Solutions UW ID#: Section: Version A

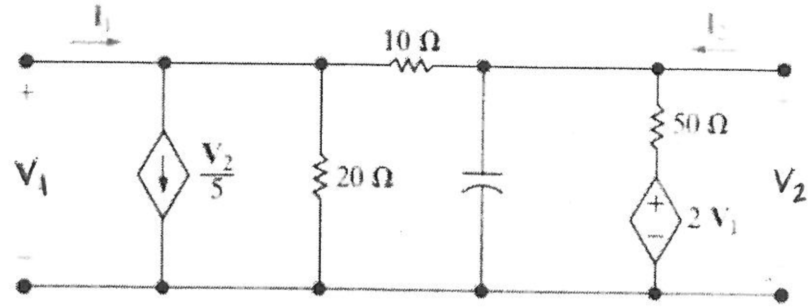
Average = 25.0
Std Dev = 8.9
Max = 43, Min = 8

Exam 3 — EE 233

Fall 2011

The test is closed book, with one sheet of notes and standard calculators (no communications) allowed. Show all work. Be sure to state all assumptions made and check them when possible. The number of points per problem are indicated in parentheses. Total of 50 points in 3 problems on 3 pages.

1. In the circuit at right, v_1 is the input and v_2 is the output. C = 20 mF.



(a) Calculate the impulse response. (7)

KCL @ V_2:

$$\frac{V_2 - V_1}{10} + \frac{V_2}{1/sC} + \frac{V_2 - 2V_1}{50} = 0$$

$$V_2\left[\frac{1}{10} + sC + \frac{1}{50}\right] = V_1\left[\frac{1}{10} + \frac{2}{50}\right] \Rightarrow \frac{V_2}{V_1} = H(s) = \frac{7}{6 + 50Cs} = \frac{7}{6+s}$$

$$h(t) = \mathcal{L}^{-1}\{H(s)\} = \underline{7e^{-6t}u(t)}$$

(b) Determine $v_2(t)$ for $v_1(t) = 6e^{-4t}u(t)$. (10)

$$v_2(t) = v_1(t) * h(t) = \int_{-\infty}^{\infty} h(\lambda)v_1(t-\lambda)d\lambda = \int_{-\infty}^{\infty} 7e^{-6\lambda}\underset{\lambda>0}{u(\lambda)}\, 6e^{-4(t-\lambda)}\underset{\lambda<t}{u(t-\lambda)}\, d\lambda$$

$$= \int_0^t 42e^{-6\lambda}e^{-4t}e^{4\lambda}d\lambda \text{ for } t>0, = 0 \text{ otherwise}$$

$$= 42e^{-4t}u(t)\int_0^t e^{-2\lambda}d\lambda = 42e^{-4t}u(t)\left[\frac{e^{-2\lambda}}{-2}\right]_0^t = -21e^{-4t}\left(e^{-2t} - 1\right)u(t)$$

$$v_2(t) = \underline{21\left(e^{-6t} - e^{-4t}\right)u(t)}$$

$$v_2(t) = \mathcal{L}^{-1}\{V_1(s)H(s)\} \quad V_1(s)H(s) = \left(\frac{6}{s+4}\right)\left(\frac{7}{s+6}\right) = \frac{K_1}{s+4} + \frac{K_2}{s+6}$$

$$K_1 = \left.\frac{6\times 7}{s+6}\right|_{s=-4} = 21 \quad K_2 = \left.\frac{6\times 7}{s+4}\right|_{s=-6} = -21$$

$$v_2(t) = \underline{\left(21e^{-4t} - 21e^{-6t}\right)u(t)}$$

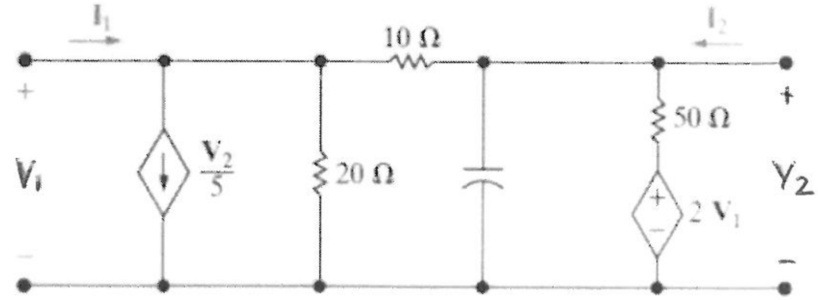
Solutions UW ID#: ______ Section: ___ Version B

Exam 3 — EE 233

Fall 2011

The test is closed book, with one sheet of notes and standard calculators (no communications) allowed. Show all work. Be sure to state all assumptions made and check them when possible. The number of points per problem are indicated in parentheses. Total of 50 points in 3 problems on 3 pages.

1. In the circuit at right, v_1 is the input and v_2 is the output. C = 5 mF.



(a) Calculate the impulse response. (7)

KCL @ V_2:

$$\frac{V_2 - V_1}{10\Omega} + \frac{V_2}{1/sC} + \frac{V_2 - 2V_1}{50\Omega} = 0$$

$$V_2\left[\frac{1}{10} + \frac{1}{50} + sC\right] = V_1\left[\frac{1}{10} + \frac{2}{50}\right] \Rightarrow \frac{V_2}{V_1} = H(s) = \frac{7}{6 + 0.250s} = \frac{28}{24 + s}$$

$$\mathcal{L}^{-1}\left\{\frac{28}{s+24}\right\} = \underline{28e^{-24t}u(t) = h(t)}$$

(b) Determine $v_2(t)$ for $v_1(t) = 12e^{-10t}u(t)$. (10)

$$v_2(t) = v_1(t) * h(t) = \int_{-\infty}^{\infty} h(\lambda)v_1(\lambda - t)\,d\lambda = \int_{-\infty}^{\infty} 28e^{-24\lambda}u(\lambda)12e^{-10(t-\lambda)}u(t-\lambda)\,d\lambda$$

$\lambda > 0$ $\lambda < t$

$$= \int_0^t (28*12)e^{-24\lambda}e^{-10t}e^{10\lambda}\,d\lambda \quad \text{for } t > 0$$

$$= u(t)336e^{-10t}\int_0^t e^{-14\lambda}\,d\lambda = 336e^{-10t}\left[\frac{e^{-14\lambda}}{-14}\right]_0^t u(t) = -24e^{-10t}\left[e^{-14t} - 1\right]u(t)$$

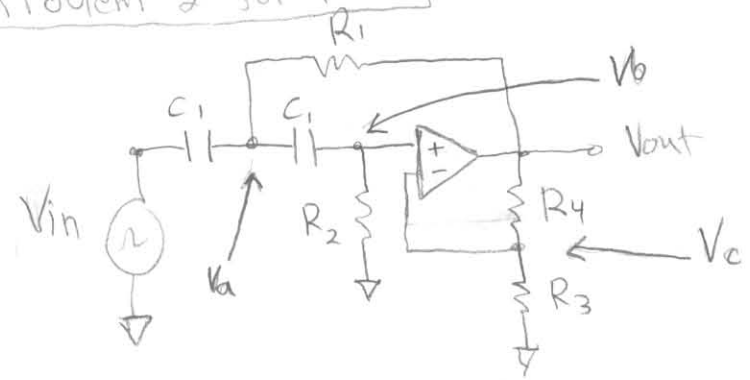
$$\underline{v_2(t) = 24\left[e^{-10t} - e^{-24t}\right]u(t)}$$

$$v_2(t) = \mathcal{L}^{-1}\{V_1(s)H(s)\} = \mathcal{L}^{-1}\left\{\frac{12 \cdot 28}{(s+10)(s+24)}\right\} = \mathcal{L}^{-1}\left\{\frac{K_1}{s+10} + \frac{K_2}{s+24}\right\}$$

$$K_1 = \left.\frac{12*28}{s+24}\right|_{s=-10} = 24 \qquad K_2 = \left.\frac{12*28}{s+10}\right|_{s=-24} = -24$$

$$\underline{v_2(t) = \left[24e^{-10t} - 24e^{-24t}\right]u(t)}$$

Problem 2 solution



① Nodal equation at Va

$$\frac{V_a - V_{in}}{\frac{1}{sC_1}} + \frac{V_a - V_b}{\frac{1}{sC_1}} + \frac{V_a - V_{out}}{R_1} = 0$$

$$(V_a - V_{in})sC_1 + (V_a - V_b)sC_1 + \frac{V_a - V_{out}}{R_1} = 0$$

$$V_a\left(sC_1 + sC_1 + \frac{1}{R_1}\right) - V_{in}sC_1 - V_b sC_1 - \frac{V_{out}}{R_1} = 0$$

$$V_a\left(2sC_1 + \frac{1}{R_1}\right) - V_b sC_1 - \frac{V_{out}}{R_1} = V_{in}sC_1$$

② Nodal equation at Vb

$$\frac{V_b - V_a}{\frac{1}{sC_1}} + \frac{V_b}{R_2} = 0$$

$$V_b sC_1 - V_a sC_1 + \frac{V_b}{R_2} = 0$$

$$V_b\left(sC_1 + \frac{1}{R_2}\right) = V_a sC_1$$

$$V_b\left(1 + \frac{1}{sC_1R_2}\right) = V_a$$

③ Nodal equation at Vc

$$\frac{V_c - V_{out}}{R_4} + \frac{V_c}{R_3} = 0$$

$$V_c\left(\frac{1}{R_4} + \frac{1}{R_3}\right) = \frac{V_{out}}{R_4}$$

$$V_c\left(\frac{R_3}{R_4R_3} + \frac{R_4}{R_4R_3}\right) = \frac{V_{out}}{R_4}$$

$$V_c\left(\frac{R_4 + R_3}{R_4R_3}\right) = \frac{V_{out}}{R_4}$$

$$V_c = \frac{R_3}{R_4 + R_3} V_{out}$$

$V_c = V_b$ so $V_a = V_{out}\left(\frac{R_3}{R_4+R_3}\right)\left(1 + \frac{1}{sC_1R_2}\right)$, plugging back into ①

$$V_{out}\left(\frac{R_3}{R_4+R_3}\right)\left(1 + \frac{1}{sC_1R_2}\right)\left(2sC_1 + \frac{1}{R_1}\right) - \left(\frac{R_3}{R_4+R_3}\right)(V_{out})sC_1 - \frac{V_{out}}{R_1} = V_{in}sC_1$$

$$V_{out}\left[\left(\frac{R_3}{R_4+R_3}\right)\left(1+\frac{1}{sC_1R_2}\right)\left(2sC_1+\frac{1}{R_2}\right)-\frac{R_3}{R_4+R_3}sC_1-\frac{1}{R_1}\right]=V_{in}\,sC_1$$

multiply by $sC_1R_2R_1$

$$V_{out}\left[\left(\frac{R_3}{R_4+R_3}\right)(sC_1R_2R_1+R_1)\left(2sC_1+\frac{1}{R_2}\right)-\left(\frac{R_3}{R_4+R_3}\right)s^2C_1^2R_2R_1-sC_1R_2\right]$$

$$=V_{in}\,s^2C_1^2R_1R_2$$

$$=V_{out}\left(\frac{R_3}{R_4+R_3}\right)\left[(sC_1R_2R_1+R_1)\left(2sC_1+\frac{1}{R_2}\right)-s^2C_1^2R_1R_2-\left(\frac{R_4+R_3}{R_3}\right)sC_1R_2\right]$$

$$=V_{out}\left(\frac{R_3}{R_4+R_3}\right)\left[2s^2C_1^2R_2R_1+sC_1R_1+2sC_1R_1+1-s^2C_1^2R_1R_2-\frac{R_4+R_3}{R_3}sC_1R_2\right]$$

$$=V_{out}\left(\frac{R_3}{R_4+R_3}\right)\left[s^2C_1^2R_1R_2+s\left(C_1R_1+2C_1R_1+-\frac{R_4+R_3}{R_3}C_1R_2\right)+1\right]$$

$$=V_{out}\left(\frac{R_3}{R_4+R_3}\right)\left[s^2C_1^2R_1R_2+s\left(3C_1R_1-C_1R_1-\frac{R_4}{R_3}C_1R_1\right)+1\right.$$

$$=V_{out}\left(\frac{R_3}{R_4+R_3}\right)\left[s^2C_1^2R_1R_2+s\left(2C_1R_1-\frac{R_4}{R_3}C_1R_1\right)+1\right]$$

so
$$\frac{V_{out}}{V_{in}}=\frac{R_4+R_3}{R_3}\left[\frac{s^2C_1^2R_1R_2}{s^2C_1^2R_1R_2+s\left(2C_1R_1-\frac{R_4}{R_3}C_1R_1\right)+1}\right]$$

this is a high pass filter

$$|H(s)|_{s\to 0}=0$$

$$|H(s)|_{s\to\infty}=\frac{R_4+R_3}{R_3}$$

Solution Problem 3

a) $$H(s) = \frac{-\frac{R_2}{R_1} s^2}{s^2 + \frac{s}{R_1 C_1} + \frac{1}{R_1 R_2 C_1 C_2}}$$

conditions: Corner frequency : ω_0 rad/s

passband gain : G_1 dB

gain at the corner frequency : G_2 dB

use CnF capacitor when necessary

$$H(s) = \frac{K s^2}{s^2 + \beta s + \omega_0^2} \Rightarrow \text{passband gain} = K = 10^{(G_1/20)}$$

↳ General 2nd order HPF

Gain at the corner frequency will define Q

at corner frequency, you got a drop of $G_1 - G_2$

then $Q = 10^{\left(\frac{-(G_1 - G_2)}{20}\right)}$ negative sign because you have a drop

then $Q = \frac{\omega_0}{\beta} \Rightarrow \beta = \frac{\omega_0}{Q}$

∴ you are going to have: (1) $\beta = \frac{1}{R_1 C_1}$ → solve for R_1

(2) $K = \frac{R_2}{R_1}$ → solve for R_2

(3) $\omega_0^2 = \frac{1}{R_1 R_2 C_1 C_2}$ → solve for C_2

3 eqns, 4 unknown, so let C_1 = cap value that is suggested above

Version A

$\omega_0 = 20{,}000$ rad/s

$G_1 = 38$ dB

$G_2 = 26$ dB

$C_1 = 5$nF

⇒ $R_1 = 2500\ \Omega$

$R_2 = 200\text{k}\Omega$

$C_2 = 1$nF

Version B

$\omega_0 = 50{,}000$ rad/s

$G_1 = 46$ dB

$G_2 = 32$ dB

$C_1 = 2$nF

⇒ $R_1 = 2000\ \Omega$

$R_2 = 400\text{k}\Omega$

$C_2 = 0.25$ nF

b) cutoff frequency:

$$|H(j\omega_c)| = \frac{|H_{max}|}{\sqrt{2}}$$

$$\left| \frac{\cancel{K}(j\omega_c)^2}{(j\omega_c)^2 + \beta j\omega_c + (\omega_o)^2} \right| = \frac{\cancel{K}}{\sqrt{2}}$$

your maximum gain is your passband gain

$$\left| \frac{-\omega_c^2}{(\omega_o^2 - \omega_c^2) + j\beta\omega_c} \right| = \frac{1}{\sqrt{2}}$$

$$\frac{\omega_c^2}{\sqrt{(\omega_o^2 - \omega_c^2)^2 + (\beta\omega_c)^2}} = \frac{1}{\sqrt{2}} \Rightarrow \frac{1}{\sqrt{(\omega_o^2 - \omega_c^2)^2 + (\beta\omega_c)^2}} = \frac{1}{\omega_c^2\sqrt{2}}$$

equate the denomonator^2

$$\Rightarrow (\omega_o^2 - \omega_c^2)^2 + \beta^2\omega_c^2 = 2\omega_c^4$$

$$\omega_o^4 - 2\omega_o^2\omega_c^2 + \omega_c^4 + \beta^2\omega_c^2 = 2\omega_c^4$$

$$\omega_c^4 + (2\omega_o^2 - \beta^2)\omega_c^2 - \omega_o^4 = 0$$

$$\omega_c^2 = \frac{-(2\omega_o^2 - \beta^2) \pm \sqrt{(2\omega_o^2 - \beta^2)^2 - 4(1)(-\omega_o^4)}}{2} \quad \text{(Quadratic equation)}$$

↳ only have 1 solution, because the second one is negative.

Version A

$$\omega_c = \sqrt{\frac{-(2.20{,}000^2 - 80{,}000^2) + \sqrt{(2.20{,}000^2 - 80{,}000^2)^2 - 4.-(20{,}000)^4}}{2}}$$

$= 74.8\ kHz \approx 75\ kHz$

Version B

$$\omega_c = \sqrt{\frac{-(2.50{,}000^2 - 250{,}000^2) + \sqrt{(2.50{,}000^2 - 250{,}000^2)^2 - 4\div(50{,}000)^4}}{2}}$$

$\approx 240\ kHz$