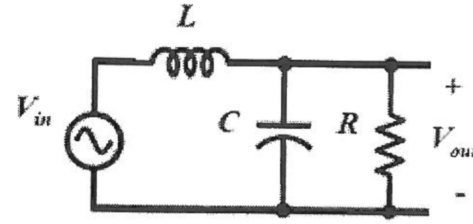
Name ~~Solutions~~ UW ID#: ______ Section: ___ Version A

Mean = 36.7
Median = 38
σ = 7.8

Exam 1 — EE 233

Fall 2011

The test is closed book, with one sheet of notes and standard calculators (no communications) allowed. Show all work. Be sure to state all assumptions made and check them when possible. The number of points per problem are indicated in parentheses. Total of 50 points in 3 problems on 3 pages.



1. In the circuit at right, $v_{in}(t) = 20 \cos(5\times10^6 t + 30^\circ)$ V, L = 20 μH, R = 75 Ω, and C = 2 nF.

(a) Determine the phasor domain values (with units) for each of the four circuit elements. (7)

$$\mathbb{V}_{in} = 20\angle 30^\circ \text{ V (peak)}$$

$$Z_L = j\omega L = j(5\times10^6 \tfrac{rad}{s})(20\times10^{-6} H) = j100\ \Omega = 100\angle 90^\circ\ \Omega$$

$$Z_C = \frac{-j}{\omega C} = -\frac{j}{(5\times10^6 \frac{rad}{s})(2\times10^{-9}F)} = -\frac{j}{10^{-2}} = -j100\ \Omega = 100\angle -90^\circ\ \Omega$$

$$Z_R = 75\ \Omega = 75\angle 0\ \Omega$$

(b) Determine $v_{out}(t)$. (10)

KCL: $$\frac{\mathbb{V}_{out} - \mathbb{V}_{in}}{Z_L} + \frac{\mathbb{V}_{out}}{Z_C} + \frac{\mathbb{V}_{out}}{Z_R} = 0$$

$$\mathbb{V}_{out}\left(\frac{1}{+j100} + \frac{1}{-j100} + \frac{1}{75}\right) = \frac{\mathbb{V}_{in}}{j100}$$

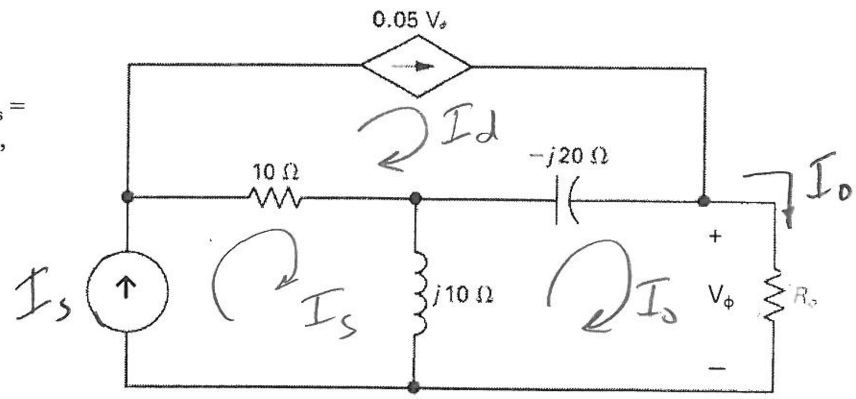
$$\mathbb{V}_{out} = \frac{75}{j100}\mathbb{V}_{in} = 0.75\angle -90^\circ \ 20\angle 30^\circ = 15\angle -60^\circ$$

$$v_{out}(t) = \underline{15 \cos(5\times10^6 t - 60^\circ) \text{ V}}$$

Alternate method: Voltage divider $$\mathbb{V}_{out} = \frac{Z_{R\|C}}{Z_L + Z_{R\|C}}\mathbb{V}_{in} \qquad Z_{R\|C} = \frac{(75)(-j100)}{75 - j100} = \frac{7500\angle -90}{125\angle -53.1} = 60\angle -36.9 = 48.0 - j36.0$$

$$\mathbb{V}_{out} = \frac{60\angle -36.9}{j100 + 48 - j36} = \frac{60\angle -36.9}{48 + j64}\mathbb{V}_{in} = \mathbb{V}_{in}\frac{60\angle -36.9}{80\angle 53.1} = 20\angle 30\ \tfrac{3}{4}\angle -90 = \underline{15\angle -60 \text{ V}}$$

2. In the circuit to the right, the current source is $I_s = 2\angle 60^\circ$ A and R_0 is 40 Ω. Calculate current in R_0 (I_0, measured clockwise) as a phasor (magnitude and phase). Note that V_φ is voltage across R_0. (15)



KVL around I_o loop
(only loop w/ current source)

$$(I_o - I_s)j10 + (I_o - I_d)(-j20) + I_o(40) = 0$$

$$I_d = 0.05V_\phi = 0.05(I_o R_o) = 0.05(40)I_o = 2I_o$$

$$I_o(j10 - j20 + 40) - I_s(j10) - I_d(-j20) = 0$$

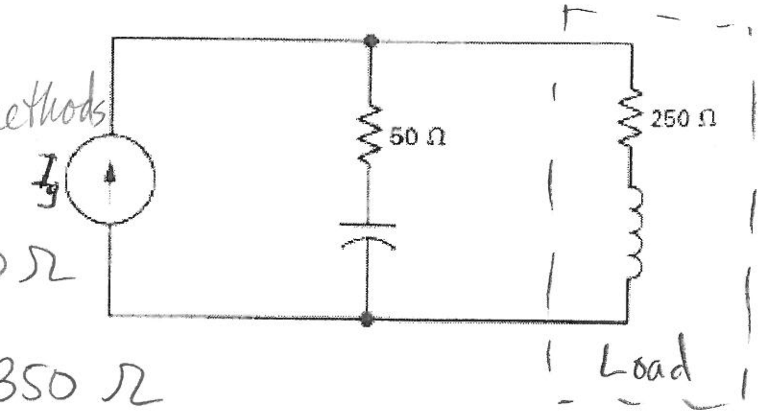
$$I_o(-j10 + 40) - 2I_o(-j20) = I_s(j10)$$

$$I_o(j30 + 40) = I_s(j10)$$

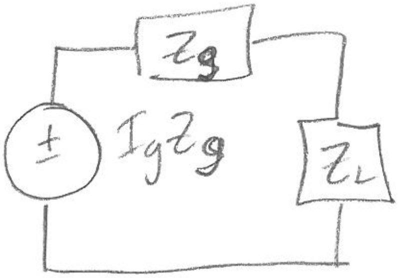
$$I_o = \frac{j10}{40 + j30} I_s = \frac{10\angle 90}{50\angle 36.9} 2\angle 60 = \underline{0.4\angle 113.1^\circ \text{ A}}$$

3. For the circuit to the right, $I_g = 20 \angle 0°$ mA(peak). The capacitor has impedance $-j50\ \Omega$ and the inductor has impedance $j350\ \Omega$.

See B solutions for alternative methods



a) What is the complex power absorbed by the load (RL series? (10)



$$Z_g = 50 - j50\ \Omega$$

$$Z_L = 250 + j350\ \Omega$$

$$V_g = I_g Z_g = (20\angle 0\ mA)(50 - j50\ \Omega) = 1000 - j1000\ mV = 1 - j\ V$$

$$I_L = \frac{V_g}{Z_g + Z_L} = \frac{1-j}{300 + j300} = \frac{1}{j300} = \frac{1}{300}\angle -90\ A$$

$$S = \frac{I_L^* V_L}{2} = \frac{|I_L|^2 Z_L}{2} = \frac{1}{2}\left(\frac{1}{300}\right)^2 (250 + j350) = \frac{250 + j350}{180000}$$

peak, not RMS

$$= \frac{5}{3600} + j\frac{7}{3600} = 0.00139 + j0.00194\ VA$$

b) What value of pure reactance added to the circuit in series with the load (RL series combination) would maximize power transfer to the load? (8)

$|I_L|$ is maximized when $|Z_g + Z_L + jX|$ is minimized

this can be done by having jX cancel net reactance of $Z_g + Z_L$

$Z_g + Z_L = 300 + j300\ \Omega$, so $jX = -300\ \Omega$ or $X = -300$

Since Z_L is not changed, power transfer is maximized when $|I_L|$ (or $|V_L|$) is maximized

End of Exam