EE233 HW1 Solution

Oct. 3rd

Due Date: Oct. 10th

Problems from the textbook: P.9.49, P.9.59, and P.9.60

1. At $t = -2ms$, a sinusoidal voltage is known to be zero and going positive. The voltage is next zero is $t = 8ms$. It is also known that the voltage is 80.9V at $t = 0ms$.
   a) What is the frequency of voltage v in hertz?

$$\frac{T}{2} = 8 + 2 = 10\,\text{ms}; \qquad T = 20\,\text{ms}$$

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}} = 50\,\text{Hz}$$

   b) What is the expression for v?

$$v = V_m \sin(\omega t + \theta)$$

$$\omega = 2\pi f = 100\pi \text{ rad/s}$$

$$100\pi(-2 \times 10^{-3}) + \theta = 0; \qquad \therefore \quad \theta = \frac{\pi}{5} \text{ rad} = 36^\circ$$

$$v = V_m \sin[100\pi t + 36^\circ]$$

$$80.9 = V_m \sin 36^\circ; \qquad V_m = 137.64\,\text{V}$$

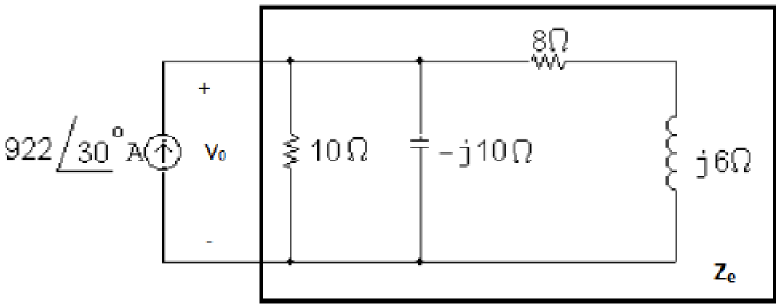
$$v = 137.64 \sin[100\pi t + 36^\circ] = 137.64 \cos[100\pi t - 54^\circ]\,\text{V}$$

2. A 10 Ω resistor and 5 μF capacitor are connected in parallel. This parallel combination is also in parallel with the series combination of $8\,\Omega$ resistor and a 300 μH inductor. These three parallel branches are driven by a sinusoidal current source whose current is $922\cos(20000t + 30^\circ)A$.

a) Draw frequency-domain equivalent circuit.

$$j\omega L = j(2 \times 10^4)(300 \times 10^{-6}) = j6\,\Omega$$

$$\frac{1}{j\omega C} = -j\frac{1}{(2 \times 10^4)(5 \times 10^{-6})} = -j10\,\Omega; \qquad \mathbf{I}_g = 922\underline{/30^\circ}\text{ A}$$



b) Reference the voltage across the current source as a rise in the direction of the current, and find the phasor voltage.

$$\mathbf{V}_o = 922\underline{/30^\circ} Z_e$$

$$Z_e = \frac{1}{Y_e}; \qquad Y_e = \frac{1}{10} + j\frac{1}{10} + \frac{1}{8+j6}$$

$$Y_e = 0.18 + j0.04\,\text{S}$$

$$Z_e = \frac{1}{0.18 + j0.04} = 5.42\underline{/-12.53^\circ}\,\Omega$$

$$\mathbf{V}_o = (922\underline{/30^\circ})(5.42\underline{/-12.53^\circ}) = 5000.25\underline{/17.47^\circ}\text{ V}$$

c) Find the steady-state expression for $v(t)$.

$$v_o = 5000.25\cos(2 \times 10^4 t + 17.47^\circ)\text{ V}$$

3. Find the impedance Z_{ab} in the circuit seen in below. Express Z_{ab} in both polar and rectangular form.

$$Z_{\text{ab}} = 1 - j8 + (2 + j4)\|(10 - j20) + (40\|j20)$$

$$= 1 - j8 + 3 + j4 + 8 + j16 = 12 + j12\,\Omega = 16.97\underline{/45^\circ}\,\Omega$$

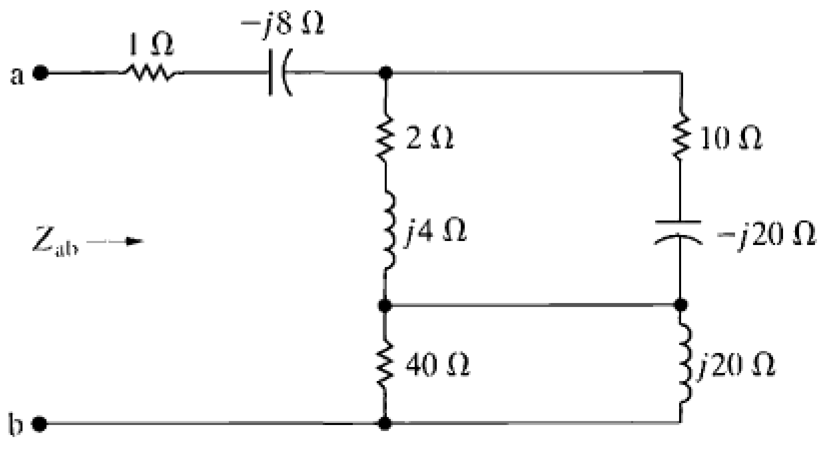
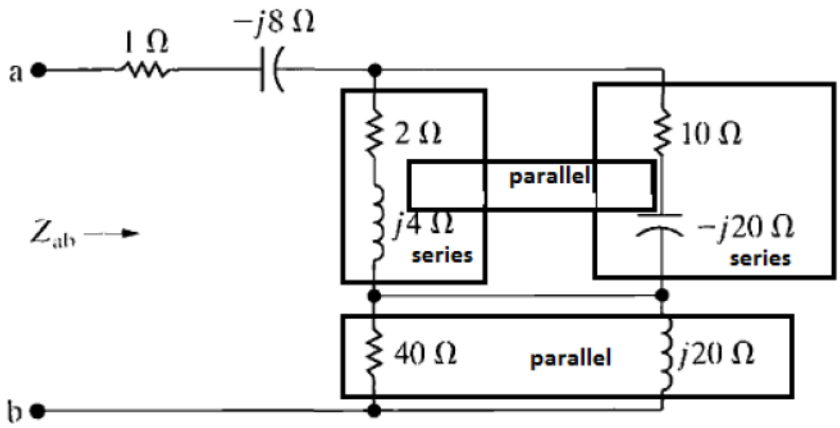


Figure Problem 3

For circuit analysis:



4. Solve:
   a) The frequency of the source voltage in the circuit below is adjusted until i_g is in phase with v_g. What is value of ω in radian per second?

$$Z_g = 500 - j\frac{10^6}{\omega} + \frac{10^3(j0.5\omega)}{10^3 + j0.5\omega}$$

$$= 500 - j\frac{10^6}{\omega} + \frac{500j\omega(1000 - j0.5\omega)}{10^6 + 0.25\omega^2}$$

$$= 500 - j\frac{10^6}{\omega} + \frac{250\omega^2}{10^6 + 0.25\omega^2} + j\frac{5 \times 10^5\omega}{10^6 + 0.25\omega^2}$$

For i_g to be in phase with v_g, Z_g have to be purely real

$$\therefore \quad \text{If } Z_g \text{ is purely real,} \quad \frac{10^6}{\omega} = \frac{5 \times 10^5\omega}{10^6 + 0.25\omega^2}$$

b) If $v_g = 20\cos(wt)[V]$ where ω is the frequency found in part a, what is the steady-state expression for v_o?

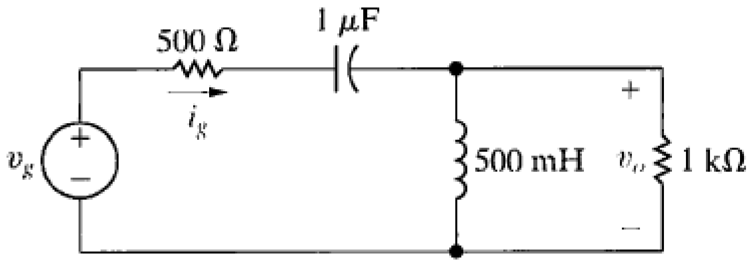


Figure Problem 4

When $\omega = 2000\,\text{rad/s}$

$$Z_g = 500 - j500 + (j1000 \| 1000) = 1000\,\Omega$$

$$\therefore \quad \mathbf{I}_g = \frac{20\underline{/0^\circ}}{1000} = 20\underline{/0^\circ}\,\text{mA}$$

$$\mathbf{V}_o = \mathbf{V}_g - \mathbf{I}_g Z_1$$

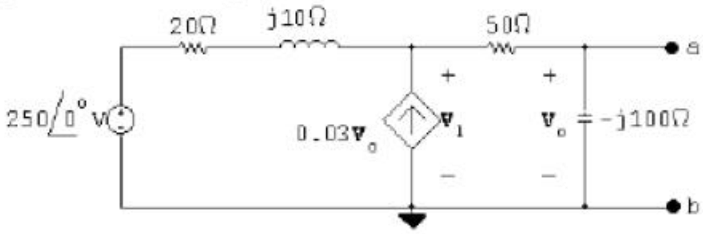
$$Z_1 = 500 - j500\,\Omega$$

$$\mathbf{V}_o = 20\underline{/0^\circ} - (0.02\underline{/0^\circ})(500 - j500) = 10 + j10 = 14.14\underline{/45^\circ}\,\text{V}$$

$$v_o = 14.14\cos(2000t + 45^\circ)\,\text{V}$$

Problem 9.49

Open circuit voltage:



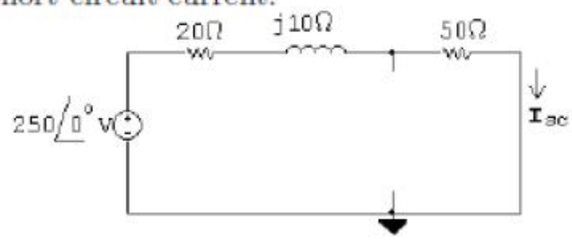
$$\frac{\mathbf{V}_1 - 250}{20 + j10} - 0.03\mathbf{V}_o + \frac{\mathbf{V}_1}{50 - j100} = 0$$

$$\therefore \quad \mathbf{V}_o = \frac{-j100}{50 - j100}\mathbf{V}_1$$

$$\frac{\mathbf{V}_1}{20 + j10} + \frac{j3\mathbf{V}_1}{50 - j100} + \frac{\mathbf{V}_1}{50 - j100} = \frac{250}{20 + j10}$$

$$\mathbf{V}_1 = 500 - j250\,\text{V}; \qquad \mathbf{V}_o = 300 - j400\,\text{V} = \mathbf{V}_{\text{Th}}$$

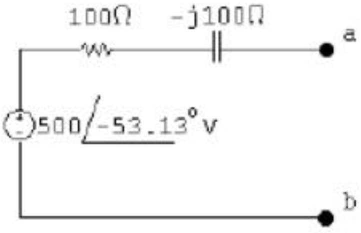
Short circuit current:



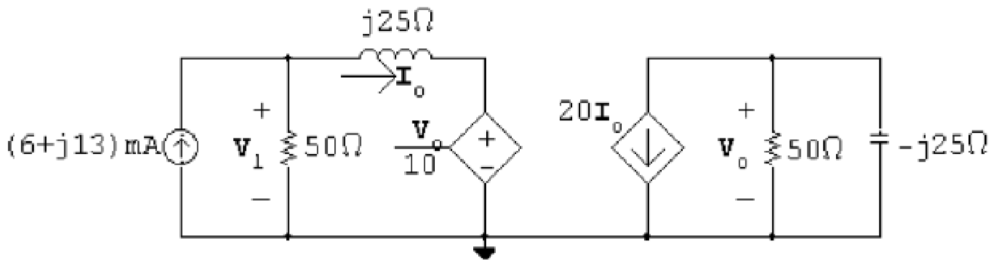
$$\mathbf{I}_{\text{sc}} = \frac{250/\underline{0^\circ}}{70 + j10} = 3.5 - j0.5\,\text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{300 - j400}{3.5 - j0.5} = 100 - j100\,\Omega$$

The Thévenin equivalent circuit:



Problem 9.59



$$\frac{\mathbf{V}_o}{50} + \frac{\mathbf{V}_o}{-j25} + 20\mathbf{I}_o = 0$$

$$(2 + j4)\mathbf{V}_o = -2000\mathbf{I}_o$$

$$\mathbf{V}_o = (-200 + j400)\mathbf{I}_o$$

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - (\mathbf{V}_o/10)}{j25}$$

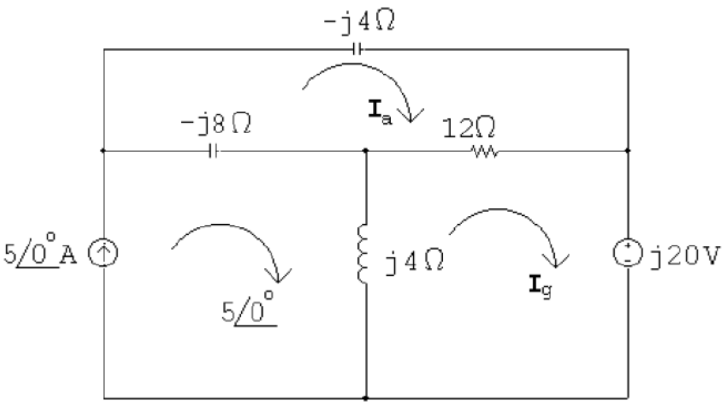
$$\therefore \quad \mathbf{V}_1 = (-20 + j65)\mathbf{I}_o$$

$$0.006 + j0.013 = \frac{\mathbf{V}_1}{50} + \mathbf{I}_o = (-0.4 + j1.3)\mathbf{I}_o + \mathbf{I}_o = (0.6 + j1.3)\mathbf{I}_o$$

$$\therefore \quad \mathbf{I}_o = \frac{0.6 + j1.3(10 \times 10^{-3})}{(0.6 + j1.3)} = 10\underline{/0^\circ}\,\text{mA}$$

$$\mathbf{V}_o = (-200 + j400)\mathbf{I}_o = -2 + j4 = 4.47\underline{/116.57^\circ}\,\text{V}$$

Problem 9.60



$(12 - j12)\mathbf{I}_a - 12\mathbf{I}_g - 5(-j8) = 0$ (Loop I_a)

$-12\mathbf{I}_a + (12 + j4)\mathbf{I}_g + j20 - 5(j4) = 0$ (Loop I_g)

Solving,

$\mathbf{I}_g = 4 - j2 = 4.47\underline{/-26.57^\circ}$ A