# EE233 HW5 Solution

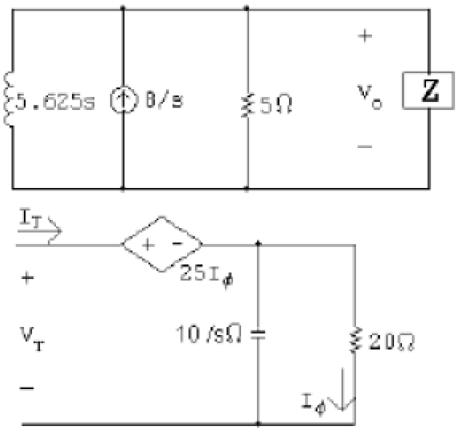
Oct. 31st

Due Date: Nov. 7th

Textbook problems: P.13.28, P.13.55

P.13.28 Solution:

(a)




$$i_L(0^-) = i_L(0^+) = \frac{24}{3} = 8A : directed\ upward$$

$$V_T = 25I_\phi + \frac{20(10/s)}{20+(10/s)} I_T = \frac{25(10/s)I_T}{20+(10/s)} + \frac{200}{10+20s} I_T$$

$$\frac{V_T}{I_T} = \frac{45}{2s+1}$$

*then,*

$$\frac{V_o}{5} + \frac{V_o(2s+1)}{45} + \frac{V_o}{5.625s} = \frac{8}{s}$$

*simplifying*

$$V_o = \frac{180}{s^2+5s+4}$$

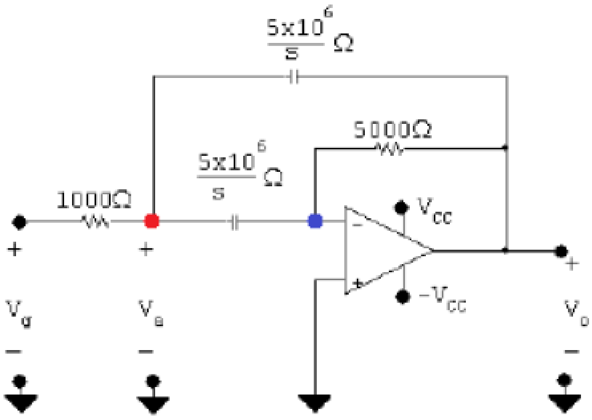
(b)

$$V_o = \frac{180}{s^2+5s+4} = \frac{60}{s+1} - \frac{60}{s+4}$$

$$v_o(t) = [60e^{-t} - 60e^{-4t}]u(t)[Volt]$$

P.13.55 Solution:

(a)




$$\frac{V_a - V_g}{1000} + \frac{sV_a}{5\times10^6} + \frac{s(V_a - V_o)}{5\times10^6} = 0 \;\; at\;\; node\;\; red$$

$$5000V_a - 5000V_g + 2sV_a - sV_o = 0$$

$$(5000 + 2s)V_a - sV_o = 5000V_g$$

$$\frac{s(0 - V_a)}{5\times10^6} + \frac{0 - V_o}{5000} = 0 \;\; at\;\; node\;\; blue$$

$$V_a = \frac{-1000}{s}V_o$$

By plugging the result at node blue to the equation at node red,

$$(2s + 5000)\left(\frac{-1000}{s}\right)V_o - sV_o = 5000V_g$$

$$1000V_o(2s + 5000) + s^2V_o = -5000sV_g$$

$$V_o(s^2 + 2000s + 5\times10^6) = -5000sV_g$$

$$\frac{V_o}{V_g} = \frac{-5000s}{s^2 + 2000s + 5\times10^6}$$

$$s_{1,2} = -1000 \pm \sqrt{10^6 - 5\times10^6} = -1000 \pm j2000$$

$$\frac{V_o}{V_g} = \frac{-5000s}{(s + 1000 - j2000)(s + 1000 + j2000)}$$

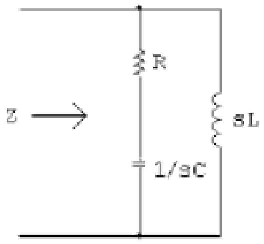
(b)

$$z_1 = 0,\; p_1 = 1000 - j2000,\; p_2 = 1000 + j2000$$

1. A 1 $k\Omega$ resistor is in series with a 625nF capacitor. This is series combination is in parallel with a 100mH inductor.
   (a) Express the equivalent s-domain impedance of these parallel branches as a rational function.
   (b) Determine the numerical values of the poles and zeros.

Solution:

(a)




$$Z = \frac{(R+1/sC)(sL)}{R+sL+(1/sC)} = \frac{(Rs)(s+1/RC)}{s^2+(R/L)s+(1/LC)}$$

*So, plugging in the values,*

$$Z = \frac{1000s(s+1600)}{s^2+10000s+16\times10^6}$$

(b)

$$Z = \frac{1000s(s+1600)}{(s+2000)(s+8000)}$$

$z_1 = 0,\quad z_2 = 1600\, rad/s$

$p_1 = 2000\, rad/s,\ p_2 = 8000\, rad/s$

2. The switch in the circuit in Figure P.2 has been closed for a long time before opening at $t = 0$.

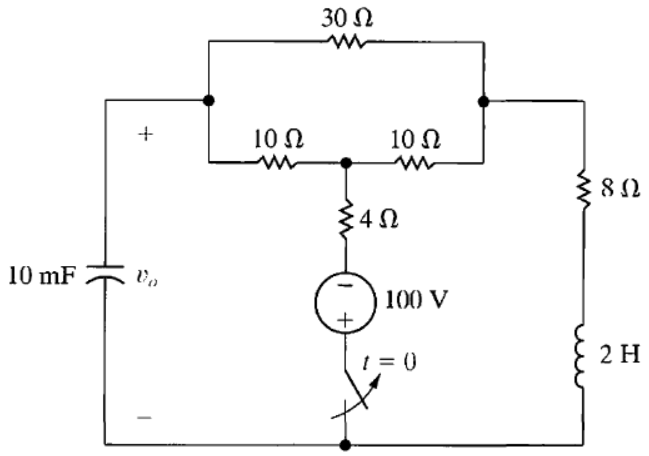



**Figure P.2.a**

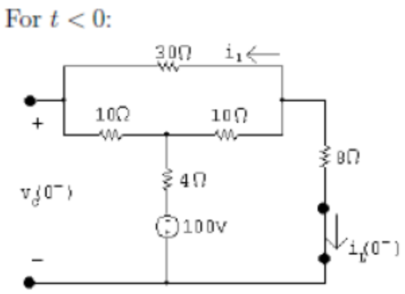
(a) Construct the s-domain equivalent circuit for $t > 0$.

(b) Find $V_o$

(c) Find $v_o$ for $t \geq 0$.

Solution:

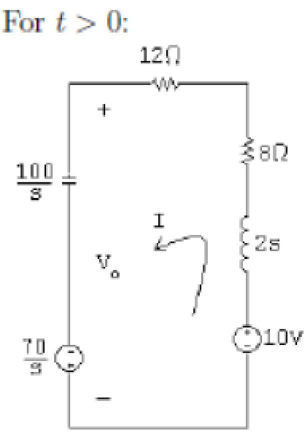
(a)




$$i_L(0^-) = \frac{-100}{4+10\,||\,40+8} = \frac{-100}{20} = -5A$$

$$i_1 = \frac{10}{50} \times 5 = 1A$$

$$v_c(0^-) = 10 \times 1 + 4 \times 5 - 100 = -70V$$




(b)

$$(20 + 2s + \frac{100}{s})I = 10 + \frac{70}{s}$$

$$so, I = \frac{5(s+7)}{s^2 + 10s + 50}$$

$$V_o = \frac{100}{s} I - \frac{70}{s} = \frac{-70(s + 20/7)}{s^2 + 10s + 50} = \frac{K}{s + 5 - j5} + \frac{K^*}{s + 5 + j5}$$

$$where\, K = 38.1\angle -156.8^\circ$$

(c) $v_o(t) = 76.2e^{-5t}\cos(5t - 156.8^\circ)u(t)[Volt]$

3. There is no energy stored in the circuit in figure P.3 at $t = 0^-$.

(a) Find $V_o$

(b) Find $v_o$

(c) Does your solution for $v_o$ make sense in terms of known circuit behavior? Explain.

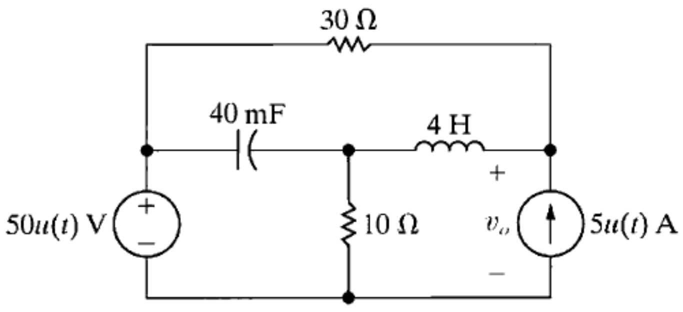
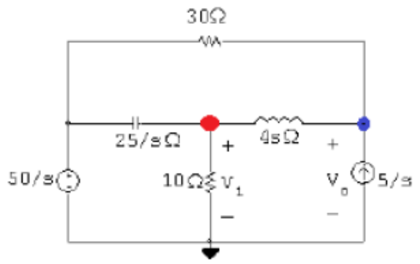



**Figure P.3**

Solution:

(a)




$$\frac{V_1}{10}+\frac{V_1-50/s}{25/s}+\frac{V_1-V_0}{4s}=0$$

$$\frac{-5}{s}+\frac{V_0-V_1}{4s}+\frac{V_0-50/s}{30}=0$$

*Then,*

$$(4s^2+10s+25)V_1-25V_0=200s$$

$$-15V_1+(2s+15)V_0=400$$

*Then, solve for* $V_0$,

$$V_0=\frac{200(8s^2+35s+50)}{8s(s+5)^2}$$

$$V_0=\frac{200(8s^2+35s+50)}{8s(s+5)^2}=\frac{K_1}{s}+\frac{K_2}{(s+5)^2}+\frac{K_3}{(s+5)}$$

$$K_1=50,\ \ K_2=-375,\ \ K_3=150$$

*So,*

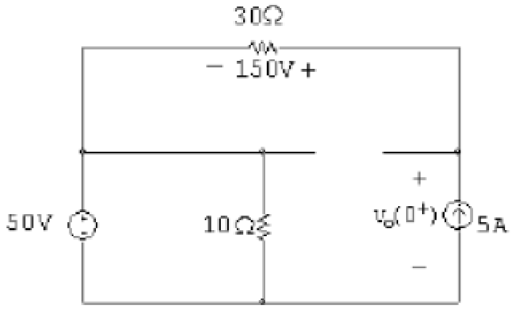
$$V_0=\frac{50}{s}-\frac{375}{(s+5)^2}+\frac{150}{(s+5)}$$

(b)

$$v_o(t)=[50-375te^{-5t}+150e^{-5t}]u(t)[Volt]$$

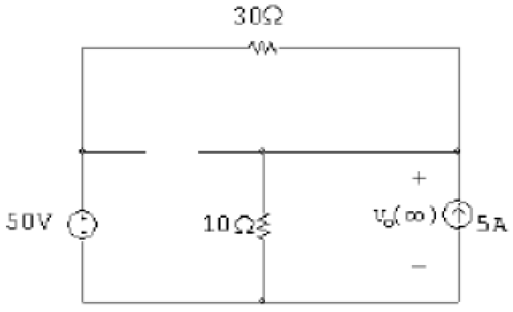
(c)

At $t = 0^+$:




$$v_o(0^+) = 50 + 150 = 200[Volt]$$

At $t = \infty$:




$$\frac{v_o(\infty)}{10} - 5 + \frac{v_o(\infty) - 50}{30} = 0$$

$$3v_o(\infty) - 150 + v_o(\infty) - 50 = 0$$

$$so, v_o(\infty) = 50[Volt]$$