Name Solutions UW ID#: ______ Section: ___ Version A

Median = 32

Mean = 31.2

$\sigma = 7.7$

Exam 2 — EE 233

Fall 2011

The test is closed book, with two sheets of notes and calculators allowed. Show all work. Be sure to state all assumptions made and **check** them when possible. The number of points per problem are indicated in parentheses. Total of 50 points in 3 problems on 3 pages (4th page has Laplace transform pairs).

1. Find the inverse Laplace transform of the following s-domain functions:

   (a) Find $f(t)$ for $t > 0^-$ if $F(s) = \dfrac{4s^3 - s^2 + 2s + 2}{2s^2 + 6s + 5}$. (8)

F(s) is improper, so divide out.

$$4s^3 - s^2 + 2s + 2 = 2s(2s^2 + 6s + 5) - 13s^2 - 8s + 2$$
$$= \left(2s - \tfrac{13}{2}\right)(2s^2 + 6s + 5) + 31s + 34.5$$

roots of $2s^2 + 6s + 5$ are $\frac{1}{4}\left[-6 \pm \sqrt{36 - 40}\right] = -\frac{3}{2} \pm j\frac{1}{2}$

$$F(s) = 2s - \frac{13}{2} + \frac{31s + 34.5}{2\left(s + \frac{3}{2} - \frac{j}{2}\right)\left(s + \frac{3}{2} + \frac{j}{2}\right)} = 2s - \frac{13}{2} + \frac{K}{s + \frac{3}{2} - \frac{j}{2}} + \frac{K^*}{s + \frac{3}{2} + \frac{j}{2}}$$

$$K = \left.\frac{31s + 34.5}{2\left(s + \frac{3}{2} + \frac{j}{2}\right)}\right|_{s = -\frac{3}{2} + \frac{j}{2}} = \frac{31\left(-\frac{3}{2} + \frac{j}{2}\right) + 34.5}{2(j)} = \frac{-12 + j\frac{31}{2}}{2j} = 9.8\angle 37.7^\circ$$

$$f(t) = 2\delta'(t) - \frac{13}{2}\delta(t) + 19.6\, e^{-\frac{3}{2}t}\cos(0.5t + 37.7^\circ)\, u(t)$$

   (b) Find $g(t)$ for $t > 0^-$ if $G(s) = \dfrac{(s+5)e^{-3s}}{(s+1)^2}$. (8)

$$G(s) = e^{-3s}\left[\frac{K_1}{(s+1)^2} + \frac{K_2}{(s+1)}\right] \qquad K_1 = (s+5)_{s=-1} = 4$$
$$= e^{-3s}\left[\frac{4}{(s+1)^2} + \frac{1}{(s+1)}\right] \qquad K_2 = \left[\frac{d}{ds}(s+5)\right]_{s=-1} = 1$$

$$g(t) = \left[4(t-3)e^{-(t-3)} + e^{-(t-3)}\right]u(t-3)$$

Check

$$\frac{4}{(s+1)^2} + \frac{1}{s+1} = \frac{4 + (s+1)}{(s+1)^2} = \frac{s+5}{(s+1)^2} \checkmark$$

Name Solutions UW ID#: ______ Section: ___ Version B

Exam 2 — EE 233

Fall 2011

The test is closed book, with two sheets of notes and calculators allowed. Show all work. Be sure to state all assumptions made and **check** them when possible. The number of points per problem are indicated in parentheses. Total of 50 points in 3 problems on 3 pages (4th page has Laplace transform pairs).

1. Find the inverse Laplace transform of the following s-domain functions:

   (a) Find $f(t)$ for $t > 0^-$ if $F(s) = \dfrac{2s^3 - 2s^2 + 2s + 2}{2s^2 + 6s + 5}$. (8)

$$2s^3 - 2s^2 + 2s + 2 = s(2s^2+6s+5) - 8s^2 - 3s + 2$$
$$= (s-4)(2s^2+6s+5) + 21s + 22$$

roots of $2s^2+6s+5$ are $\frac{1}{4}\left[-6 \pm \sqrt{36-40}\right] = -\frac{3}{2} \pm j\frac{1}{2}$

$$F(s) = s - 4 + \frac{21s+22}{2(s+\frac{3}{2}-j/2)(s+3/2+j/2)} = s - 4 + \frac{K}{s+\frac{3}{2}-j/2} + \frac{K^*}{s+3/2+j/2}$$

$$K = \left.\frac{21s+22}{2(s+3/2+j/2)}\right|_{s=-\frac{3}{2}+\frac{j}{2}} = \frac{21(-\frac{3}{2}+\frac{j}{2})+22}{2j} = \frac{-\frac{19}{2}+j\frac{21}{2}}{2j} = 7.08\angle 42.1^\circ$$

$$f(t) = \delta'(t) - 4\delta(t) + 14.16e^{-3/2t}\cos(0.5t + 42.1^\circ)\,u(t)$$

   (b) Find $g(t)$ for $t > 0^-$ if $G(s) = \dfrac{(s+3)e^{-5s}}{(s+1)^2}$. (8)

$$G(s) = e^{-5s}\left[\frac{K_1}{(s+1)^2} + \frac{K_2}{s+1}\right]$$
$$= e^{-5s}\left[\frac{2}{(s+1)^2} + \frac{1}{s+1}\right]$$

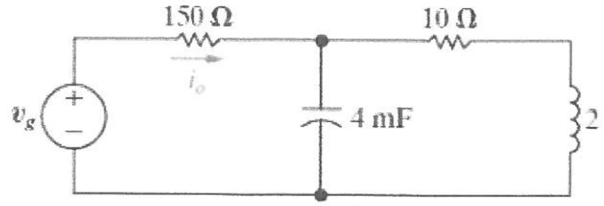
$$K_1 = (s+3)_{s=-1} = 2$$
$$K_2 = \left[\frac{d}{ds}(s+3)\right]_{s=-1} = 1$$

$$g(t) = \left[2(t-5)e^{-(t-5)} + e^{-(t-5)}\right]u(t-5)$$

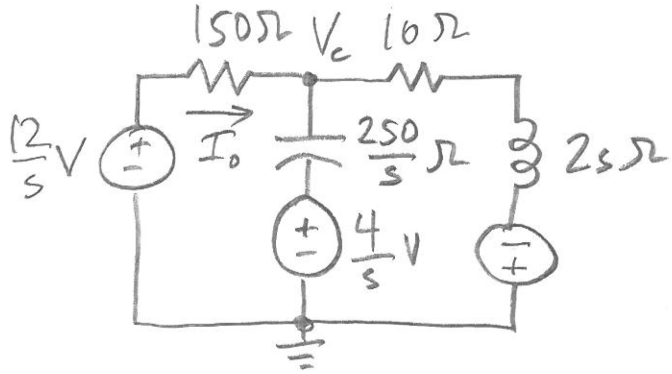
Check

$$\frac{2}{(s+1)^2} + \frac{1}{(s+1)} = \frac{2+s+1}{(s+1)^2} = \frac{s+3}{(s+1)^2}$$ ✓

2. In the circuit to the right, $v_g(t) = 12u(t)\,V$, $i_L(0^-) = 0.4\,A$ and $v_C(0^-) = 4\,V$



(a) Draw the s-domain circuit valid for $t > 0$. (7)



$$\mathcal{L}\{12u(t)\} = \frac{12}{s}$$

$$Z_c(s) = \frac{1}{sC} = \frac{1}{0.004s} = \frac{250}{s}\,\Omega$$

$$Z_L(s) = sL = 2s\,\Omega$$

$$LI_{L0} = 2(0.4) = 0.8\,V$$

(b) Find $I_0(s)$ (Laplace transform of 150Ω resistor current). (7)

$$I_o = \left(\frac{12V}{s} - V_c\right)/150\Omega$$

$$V_c(s) = \frac{12}{s} - 150 I_o(s)$$

KCL @ V_c:

$$\frac{12/s - V_c}{150} + \frac{4/s - V_c}{250/s} + \frac{-0.8 - V_c}{10+2s} = 0$$

$$I_o + \frac{(4/s + 150I_o - 12/s)s}{250} + \frac{-0.8 + 150I_o - 12/s}{10+2s} = 0$$

$$I_o\left(1 + 0.6s + \frac{75}{5+s}\right) = \frac{8}{250} + \frac{0.4 + 6/s}{5+s}$$

$$I_o(5 + s + 3s + 0.6s^2 + 75) = 0.16 + 0.032s + 0.4 + 6/s$$

$$I_o(s) = \frac{0.032s + 0.56 + 6/s}{0.6s^2 + 4s + 80} = \frac{0.16s^2 + 2.8s + 30}{s(3s^2 + 20s + 400)}$$

(c) Check your answer with the Initial and Final Value Theorems (compare to behavior of the time-domain circuit). (5)

$$i(0^+) = \frac{v_g(0^+) - v_c(0^+)}{150\Omega} = \frac{12-4}{150} = \frac{8}{150} = \frac{4}{75}\,A$$

$$\lim_{s\to\infty}(sI_o(s)) = \frac{0.16}{3} = \frac{16}{300} = \frac{4}{75}\,A$$

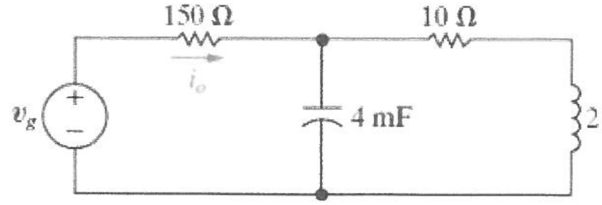
IVT checks

$$i(\infty) = 12V/(150\Omega + 10\Omega) = 12/160 = 3/40 = 0.075A$$

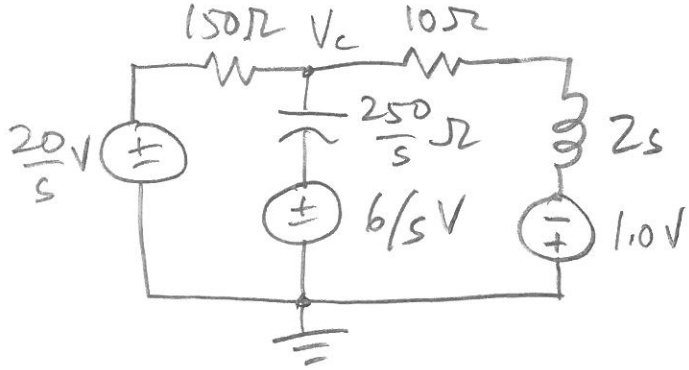
$$\lim_{s\to 0}(sI_o(s)) = \frac{30}{400} = \frac{3}{40} = 0.075A$$

FVT Checks

2. In the circuit to the right, $v_g(t) = 20u(t)\,V$, $i_L(0^-) = 0.5\,A$ and $v_C(0^-) = 6\,V$



(a) Draw the s-domain circuit valid for $t > 0$. (7)



$$\mathcal{L}\{20u(t)\} = \frac{20}{s}$$

$$Z_c(s) = 1/sC = \frac{1}{0.004s} = \frac{250}{s}$$

$$Z_L(s) = sL = 2s$$

$$LI_{L0} = 2(0.5) = 1.0$$

(b) Find $I_0(s)$ (Laplace transform of 150Ω resistor current). (7)

$$I_o = \left(\frac{20}{s} - V_c\right)/150\Omega$$

$$V_c(s) = \frac{20}{s} - 150 I_o(s)$$

KCL @ V_c:

$$\frac{20/s - V_c}{150} + \frac{6/s - V_c}{250/s} + \frac{-1.0 - V_c}{10+2s} = 0$$

$$I_o + \frac{(6/s + 150I_o - 20/s)s}{250} + \frac{150I_o - \frac{20}{s} - 1.0}{10+2s} = 0$$

$$I_o\left[1 + 0.6s + \frac{75}{5+s}\right] = \frac{14}{250} + \frac{0.5 + 10/s}{5+s} = \frac{0.056(5+s) + 0.5 + \frac{10}{s}}{5+s}$$

$$I_o\left[5 + s + 3s + 0.6s^2 + 75\right] = 0.28 + 0.056s + 0.5 + 10/s$$

$$I_o(s) = \frac{0.056s + 0.78 + 10/s}{0.6s^2 + 4s + 80} = \frac{0.056s^2 + 0.78s + 10}{s(0.6s^2 + 4s + 80)}$$

(c) Check your answer with the Initial and Final Value Theorems (compare to behavior of the time-domain circuit). (5)

$$i(0^+) = \frac{v_g(0^+) - v_c(0^+)}{150\Omega} = \frac{20-6}{150} = \frac{14}{150} = \frac{7}{75}\,A$$

$$\lim_{s\to\infty}(sI_o(s)) = \frac{0.056}{0.6} = \frac{56}{600} = \frac{7}{75}\,A$$

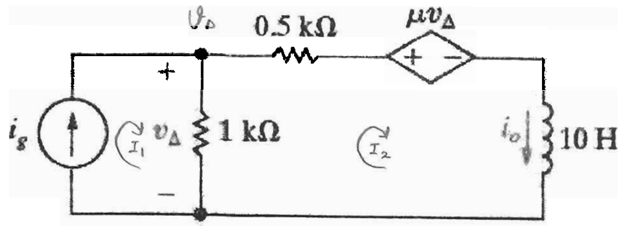
IVT checks

$$i(\infty) = 20V/(150\Omega + 10\Omega) = \frac{20}{160} = \frac{1}{8}\,A$$

$$\lim_{s\to 0}(sI_o(s)) = \frac{10}{80} = \frac{1}{8}\,A$$

FVT Checks

3. (a) Find the transfer function $H(s) = I_o(s)/I_g(s)$ for the circuit to the right. Assume no stored energy in circuit at $t = 0^-$. (7)



(a) Node Voltage at node v_Δ

$$\frac{v_\Delta}{1k} + \frac{v_\Delta - \mu v_\Delta}{0.5k + 10s} - I_g = 0$$

$$\Rightarrow I_g = \frac{v_\Delta}{1k} + \frac{v_\Delta(1-\mu)}{0.5k + 10s}$$

since $I_o = \dfrac{v_\Delta - \mu v_\Delta}{0.5k + 10s}$

then
$$\frac{I_o}{I_g} = \frac{\dfrac{\cancel{v_\Delta}(1-\mu)}{\cancel{0.5k+10s}}}{\dfrac{\cancel{v_\Delta}(0.5 + \frac{10}{1000}s + 1 - \mu)}{\cancel{0.5k+10s}}}$$

$$= \frac{1-\mu}{1.5 + \frac{10}{1000}s - \mu}$$

$$\mu = 3 \Rightarrow \frac{I_o}{I_g} = \frac{-2}{0.01s - 1.5} = \frac{200}{150 - s}$$

(b) Mesh current

$I_2: (I_2 - I_1)1k + 0.5k \cdot I_2 + \mu v_\Delta + I_2 \, 10 s = 0$

$\Rightarrow I_2(1.5k + 10s) - I_1 1k + \mu v_\Delta = 0$

$v_\Delta = (I_1 - I_2)1k$

we have:

$I_2(1.5k + 10s) - I_1 1k + \mu(I_1 - I_2)1k = 0$

$I_2(1.5k + 10s - \mu \cdot 1k) = I_1(1k - \mu \cdot 1k)$

Since $I_1 = I_g$ & $I_2 = I_o$

$$\Rightarrow \frac{I_o}{I_g} = \frac{1k(1-\mu)}{1k(0.01s + 1.5 - \mu)}$$

$$\mu = 3 \Rightarrow \frac{I_o}{I_g} = \frac{-2}{0.01s - 1.5} = \frac{200}{150 - s}$$

(b) If $i_g(t) = 5t^2e^{-4t}u(t)\,mA$, calculate $I_o(s)$. (8)

Version A: $i_g(t) = 4t^2 e^{-5t} u(t)$ mA using table: $t^n e^{-bt} \Longleftrightarrow \dfrac{n!}{(s+b)^{n+1}}$

$I_g(s) = 4 \cdot \dfrac{2!}{(s+5)^3}$ mA

then $I_o(s) = I_g(s) \cdot H(s)$

$$= \frac{8}{(s+5)^3} \cdot \frac{200}{150-s} = \frac{1600}{(s+5)^3(150-s)} \times 10^{-3}$$

B: $i_g(t) = 5t^2 e^{-4t} u(t)$ mA

$I_g(s) = 5 \dfrac{2!}{(s+4)^3}$ mA

$$I_o(s) = \frac{10}{(s+4)^3} \cdot \frac{200}{150-s} = \frac{2000}{(s+4)^3(150-s)} \times 10^{-3}$$

End Of Exam