $H(j\omega) \cong \frac{j\omega(10^5)}{-\omega^2} = -\frac{j10^5}{\omega}$

$\left|\frac{jW}{10^3}\right| = \left|-\frac{j10^5}{\omega}\right| = 10$ @ $W = 10^4$

At high frequencies ($\omega > 10^5$): $H(j\omega) \cong 1$ (0dB)

$20\log_{10} Q = 20\log_{10} 5 = 14\text{dB}$



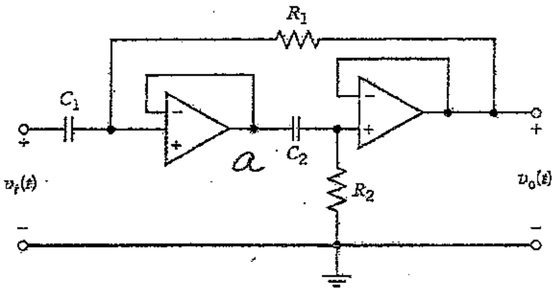
$180Q \frac{\text{deg}}{\text{dec}} = 900 \frac{\text{deg}}{\text{dec}}$

Note that we did not discuss how to plot Bode angle plots this quarter, so I would not ask for this to be drawn on the final. However, you should be able to calculate phase shift at any given frequency.

5. A filter is constructed using the circuit at the right.

   Find the transfer function $H(s)$ in terms of R_1, R_2, C_1, and C_2.

   What type of filter is it? (20)



$$v_+^1 = v_-^1 = v_a, \qquad v_+^2 = v_-^2 = v_o$$

At node v_+^1: $(V_i - V_a)sC_1 = \frac{(V_a - V_0)}{R_1}$ [1]

At node v_+^2: $(V_a - V_o)sC_2 = V_o/R_2 \Rightarrow V_a = V_0\left(1 + \frac{1}{sR_2C_2}\right)$

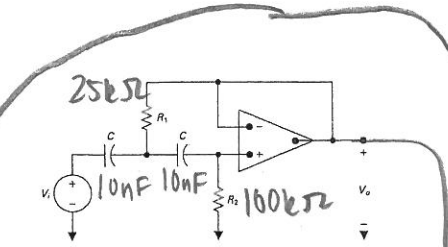
in [1]

$$V_i - \frac{V_o(1+sR_2C_2)}{sR_2C_2} = \frac{1}{sR_1C_1}\left[\frac{V_o}{sR_2C_2}\right]$$

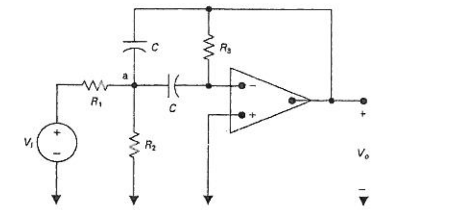
$$V_i = V_o\left[\frac{1 + sR_1C_1(1+sR_2C_2)}{s^2R_1R_2C_1C_2}\right]$$

$$\frac{V_o}{V_i} = \frac{s^2R_1R_2C_1C_2}{s^2R_1R_2C_1C_2 + sR_1C_1 + 1} = \frac{s^2}{s^2 + \frac{1}{R_2C_2}s + \frac{1}{R_1R_2C_1C_2}}$$

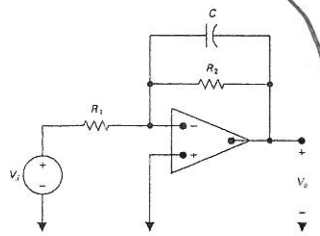
6. Given these filter circuits with associated transfer functions:



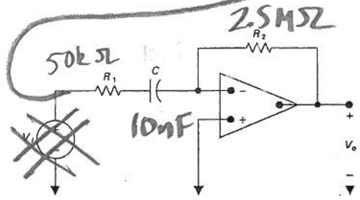
$$\frac{s^2}{s^2 + 2s/(R_2C) + 1/(R_1R_2C^2)}$$



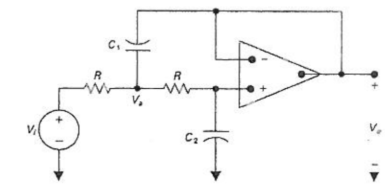
$$\frac{-s/(R_1C)}{s^2 + 2s/(R_3C) + (R_1+R_2)/(R_1R_2R_3C^2)}$$



$$\frac{-1/(R_1C)}{s + 1/(R_2C)}$$



$$\frac{-sR_2/R_1}{s + 1/(R_1C)}$$



$$\frac{1/(R^2C_1C_2)}{s^2 + 2s/(RC_1) + 1/(R^2C_1C_2)}$$

Using the above active filter blocks, design a Butterworth highpass filter with cutoff frequency of $\omega_c = 2000$ rad/s and a pass-band gain of magnitude 34dB. If the gain must be below 0dB for frequencies below 500 rad/s, what is the minimum filter order? Specify all component values, using 10nF capacitors whenever possible without increasing the part count. (30)

Butterworth: $\omega_{cutoff} = \omega_{corner} = 2000 \frac{rad}{s}$

$\frac{\omega_s}{\omega_c} = \frac{500}{2000} = \frac{1}{4}$ $\log_{10}\left(\frac{1}{4}\right) = -0.6$ (60% of a decade)

Since gain must drop by 34dB, required order $\frac{34dB}{0.6(20dB)} = 2.83 \le n$

Thus, a $\underline{3^{rd}}$ order HPF is required. More accurate Butterworth equation gives $n \ge \log_{10}(10^{34/10}-1)/2\log_{10}(4) = 2.82$,

Circuit is illustrated above: → 2nd Order HPF → 1st order HPF →

For 2nd order HPF $\omega_0 = \sqrt{\frac{1}{R_1R_2C^2}} = 2000 = \omega_c$, so $R_1R_2C^2 = 2.5\times10^{-7}$

For 3rd order Butterworth, Q of 2nd order stage is 1 (no drop, 3dB drop from 1st order)

$\frac{\omega_0}{Q} = \frac{2000}{1} = \frac{2}{R_2C}$. These can both be satisfied with $\underline{C = 10nF}$

$R_2 = \frac{2}{2000C} = \frac{2}{2\times10^3(10^{-8})} = 10^5\,\Omega = \underline{100k\Omega}$, $R_1 = \frac{2.5\times10^{-7}}{(10^5)(10^{-8})^2} = 2.5\times10^4\,\Omega = \underline{25k\Omega}$

For 1st order HPF $\omega_0 = 1/R_1C = 2000$, so $R_1C = 5\times10^{-4}$, using $\underline{C = 10nF}$

$R_1 = \frac{5\times10^{-4}}{10^{-8}} = 5\times10^4 = \underline{50k\Omega}$. Passband gain of 2nd order HPF = 1, so

1st order needs gain of 34dB or $50 = \frac{R_2}{R_1}$, so $\underline{R_2 = 2.5M\Omega}$

End Of Exam