EE233 HW0 Solution

Sept. 28th

Due Date: Oct. 3rd

1. Find the Norton equivalent with respect to the terminals a, b for the circuit in the following figure.

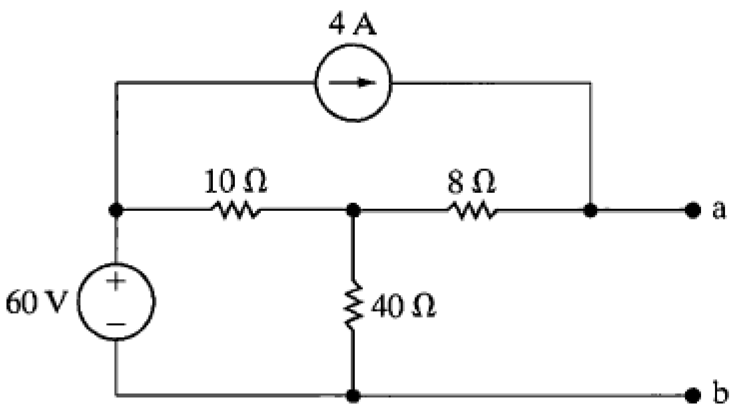
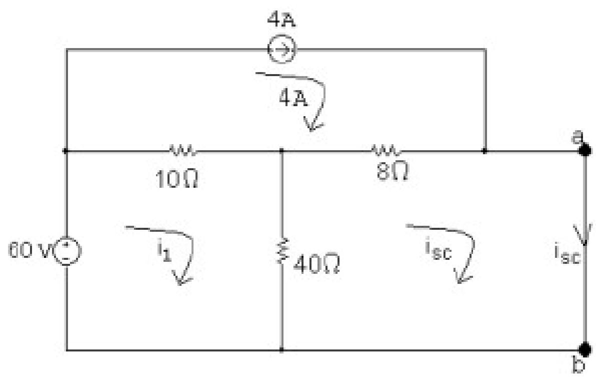


Figure Problem 1

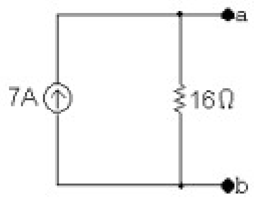
Solution



$$i_{SC} = 7A$$

$$R_{th} = 8 + \frac{10 \times 40}{50} = 16\Omega$$

The Norton equivalent circuit is:



2. The op-amp in the circuit below is ideal.
   a) Calculate v_o when v_g equals 4V.
   b) Specify the range of values of v_g so that the op-amp operates in linear mode.
   c) Assume that v_g equals 2V and that the 63Ω resistor is replaced with variable resistor. What value of the variable resistor will cause the op-amp to saturate?

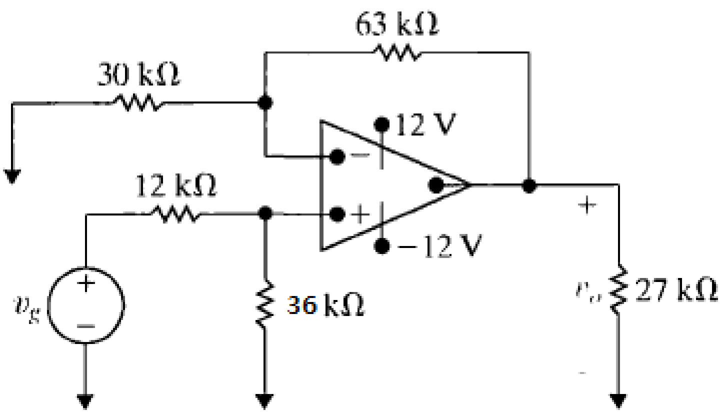


Figure Problem 2

<u>**Solution**</u>

a) The first step is to find V_+ which denotes the voltage at the positive terminal of opamp.

$$4\left(\frac{36k\Omega}{36k\Omega + 12k\Omega}\right) = 3V$$

Now, we know V_- will also be 3V and no current flows into it. So all the current flows through the $30k\Omega$ resistor.

$$\frac{3V}{30k\Omega} = 0.1mA$$

Then,

$$V_o = (0.1mA)(63k\Omega) + 3V$$
$$V_o = 9.3V$$

b) The opamp output cannot go above or below its supply power rails. We need to find the corresponding input voltages which can easily be done if we know the relationship between V_g and V_o.

$$V_g\left(\frac{36k\Omega}{36k\Omega + 12k\Omega}\right)\left(1 + \frac{63k\Omega}{30k\Omega}\right) = V_o$$
$$2.33V_g = V_o$$

So, when $V_o = 12V$, $V_g = 5.15V$ and when $V_o = -12V$, $V_g = -5.15V$

c)

$$2\left(\frac{36k\Omega}{36k\Omega + 12k\Omega}\right)\left(1 + \frac{R}{30k\Omega}\right)V = 12V$$
$$1.5 + \frac{1.5R}{30k\Omega}V = 12V$$
$$R = 210k\Omega$$

3. The three inductors in the circuit below are connected across the terminals of a black box at $t = 0$. The resulting voltage for $t > 0$ is known to be $v_o = 2000e^{-100t}V$.

If $i_1(0) = -6A \; and \; i_2(0) = 1A$, find

a) $i_0(0)$

b) $i_0(t), t \geq 0$

c) $i_1(t), t \geq 0$

d) $i_2(t), t \geq 0$

e) the initial energy stored in the three inductors

f) the total energy delivered to the black box

g) the energy trapped in the ideal inductors

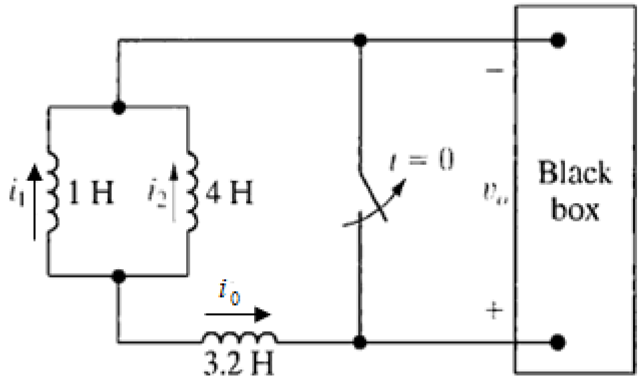
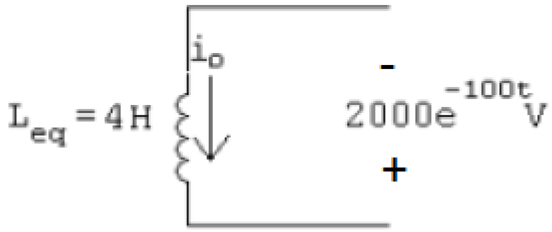


Figure Problem 3

Solution

a) $i_0(0) = -i_1(0) - i_2(0) = 6 - 1 = 5A$

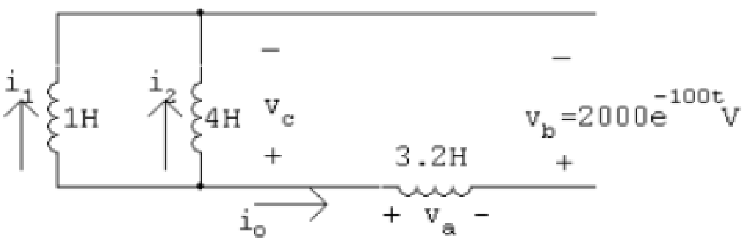
b) The equivalent circuit becomes



So,

$$i_0(t) = -\frac{1}{4}\int_0^t 2000e^{-100\tau}d\tau + 5 = 5e^{-100t}A, t \geq 0$$

c)



$$v_a(t) = 3.2(-500e^{-100t})[V] = -1600e^{-100t}\,[V]$$

$$v_c(t) = v_a(t) + v_b(t) = -1600e^{-100t} + 2000e^{-100t}\,[V]$$

$$= 400e^{-100t}\,[V]$$

$$i_1(t) = \frac{1}{1}\int_0^t 400\,e^{-100\tau}d\tau - 6 = -4e^{-100t} - 2[A], t \geq 0$$

d)

$$i_2(t) = \frac{1}{4}\int_0^t 400\,e^{-100\tau}d\tau + 1 = -e^{-100t} + 2[A], t \geq 0$$

e)

$$w(0) = \frac{1}{2}(1)(6^2) + \frac{1}{2}(4)(1^2) + \frac{1}{2}(3.2)(5^2) = 60[J]$$

f)

$$w_{del} = \frac{1}{2}(4)(5^2) = 50[J]$$

g)

$$w_{trap} = 60 - 50 = 10[J]$$

4. After the circuit below has been in operation for a long time, a screwdriver is inadvertently connected across the terminals a, b. Assume the resistance of the screwdriver is negligible.
   a) Find the current in the screwdriver at $t = 0^+$ and $t = \infty$.
   b) Derive the expression for the current in the screwdriver for $t \geq 0^+$.

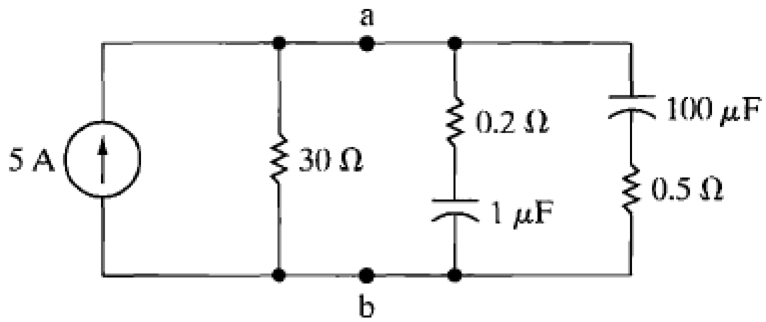
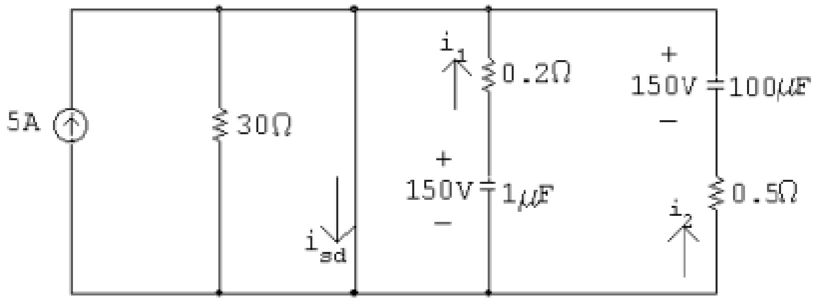


Figure Problem 4

__Solution__

a) At $t = 0^+$, the voltage on each capacitor will be 150V, positive at the upper terminal. Hence,



So, $i_{sd}(0^+) = 5 + \frac{150}{0.2} + \frac{150}{0.5}[A] = 1055[A]$

Also, at $t = \infty$, both capacitors will have completely discharged.

So, $i_{sd}(\infty) = 5[A]$

b)

$$\begin{aligned}
&i_{sd}(t) = 5 + i_1(t) + i_2(t)\\
&and\ \tau_1 = 0.2(10^{-6}) = 0.2\mu s\\
&\tau_2 = 0.5(100\times10^{-6}) = 50\mu s\\
&So,\\
&i_1(t) = 750e^{-5\times10^6 t}[A], t \ge 0^+\\
&i_2(t) = 300e^{-20000t}[A], t \ge 0^+\\
&i_{sd}(t) = 5 + 750e^{-5\times10^6 t} + 300e^{-20000t}[A], t \ge 0^+
\end{aligned}$$

5. The switch in the circuit below has been in position ***a*** for a long time. At $t = 0$, the switch moves instantaneously to position ***b***. Find

   a) $v_o(0^+)$

   b) $dv_o(0^+)/dt$

   c) $v_o(t)\ for\, t \ge 0^+$

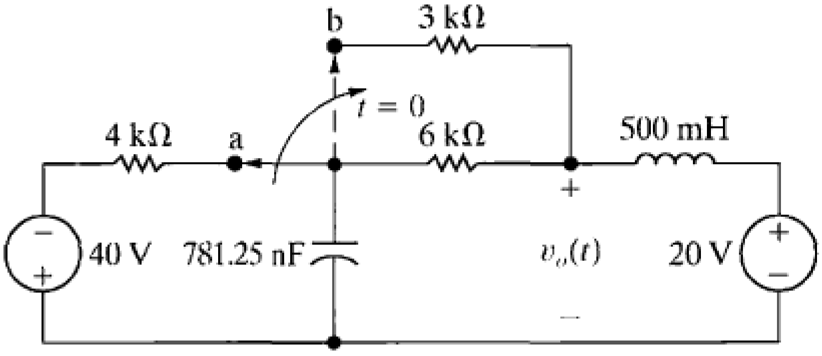
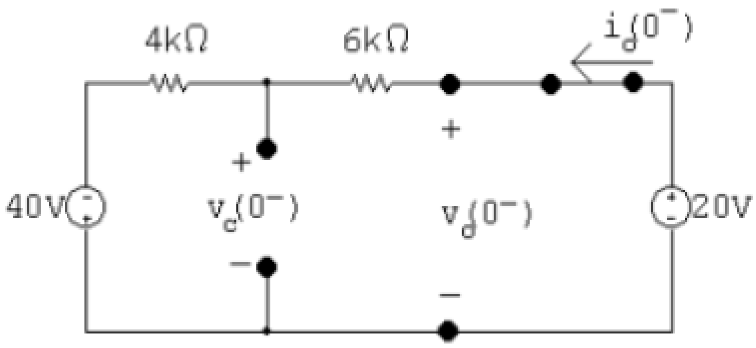


Figure Problem 5

Solution

a) At $t < 0$,

The circuit is:

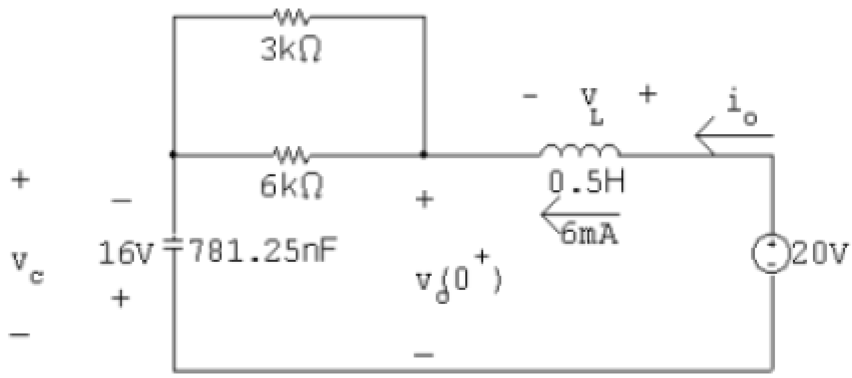


So,

$$i_o(0^-) = \frac{60}{10000} = 6[mA]$$
$$v_c(0^-) = 20 - (6000)(0.006)[V] = -16[V]$$

At $t = 0^+$,

The circuit is:



So,

$$v_o(0^+) = (0.006)(2000) - 16[V] = 12 - 16[V] = -4[V]$$
and
$$v_L(0^+) = 20 - (-4)[V] = 24[V]$$

b)

$$v_o(t) = 2000 i_o(t) + v_c(t)$$
$$\frac{dv_o(t)}{dt} = 2000\frac{di_o(t)}{dt} + \frac{dv_c(t)}{dt}$$
$$\frac{dv_o(0^+)}{dt} = 2000\frac{di_o(0^+)}{dt} + \frac{dv_c(0^+)}{dt}$$
$$v_L(0^+) = L\frac{di_o(0^+)}{dt}$$
$$So, \frac{di_o(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{24}{0.5} = 48[A/s]$$
$$and\ C\frac{dv_c(0^+)}{dt} = i_o(0^+)$$
$$Hence,$$
$$\frac{dv_c(0^+)}{dt} = \frac{6\times10^{-3}}{781.25\times10^{-9}} = 7680$$
$$\frac{dv_o(0^+)}{dt} = (2000)(48) + 7680 = 103680[V/s]$$

c)

$$\omega_o^2 = \frac{1}{LC} = 2.56\times10^6, so\ \omega_o = 1600[rad/s]$$
$$\alpha = \frac{R}{2L} = 2000[rad/s]$$
since $\alpha^2 > \omega_o^2, overdamped$

$$So,$$
$$s_{1,2} = -2000 \pm j1200[rad/s]$$
$$then,$$
$$v_o(t) = V_f + A_1' e^{-800t} + A_2' e^{-3200t}$$
$$V_f = v_o(\infty) = 20[V]$$
$$20 + A_1' + A_2' = -4$$
$$-800A_1' - 3200A_2' = 103680$$
$$Hence,$$
$$A_1' = 11.2, A_2' = -35.2$$
$$v_o(t) = 20 + 11.2e^{-800t} - 35.2e^{-3200t}[V], t \ge 0^+$$

6. Calculate the following complex number: $(1-3j)+\frac{2+2j}{1-2j}$

   Give answer in both Cartesian (rectangular) and angular (polar, magnitude and phase) form.

Solution

$$
\begin{aligned}
&(1-3j)+\frac{2+2j}{1-2j} \\
&=(1-3j)+\frac{2+2j}{1-2j}\cdot\frac{1+2j}{1+2j} \\
&=1-3j-0.4+1.2j \\
&=0.6-1.8j \\
&=1.89\angle-71^{\circ}
\end{aligned}
$$