

TABLE 12.1 An Abbreviated List of Laplace Transform Pairs

Type	$f(t)$ ($t > 0$)	$F(s)$
(impulse)	$\delta(t)$	1
(step)	$u(t)$	$\frac{1}{s}$
(ramp)	t	$\frac{1}{s^2}$
(exponential)	e^{-at}	$\frac{1}{s+a}$
(sine)	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
(cosine)	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
(damped ramp)	te^{-at}	$\frac{1}{(s+a)^2}$
(damped sinc)	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
(damped cosine)	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$

TABLE 18.1 Parameter Conversion Table

$z_{11} = \frac{y_{22}}{\Delta y} = \frac{a_{11}}{a_{21}} = \frac{b_{22}}{b_{21}} = \frac{\Delta h}{h_{22}} = \frac{1}{g_{11}}$	$b_{21} = \frac{1}{z_{12}} = -\frac{\Delta y}{y_{12}} = \frac{a_{21}}{\Delta a} = \frac{h_{22}}{h_{12}} = -\frac{g_{11}}{g_{12}}$
$z_{12} = -\frac{y_{12}}{\Delta y} = \frac{\Delta a}{a_{21}} = \frac{1}{b_{21}} = \frac{h_{12}}{h_{22}} = -\frac{g_{12}}{g_{11}}$	$b_{22} = \frac{z_{11}}{z_{12}} = \frac{y_{22}}{y_{12}} = \frac{a_{11}}{\Delta a} = \frac{\Delta h}{h_{12}} = -\frac{1}{g_{12}}$
$z_{21} = \frac{-y_{21}}{\Delta y} = \frac{1}{a_{21}} = \frac{\Delta b}{b_{21}} = -\frac{h_{21}}{h_{22}} = \frac{g_{21}}{g_{11}}$	$h_{11} = \frac{\Delta z}{z_{22}} = \frac{1}{y_{11}} = \frac{a_{12}}{a_{22}} = \frac{b_{12}}{b_{11}} = \frac{g_{22}}{\Delta g}$
$z_{22} = \frac{y_{11}}{\Delta y} = \frac{a_{22}}{a_{21}} = \frac{b_{11}}{b_{21}} = \frac{1}{h_{22}} = \frac{\Delta g}{g_{11}}$	$h_{12} = \frac{z_{12}}{z_{22}} = -\frac{y_{12}}{y_{11}} = \frac{\Delta a}{a_{22}} = \frac{1}{b_{11}} = -\frac{g_{12}}{\Delta g}$
$y_{11} = \frac{z_{22}}{\Delta z} = \frac{a_{22}}{a_{12}} = \frac{b_{11}}{b_{12}} = \frac{1}{h_{11}} = \frac{\Delta g}{g_{22}}$	$h_{21} = -\frac{z_{21}}{z_{22}} = \frac{y_{21}}{y_{11}} = -\frac{1}{a_{22}} = -\frac{\Delta b}{b_{11}} = -\frac{g_{21}}{\Delta g}$
$y_{12} = -\frac{z_{12}}{\Delta z} = \frac{\Delta a}{a_{12}} = -\frac{1}{b_{12}} = \frac{h_{12}}{h_{11}} = \frac{g_{12}}{g_{22}}$	$h_{22} = \frac{1}{z_{22}} = \frac{\Delta y}{y_{11}} = \frac{a_{21}}{a_{22}} = \frac{b_{21}}{b_{11}} = \frac{g_{11}}{\Delta g}$
$y_{21} = -\frac{z_{21}}{\Delta z} = -\frac{1}{a_{12}} = \frac{\Delta b}{b_{12}} = \frac{h_{21}}{h_{11}} = -\frac{g_{21}}{g_{22}}$	$g_{11} = \frac{1}{z_{11}} = \frac{\Delta y}{y_{22}} = \frac{a_{11}}{a_{21}} = \frac{b_{22}}{b_{21}} = \frac{h_{22}}{\Delta h}$
$y_{22} = \frac{z_{11}}{\Delta z} = \frac{a_{11}}{a_{12}} = \frac{b_{22}}{b_{12}} = \frac{\Delta h}{h_{11}} = \frac{1}{g_{22}}$	$g_{12} = -\frac{z_{12}}{z_{11}} = \frac{y_{12}}{y_{22}} = \frac{\Delta a}{a_{11}} = \frac{1}{b_{22}} = -\frac{h_{12}}{\Delta h}$
$a_{11} = \frac{z_{11}}{z_{21}} = \frac{y_{22}}{y_{21}} = \frac{b_{22}}{\Delta b} = \frac{\Delta h}{h_{21}} = \frac{1}{g_{21}}$	$g_{21} = \frac{z_{21}}{z_{11}} = \frac{y_{21}}{y_{22}} = \frac{1}{a_{11}} = \frac{\Delta b}{b_{22}} = \frac{h_{21}}{\Delta h}$
$a_{12} = \frac{\Delta z}{z_{21}} = -\frac{1}{y_{21}} = \frac{b_{12}}{\Delta b} = -\frac{h_{11}}{h_{21}} = \frac{g_{22}}{g_{21}}$	$g_{22} = \frac{\Delta z}{z_{11}} = \frac{1}{y_{22}} = \frac{a_{12}}{a_{11}} = \frac{b_{12}}{b_{22}} = \frac{h_{11}}{\Delta h}$
$a_{21} = \frac{1}{z_{21}} = \frac{\Delta y}{y_{21}} = \frac{b_{21}}{\Delta b} = -\frac{h_{22}}{h_{21}} = \frac{g_{11}}{g_{21}}$	$\Delta z = z_{11}z_{22} - z_{12}z_{21}$
$a_{22} = \frac{z_{22}}{z_{21}} = -\frac{y_{11}}{y_{21}} = \frac{b_{11}}{\Delta b} = -\frac{1}{h_{21}} = \frac{\Delta g}{g_{21}}$	$\Delta y = y_{11}y_{22} - y_{12}y_{21}$
$b_{11} = \frac{z_{22}}{z_{12}} = \frac{y_{11}}{y_{12}} = \frac{a_{22}}{\Delta a} = \frac{1}{h_{12}} = -\frac{\Delta g}{g_{12}}$	$\Delta a = a_{11}a_{22} - a_{12}a_{21}$
$b_{12} = \frac{\Delta z}{z_{12}} = -\frac{1}{y_{12}} = \frac{a_{12}}{\Delta a} = \frac{h_{11}}{h_{12}} = -\frac{g_{22}}{g_{12}}$	$\Delta b = b_{11}b_{22} - b_{12}b_{21}$
	$\Delta h = h_{11}h_{22} - h_{12}h_{21}$
	$\Delta g = g_{11}g_{22} - g_{12}g_{21}$

TABLE 12.2 An Abbreviated List of Operational Transforms

Operation	$f(t)$	$F(s)$
Multiplication by a constant	$Kf(t)$	$KF(s)$
Addition/subtraction	$f_1(t) + f_2(t) - f_3(t) + \dots$	$F_1(s) + F_2(s) - F_3(s) + \dots$
First derivative (time)	$\frac{df(t)}{dt}$	$sF(s) - f(0^-)$
Second derivative (time)	$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0^-) - \frac{df(0^-)}{dt}$
n th derivative (time)	$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - s^{n-1}f(0^-) - s^{n-2}\frac{df(0^-)}{dt} - s^{n-3}\frac{d^2f(0^-)}{dt^2} - \dots - \frac{d^{n-1}f(0^-)}{dt^{n-1}}$
Time integral	$\int_0^t f(x) dx$	$\frac{F(s)}{s}$
Translation in time	$f(t-a)u(t-a), a > 0$	$e^{-as}F(s)$
Translation in frequency	$e^{-at}f(t)$	$F(s+a)$
Scale changing	$f(at), a > 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
First derivative (s)	$tf(t)$	$-\frac{dF(s)}{ds}$
n th derivative (s)	$t^n f(t)$	$(-1)^n \frac{d^n F(s)}{ds^n}$
s integral	$\frac{f(t)}{t}$	$\int_s^\infty F(u) du$

$$\begin{aligned}
 V_1 &= z_{11}I_1 + z_{12}I_2, \\
 V_2 &= z_{21}I_1 + z_{22}I_2; \\
 I_1 &= y_{11}V_1 + y_{12}V_2, \\
 I_2 &= y_{21}V_1 + y_{22}V_2; \\
 V_1 &= a_{11}V_2 - a_{12}I_2, \\
 I_1 &= a_{21}V_2 - a_{22}I_2; \\
 V_2 &= b_{11}V_1 - b_{12}I_1, \\
 I_2 &= b_{21}V_1 - b_{22}I_1; \\
 V_1 &= h_{11}I_1 + h_{12}V_2, \\
 I_2 &= h_{21}I_1 + h_{22}V_2; \\
 I_1 &= g_{11}V_1 + g_{12}I_2, \\
 V_2 &= g_{21}V_1 + g_{22}I_2.
 \end{aligned}$$

TABLE 15.1 Normalized (so that $\omega_c = 1$ rad/s) Butterworth Polynomials up to the Eighth Order

n	n th-Order Butterworth Polynomial
1	$(s + 1)$
2	$(s^2 + \sqrt{2}s + 1)$
3	$(s + 1)(s^2 + s + 1)$
4	$(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)$
5	$(s + 1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$
6	$(s^2 + 0.518s + 1)(s^2 + \sqrt{2} + 1)(s^2 + 1.932s + 1)$
7	$(s + 1)(s^2 + 0.445s + 1)(s^2 + 1.247s + 1)(s^2 + 1.802s + 1)$
8	$(s^2 + 0.390s + 1)(s^2 + 1.111s + 1)(s^2 + 1.6663s + 1)(s^2 + 1.962s + 1)$