# EE233 HW3 Solution

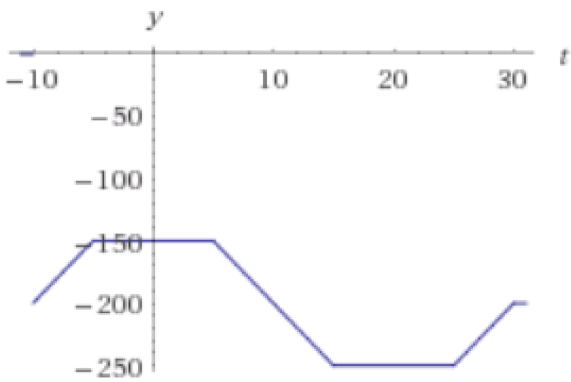
Oct. 17th

Due Date: Oct. 24th

1. Make a sketch of $f(t)$ for -10s$\leq$t$\leq$30s when $f(t)$ is given as follows.

$$f(t) = (10t-100)u(t+10)-(10t+50)u(t+5)+(50-10t)u(t-5)$$
$$-(150-10t)u(t-15)+(10t-250)u(t-25)-(10t-300)u(t-30)$$

Solution:




2. The triangular pulses shown in below figure P.2.a are equivalent to the rectangular pulses in figure P.2.b, because they both encloses the same area $1/\varepsilon$, and they both approach infinity proportional to $1/\varepsilon^2$ as $\varepsilon \to 0$. Use this triangular pulse representation for $\delta'(t)$ to find the Laplace transform of $\delta''(t)$.

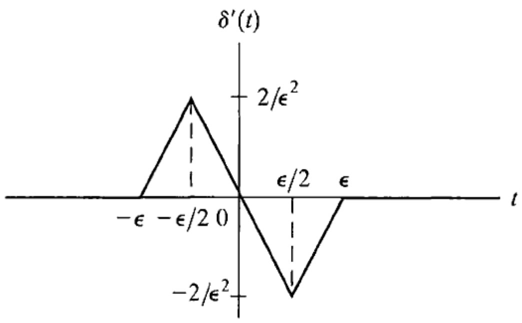



**Figure P.2.a**

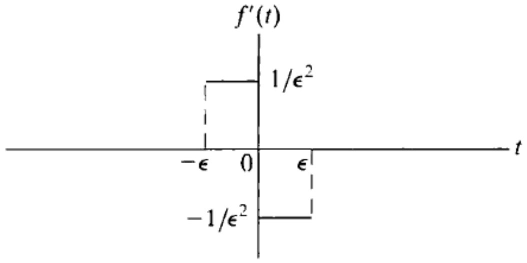
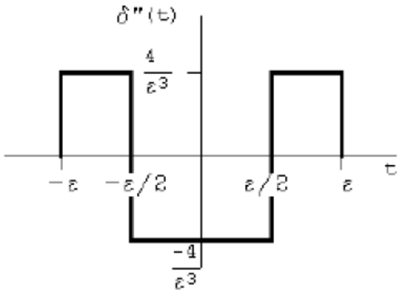



**Figure P.2.b**

Solution:




$$F(s)=\int_{-\varepsilon}^{-\varepsilon/2}\frac{4}{\varepsilon^3}e^{-st}dt+\int_{-\varepsilon/2}^{\varepsilon/2}\frac{-4}{\varepsilon^3}e^{-st}dt+\int_{\varepsilon/2}^{\varepsilon}\frac{4}{\varepsilon^3}e^{st}dt$$

$$=\frac{4}{s\varepsilon^3}[e^{s\varepsilon}-2e^{s\varepsilon/2}+2e^{-s\varepsilon/2}-e^{s\varepsilon}]$$

Using L'Hospital's rule,

$$\lim_{\varepsilon\to 0}\frac{2s}{3}[e^{s\varepsilon}-\frac{s}{4}e^{s\varepsilon/2}-\frac{s}{4}e^{-s\varepsilon/2}+e^{s\varepsilon}]=s^2$$

$$so, L\{\delta^{"}(t)\}=s^2$$

3. (a) Find the Laplace transform of the function illustrated in figure below.

   (b) Find the Laplace transform of the first derivative of the function illustrated in the figure below.

   (c) Find the Laplace transform of the second derivative of the function illustrated in the figure below.

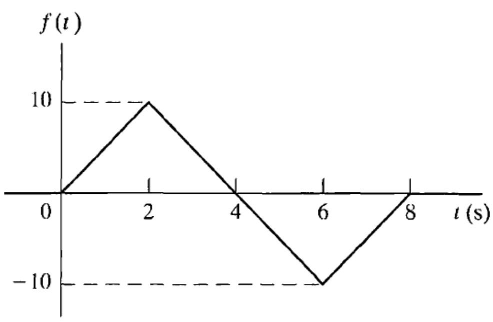



**Figure P.3**

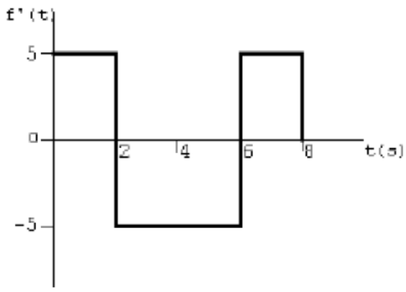
Solution:

(a) $f(t) = 5t[u(t) - u(t-2)] + (20-5t)[u(t-2) - u(t-6)] + (5t-40)[u(t-6) - u(t-8)]$

So, using properties of Laplace transform,

$$F(s) = \frac{5[1-2e^{-2s} + 2e^{-6s} - e^{-8s}]}{s^2}$$

(b) The derivative looks like:



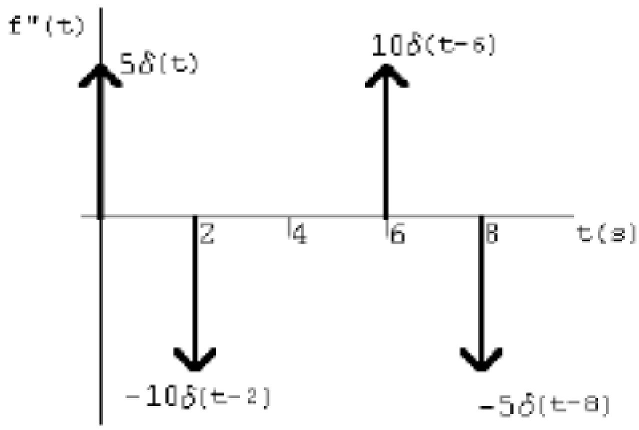

So,

$$f^{'}(t) = 5[u(t) - u(t-2)] - 5[u(t-2) - u(t-6)] + 5[u(t-6) - u(t-8)]$$

*so,*

$$L\{f^{'}(t)\} = \frac{5[1-2e^{-2s} + 2e^{-6s} - e^{-8s}]}{s}$$

(c) The second derivative looks like




So,

$$f^{"}(t) = 5\delta(t) - 10\delta(t-2) + 10\delta(t-6) - 5\delta(t-8)$$

*so,*

$$L\{f^{"}(t)\} = 5[1-2e^{-2s} + 2e^{-6s} - e^{-8s}]$$

4. The switch in the circuit in figure below has been in position a for a long time. At $t=0$, the switch moves instantaneously to position b.

a) Derive the integrodifferential equation that governs the behavior of the voltage

$v_o$ for $t \geq 0^+$.

b) Show that:

$$V_o(s) = \frac{V_{dc}[s+\left(\frac{R}{L}\right)]}{[s^2+\left(\frac{R}{L}\right)s+\left(\frac{1}{LC}\right)]}$$

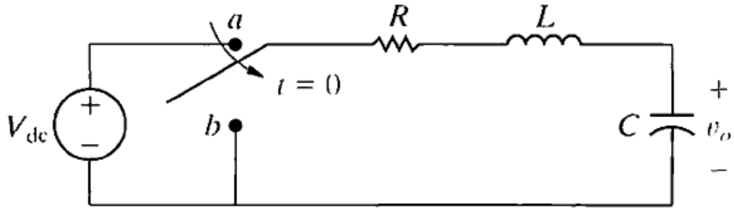



**Figure P.4**

Solution:

(a)

For $t \geq 0^+$:

$$Ri_o + L\frac{di_o}{dt} + v_o = 0$$

$$i_o = C\frac{dv_o}{dt} \qquad \frac{di_o}{dt} = C\frac{d^2v_o}{dt^2}$$

$$\therefore \quad RC\frac{dv_o}{dt} + LC\frac{d^2v_o}{dt^2} + v_o = 0$$

or

$$\frac{d^2v_o}{dt^2} + \frac{R}{L}\frac{dv_o}{dt} + \frac{1}{LC}v_o = 0$$

(b) Take the Laplace transform of the equation in part (a) and use the initial condition. Then,

$$s^2V_o(s) - sV_{dc} - 0 + \frac{R}{L}[sV_o(s) - V_{dc}] + \frac{1}{LC}V_o(s) = 0$$

$$V_o(s)[s^2 + \frac{R}{L}s + \frac{1}{LC}] = V_{dc}[s + \frac{R}{L}]$$

$$V_o(s) = \frac{V_{dc}[s+\frac{R}{L}]}{[s^2+\frac{R}{L}s+\frac{1}{LC}]}$$

5. The circuit parameters in the circuit in below figure have the following values:

$R = 1k\Omega, L = 12.5H, C = 2\mu F, I_{dc} = 30mA$

(a) Find $v_o(t)$ for $t \geq 0$.

(b) Find $i_o(t)$ for $t \geq 0$.

(c) Does your solution for $i_o(t)$ make sense when t=0? Explain.

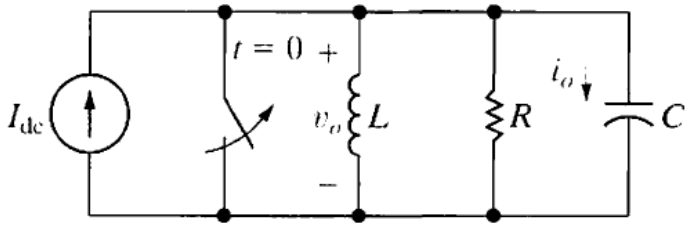



**Figure P.5**

Solution:

(a)

$$\frac{1}{RC}=\frac{1}{(10^3)(2\times10^{-6})}=500$$

$$\frac{1}{LC}=\frac{1}{(12.5)(2\times10^{-6})}=40000$$

$$V_o(s)=\frac{500000I_{dc}}{s^2+500s+40000}=\frac{15000}{(s+400)(s+100)}=\frac{50}{(s+100)}-\frac{50}{(s+400)}$$

$$v_o(t)=[50e^{-100t}-50e^{-400t}]u(t)[Volt]$$

(b)

$$I_o(s)=\frac{0.03s}{s^2+500s+40000}=\frac{-0.01}{(s+100)}+\frac{0.04}{(s+400)}$$

$$i_o(t)=[40e^{-400t}-10e^{-100t}]u(t)[mA]$$

(c)

$i_o(0)=30mA$,

Yes, the initial inductor current is zero by hypothesis. The initial resistor current is zero because the initial capacitor voltage is zero by hypothesis. Thus at t=0, the source current appears in the capacitor.