EE 233 Homework8 solution

Autumn 2016

November 30, 2016

1 Problem 15.28

(a) The lower cutoff frequency will belong to the high pass stage, and the upper cut off to the low pass stage. For the high pass:

$$H(s) = \frac{S}{S + 1/R_H C_H}$$

where $\omega_{C_H} = \frac{1}{R_H C_H}$

$$R_H = 1/(2\pi f_H C_H) = 1/(2\pi \times 200 \times 50nF) = 15.9k\Omega$$

For the low pass:

$$H(S) = \frac{1/R_L C_L}{S + 1/R_L C_L}$$

where $\omega_{C_L} = \frac{1}{R_L C_L}$

$$R_L = 1/(2\pi f_L C_L) = 1/(2\pi \times 2000 \times 50nF) = 1.59k\Omega$$

The bandpass filter gain is 20dB, which means

$$K = R_f/R_i = 10$$

The designed bandpass filter is shown below:

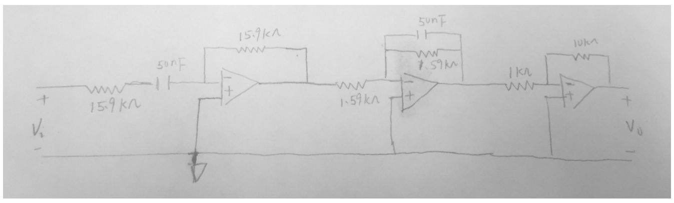


Figure 1: bandpass filter for problem 15.28

(b) Combining the 2 stages gives:

$$H(s) = 10 \cdot \frac{s}{s + 1/R_H C_H} \cdot \frac{1/R_L C_L}{s + 1/R_L C_L}$$

$$H(s) = 10 \cdot \frac{s}{s + 2\pi(200)} \cdot \frac{2\pi(2000)}{s + 2\pi(5000)} = \frac{40000\pi s}{(s + 400\pi)(s + 4000\pi)}$$

(c)

$$\begin{aligned}
\omega_0 &= \sqrt{\omega_{C_1}\omega_{C_2}} = \sqrt{400\pi \cdot 4000\pi} = 3973.8 rad/s \\
H(j\omega) &= \frac{jw(40000\pi)}{(jw + 400\pi)(jw + 4000\pi)} \\
H(j\omega_0) &= \frac{(40000\pi)(3973.8\angle 90^\circ)}{(3973.8j + 400\pi)(3973.8j + 4000\pi)} \\
&= \frac{(40000\pi)(3973.8\angle 90^\circ)}{(4167.8\angle 72.45^\circ)(13179.7\angle 17.55^\circ)} \\
&= 9.091\angle 0^\circ
\end{aligned}$$

(d) $gain = 20log(9.091) = 19.17dB$
(e) The graph is shown below:

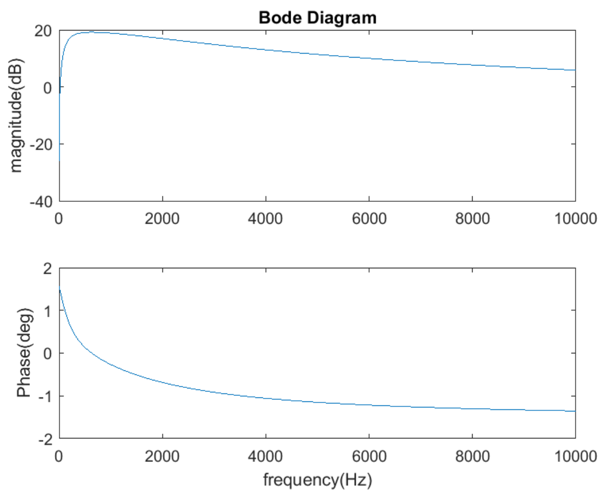


Figure 2: Bode diagram plot for problem 15.28

2 Problem 15.45

From Problem 15.44, you just need to find the order of the low pass filter and high pass filter.
HPF: $slope = \frac{40}{log(200/40)} = 57.23dB/dec \therefore n = 3$

LPF: $slope = \frac{-40}{log(12500/2500)} = -57.23dB/dec \therefore n = 3$
P15.45 (a) Unscaled high-pass stage

$$H_{hp}(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

The frequency scaling factor is $k_f = (\omega_0^{'}/\omega_0) = 400\pi$. Therefore the scaled transfer function is

$$H_{hp}^{'}(s) = \frac{(s/400\pi)^3}{(s/400\pi+1)[(s/400\pi)^2+s/400\pi+1]} = \frac{s^3}{(s+400\pi)(s^2+400\pi s+400^2\pi^2)}$$

Unscaled low-pass stage:

$$H_{hp}(s) = \frac{1}{(s+1)(s^2+s+1)}$$

The frequency scaling factor is $k_f = (\omega_0^{'}/\omega_0) = 5000\pi$. Therefore the scaled transfer function is

$$H_{lp}^{'}(s) = \frac{1}{(s/5000\pi+1)[(s/5000\pi)^2+s/5000\pi+1]} = \frac{(5000\pi)^3}{(s+5000\pi)(s^2+5000\pi s+25\times10^6\pi^2)}$$

Thus the transfer function for the filter is

$$H^{'}(s) = 100H_{hp}^{'}(s)H_{lp}^{'}(s) = \frac{125\times10^{11}\pi^3 s^3}{D_1D_2D_3D_4}$$

where

$$\begin{aligned} D_1 &= s+400\pi \\ D_2 &= s+5000\pi \\ D_3 &= s^2+400\pi s+16\times10^4\pi^2 \\ D_4 &= s^2+5000\pi s+25\times10^6\pi^2 \end{aligned}$$

(b) at 40 Hz, $\omega = 80\pi rad/s$

$$\begin{aligned} D_1(j80\pi) &= 80\pi(5+j) \\ D_2(j80\pi) &= 80\pi(62.5+j) \\ D_3(j80\pi) &= 3.2\times10^4\pi^2(4.8+j) \\ D_4(j80\pi) &= 4\times10^5\pi^2(62.484+j) \end{aligned}$$

Therefore

$$\begin{aligned} D_1D_2D_3D_4(j80\pi) &= 8.192\times10^{13}\pi^6(97659.4\angle24.91^\circ) \\ H^{'}(j80\pi) &= \frac{(125\pi^3\times10^{11})(512000\pi^3\angle-90^\circ)}{8.192\times10^{13}\pi^6(97659.4\angle24.91^\circ)} = 0.8\angle-114.91^\circ \\ \therefore gain &= 20\log_{10}(|H^{'}(j80\pi)|) = 20\log_{10}(0.8) = -1.94dB \end{aligned}$$

at 1000Hz, $\omega = 2000\pi\ rad/s$ Then

$$\begin{aligned} D_1(j2000\pi) &= 400\pi(1+5j) \\ D_2(j2000\pi) &= 2000\pi(2.5+j) \\ D_3(j2000\pi) &= 8\times10^5\pi^2(-4.8+j) \\ D_4(j2000\pi) &= 10^7\pi^2(2.1+j) \end{aligned}$$

$$D_1D_2D_3D_4(j2000\pi) = 64 \times 10^{17}\pi^6(156.575\angle - 65.81^\circ)$$

$$H^{'}(j2000\pi) = \frac{(125\pi^3 \times 10^{11})(8 \times 10^9\pi^3\angle - 90^\circ)}{64 \times 10^{17}\pi^6(156.575\angle - 65.81^\circ)} = 99.79\angle - 24.19^\circ$$

$$\therefore gain = 20\log_{10}(|H^{'}(j2000\pi)|) = 20\log_{10}(99.79) = 39.98\ dB$$

(c) From the transfer function the gain is down 39.98+ 1.94 or 41.92 dB at 40 Hz, which is greater than 40 dB. 1000 Hz is in the passband for this bandpass filter. Hence we expect the gain at 1000 Hz to nearly equal 40 dB as specified in problem 15.44. Thus our scaled transfer function confirms that the filter meets the specifications.

Problem 15.53

From 15.50 we have something like

$$H(s) = \frac{-Kb_o}{s^2 + b_1 s + b_o}$$

where $K = \frac{G_1}{G_2}$; $\quad b_o = \frac{G_2 G_3}{C_1 C_2}$ $\qquad\qquad G_i = \frac{1}{R_i}$

and $b_1 = \frac{G_1 + G_2 + G_3}{C_2}$

Rearranging we see that

$$G_1 = KG_2$$

$$G_3 = \frac{b_o C_1 C_2}{G_2} = \frac{b_o C_1}{G_2}$$

since by hypothesis $C_2 = 1\,\mathrm{F}$

$$b_1 = \frac{G_1 + G_2 + G_3}{C_2} = G_1 + G_2 + G_3$$

$$\therefore\ b_1 = KG_2 + G_2 + \frac{b_o C_1}{G_2}$$

$$b_1 = G_2(1+K) + \frac{b_o C_1}{G_2}$$

Solving this quadratic equation for G_2 we get

$$G_2 = \frac{b_1}{2(1+K)} \pm \sqrt{\frac{b_1^2 - b_o C_1 4(1+K)}{4(1+K)^2}}$$

$$= \frac{b_1 \pm \sqrt{b_1^2 - 4b_o(1+K)C_1}}{2(1+K)}$$

For G_2 to be realizable

$$C_1 < \frac{b_1^2}{4b_o(1+K)}$$

From Problem 15.52 we have

$$H(s) = \frac{\frac{-C_1}{C_2}s^2}{\left[s^2 + \frac{G_1}{C_2C_3}(C_1 + C_2 + C_3)s + \frac{G_1G_2}{C_2C_3}\right]}$$
$$= \frac{-Ks^2}{s^2 + b_1s + b_o}$$

Therefore the circuit implements a second-order high-pass filter with a passband gain of C_1/C_2.

$C_1 = K$: (Choose $C_1 = K$, $C_2 = 1$, $C_3 = 1$)

$$b_1 = \frac{G_1}{(1)(1)}(K + 2) = G_1(K + 2)$$

$$\therefore\ G_1 = \frac{b_1}{K + 2}; \qquad R_1 = \left(\frac{K + 2}{b_1}\right)$$

$$b_o = \frac{G_1G_2}{(1)(1)} = G_1G_2$$

$$\therefore\ G_2 = \frac{b_o}{G_1} = \frac{b_o}{b_1}(K + 2)$$

$$\therefore\ R_2 = \frac{b_1}{b_o(K + 2)}$$

problem 1553 [a] low-pass filter

$$\text{slope} = \frac{-24}{\log(2k/1k)} = -79.726 \ \text{dB/dec} \quad \therefore n = 4$$

In the first prototype second-order section: we set $b_1 = 0.765$, $b_0 = 1$. $C_2 = 1F$, $k = 1$ then we can determine R_1, R_2, R_3, C_1, C_2

$$C_1 \le \frac{b_1^2}{4b_0(1+k)} \le \frac{(0.765)^2}{4(1+1)} \le 0.0732$$

choose $C_1 = 0.03$ F

$$G_2 = \frac{0.765 \pm \sqrt{(0.765)^2 - 4(2)(0.03)}}{4} = \frac{0.765 \pm 0.588}{4}$$

Arbitrily select the larger value for G_2, then

$G_2 = 0.338$ S $\quad \therefore R_2 = \frac{1}{G_2} = 2.96\Omega$

$G_1 = kG_2 = G_2 = 0.338$ S $\quad \therefore R_1 = \frac{1}{G_1} = 2.96\Omega$

$$G_3 = \frac{b_0 C_1}{G_2} = \frac{1(0.03)}{0.338} = 0.089 \quad \therefore R_3 = 1/G_3 = 11.3\Omega$$

* Therefore in the first second order prototype circuit

$R_1 = R_2 = 2.96\Omega \qquad R_3 = 11.3\Omega$

$C_1 = 0.03$ F $\qquad C_2 = 1$ F

in the second second-order prototype circuit; we set

$b_1 = 1.848 \qquad b_0 = 1 \qquad C_2 = 1$ F

$$\therefore C_1 \le \frac{(1.848)^2}{8} \le 0.427$$

choose $C_1 = 0.30$ F

$$G_2 = \frac{1.848 \pm \sqrt{1.848^2 - 8 \times 0.3}}{4} = \frac{1.848 \pm 1.008}{4}$$

* Arbitrarily select the larger value, then

$G_2 = 0.7139$ S, $\therefore R_2 = \frac{1}{G_2} = 1.4008\Omega$

$G_1 = kG_2 = 0.7139$ S, $\therefore R_1 = \frac{1}{G_1} = 1.4008\Omega$

$$G_3 = \frac{b_0 C_1}{G_2} = \frac{(1) \times (0.30)}{0.7139} = 0.4202 \ \text{S} \quad \therefore R_3 = 1/G_3 = 2.3796\Omega$$

In the low-pass section of the filter

$$k_f = \frac{w_0'}{w_0} = 2\pi(1000) = 2000\pi$$

$$k_m = \frac{C}{C' k_f} = \frac{1}{(25\times10^{-9})k_f} = \frac{2\times10^4}{\pi}.$$

Therefore in the first scaled second-order section

$R_1' = R_2' = 2.96\, k_m = 18.8\, k\Omega$

$R_3' = 11.3\, k_m = 71.9\, k\Omega$.

$$C_1' = \frac{0.03}{k_f k_m} = \frac{0.03}{4\times10^7} = 7.5e^{-10}\, F = 750\, pF$$

$C_2' = 25\, nF$. (this is the value supplied in problem)

In the second scaled second-order section

$R_1' = R_2' = 1.4008\, k_m = 8.9\, k\Omega$. $R_3' = 2.38\, k_m = 15.1\, k\Omega$.

$C_1' = \frac{0.3}{k_f k_m} = 7.5\, nF$. $C_2'' = 25\, nF$

* High-pass filter selection:

$$\text{slope} = \frac{-24}{\log(8k/4k)} = -79.7\, dB/dec \quad \therefore n = 4.$$

In the first prototype second-order section, we select.

$b_1 = 0.765$; $b_0 = 1$. $C_2 = C_3 = 1\, F = C_1$, $K = 1$.

$$R_1 = \frac{K+2}{b_1} = \frac{3}{0.765} = 3.92\,\Omega \qquad R_2 = \frac{b_1}{b_0(K+2)} = \frac{0.765}{3} = 0.255\,\Omega.$$

In the second prototype second-order ~~filter~~ section, we set $b_1 = 1.848$, $b_0 = 1$;

$C_2 = C_3 = 1F = C_1$ ~~C~~ $K = 1F$

$$R_1 = \frac{K+2}{b_1} = \frac{3}{1.848} = 1.623\,\Omega \qquad R_2 = \frac{b_1}{b_0(K+2)} = \frac{1.848}{3} = 0.616\,\Omega$$

In the high-pass section of the filter, we still do scaling.

$$k_f = \frac{w_0'}{w_0} = 2\pi(8000) = 16000\pi.$$

$$k_m = \frac{C}{C' k_f} = \frac{1}{(25\times10^{-9})\times(16000\pi)} = \frac{795.77}{\pi}.$$

In the first scaled second-order section

$R_1' =$ ~~1.623~~ 3.92 Km $=$ ~~411.11~~ 992.94 Ω

$R_2' = 0.255$ Km $= 64.59$ Ω

$C_1' = C_2' = C_3' = 25$ nF

In the second scaled second-order section

$R_1' = 1.623$ km $= 411.11$ Ω

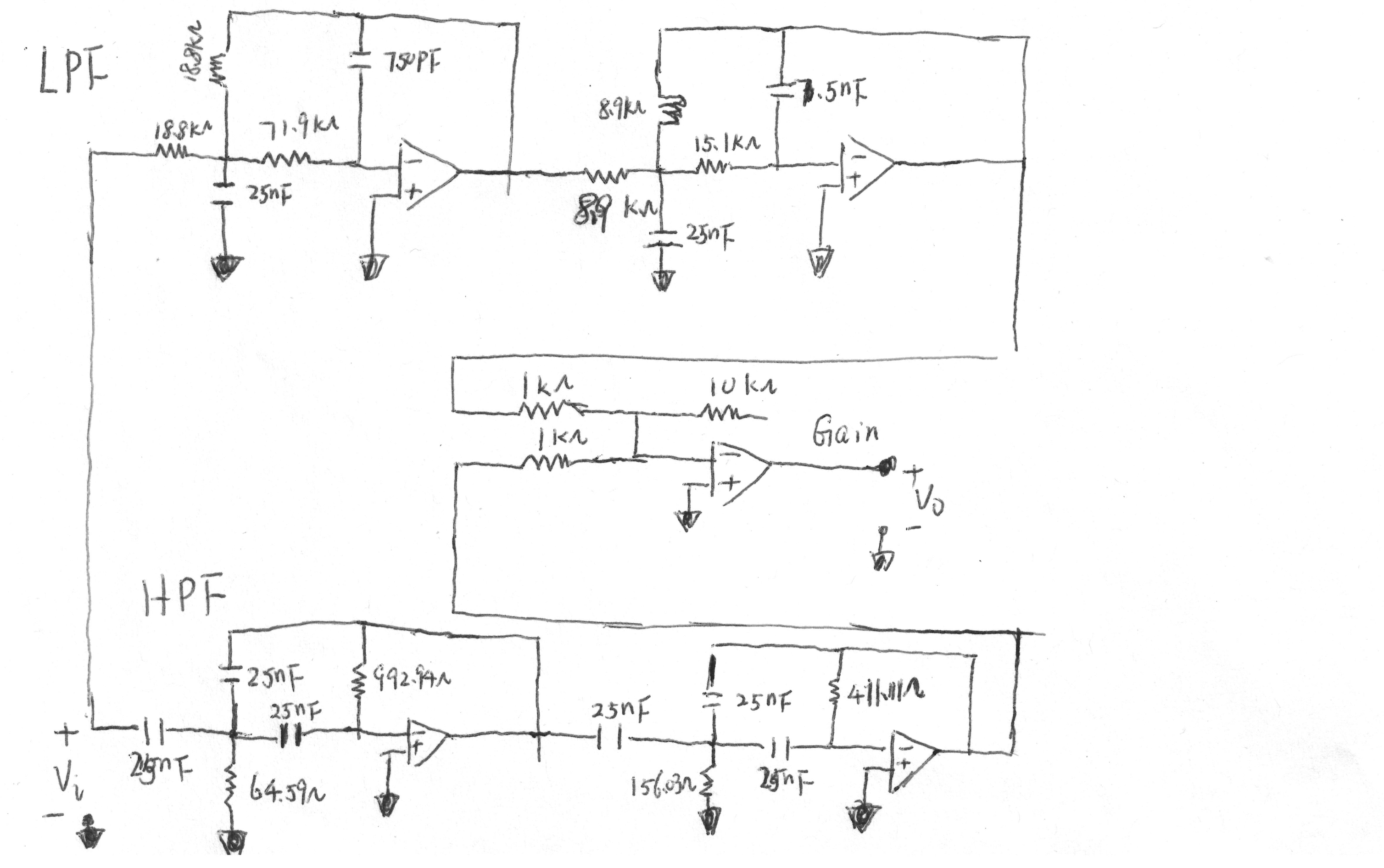
$R_2' = 0.616$ km $= 156.03$ Ω.

$C_1' = C_2' = C_3' = 1$ 0 F

* In the gain section, since we want the gain to be 20 dB.

so we can choose $R_i = 1$ kΩ and $R_f = 10$ kΩ

[b] the circuit is shown below, we have low pass filter and high-pass filter in parrellel, and each filter has two cascading second-order filter.



15.55 (a) At low frequencies the capacitors can be thought of as open circuits, and the gain of the circuit is approximately 1. At high frequencies, the capacitors will act like short circuits, and the outputs will be zero. So the circuit acts like a unity gain low pass filter.

(b) Node A is to the right of R_1 and Node B is to the right of R_2. Node A:

$$\begin{aligned} 0 &= \frac{V_a - V_{in}}{R_1} + \frac{V_a - V_{out}}{\frac{1}{SC_1}} \\ &= \frac{V_a - V_{in}}{R_1} + (V_a - V_{out})SC_1 \\ &= \frac{V_a}{R_1} - \frac{V_{in}}{R_1} + V_a(SC_1) - V_{out}(SC_1) \\ V_a(SC_1 + \frac{1}{R_1}) - \frac{V_{in}}{R_1} &= V_{out}(SC_1) \end{aligned}$$

Node B:

$$\begin{aligned} 0 &= \frac{V_b - V_a}{R_2} + \frac{V_b}{\frac{1}{SC_2}}, (V_b = V_{out}) \\ &= \frac{V_b}{R_2} - \frac{V_a}{R_2} + V_b(SC_2) \\ \frac{V_a}{R_2} &= V_b(SC_2 + \frac{1}{R_2}) \\ V_a &= V_b(SR_2C_2 + 1) \\ &= V_{out}(SR_2C_2 + 1) \end{aligned}$$

$$\begin{aligned} V_{out}(SC_1) &= V_{out}(SC_2R_2 + 1)(SC_1 + \frac{1}{R_1}) - \frac{V_{in}}{R_1} \\ V_{out}(SC_1R_1) &= V_{out}(SC_2R_2 + 1)(SC_1R_1 + 1) - V_{in} \\ V_{in} &= V_{out}[S^2C_2R_2C_1R_1 + SC_1R_1 + SC_2R_2 + 1] - V_{out}(SC_1R_1) \\ V_{in} &= V_{out}[S^2C_2R_2C_1R_1 + SC_1R_1 + SC_2R_2 + 1 - SC_1R_1] \\ V_{in} &= V_{out}[S^2C_2R_2C_1R_1 + SC_2R_2 + 1] \\ \frac{V_{out}}{V_{in}} &= \frac{1}{S^2C_2R_2C_1R_1 + SC_2R_2 + 1} \\ &= \frac{\frac{1}{R_2R_1C_2C_1}}{S^2 + S(\frac{1}{R_1C_1} + \frac{1}{R_1R_2C_2C_1})} \\ &= \frac{b_0}{S^2 + b_1S + b_0} \end{aligned}$$

Or written with $G_1 = \frac{1}{R_1}$, $G_2 = \frac{1}{R_2}$,

$$\frac{\frac{G_1G_2}{C_1C_2}}{S^2 + S(\frac{G_1}{C_1}) + \frac{G_1G_2}{C_1C_2}}$$

(c) There are 2 restraints from b_0, and b_1, and 4 circuit components. This gives a total of 2 free choices.

(d) From inspection, we see $b_1 = \frac{G_1}{C_1}$ and $b_0 = \frac{G_1G_2}{C_1C_2}$

(e) No, all physically realizable capacitors yield physically realizable resistors.

(f)

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{R_2 R_1 C_2 C_1}}{S^2 + S(\frac{1}{R_1 C_1} + \frac{1}{R_1 R_2 C_2 C_1})}$$
$$= \frac{b_0}{S^2 + b_1 S + b_0}$$

The corner frequency in radians per second is $\sqrt{\frac{1}{R_1 R_2 C_1 C_2}} = \sqrt{b_0}$ and the pass band gain is 1. So there must be a relationship between b_1 and b_0 for a gain of 1 at the corner frequency.

$$H(j\omega) = \frac{b_0}{(j\omega)^2 + j\omega b_1 + b_0}$$

If we take the magnitude and set it equal to 1,

$$1 = |H(j\omega)| = \frac{b_0}{\sqrt{(b_0 - \omega_0^2)^2 + (\omega_0 b_1)^2}}$$

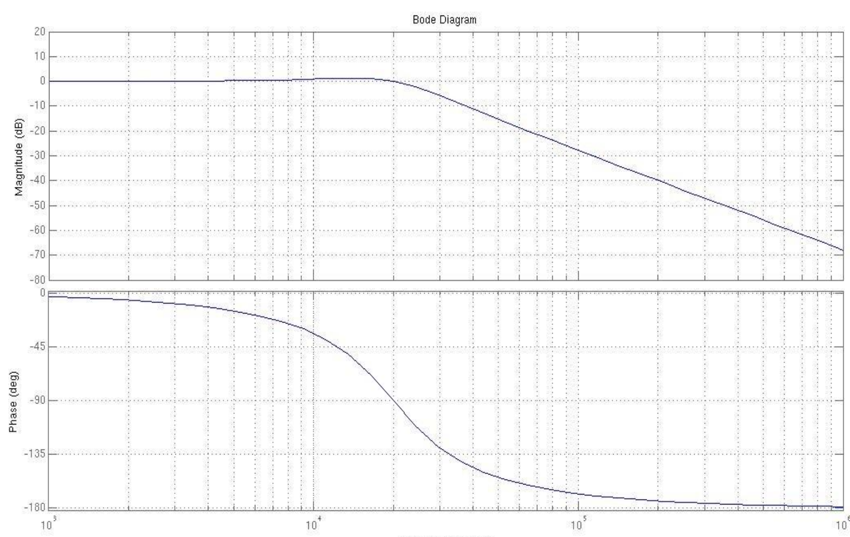
Substitute $w_0 = \sqrt{b_0}$

$$1 = \frac{b_0}{\sqrt{(b_0 - b_0)^2 + (\sqrt{b_0} b_1)^2}}$$
$$= \frac{b_0}{\sqrt{(b_0 b_1^2)}}$$
$$= \frac{1}{b_1} \cdot \frac{b_0}{\sqrt{b_0}}$$
$$= \frac{\sqrt{b_0}}{b_1}$$

So $b_1 = \sqrt{b_0}$ for gain at corner frequency equal to passband gain 1. $b_0 = (20{,}000)^2$ so $b_1 = 20{,}000$. This means we can set $R_1 = R_2$ and $C_1 = C_2$,

$w_0 = 1/RC$, Let's choose $c = 10nF$.

$$R = \frac{1}{\omega_0 C} = \frac{1}{(20000)(10nF)} = 5k\Omega.$$



Problem 1

[a] $slope = \dfrac{-48}{\log\left(32/8\right)} = -79.72\ dB/dec \quad \therefore\ n = 4$

From Table 15.1 the transfer function is

$$H(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The capacitor values for the first stage prototype circuit are

$$\frac{2}{C_1} = 0.765 \quad \therefore \quad C_1 = 2.61\,\mathrm{F}$$

$$C_2 = \frac{1}{C_1} = 0.38\,\mathrm{F}$$

The values for the second stage prototype circuit are

$$\frac{2}{C_1} = 1.848 \quad \therefore \quad C_1 = 1.08\,\mathrm{F}$$

$$C_2 = \frac{1}{C_1} = 0.92\,\mathrm{F}$$

The scaling factors are

$$k_m = \frac{R'}{R} = 1000; \qquad k_f = \frac{\omega_o'}{\omega_o} = 16{,}000\pi$$

Therefore the scaled values for the components in the first stage are

$$R_1 = R_2 = R = 1000\,\Omega$$

$$C_1 = \frac{2.61}{(16{,}000\pi)(1000)} = 52.01\,\mathrm{nF}$$

$$C_2 = \frac{0.38}{(16{,}000\pi)(1000)} = 7.61\,\mathrm{nF}$$

The scaled values for the second stage are

$$R_1 = R_2 = R = 1000\,\Omega$$

$$C_1 = \frac{1.08}{(16{,}000\pi)(1000)} = 21.53\,\mathrm{nF}$$

$$C_2 = \frac{0.92}{(16{,}000\pi)(1000)} = 18.38\,\mathrm{nF}$$

[b]