EE 233 Homework2 solution

Autumn 2016

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1 Problem 1

(a)

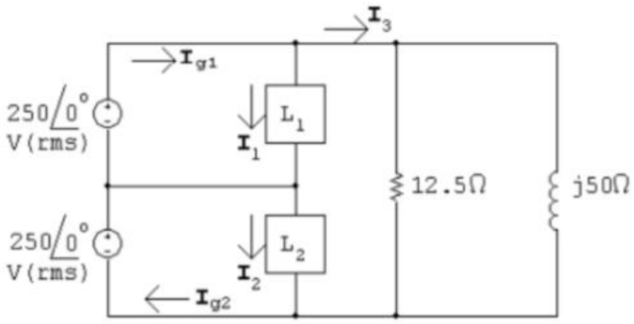


Figure 1

$$250I_1^* = 750 + j2500; \quad \therefore I_1 = 30 - j10A(rms)\ (this\ is\ from\ S = V_{rms}I_{rms}^*)$$

$$250I_2^* = 2800 - j9600; \quad \therefore I_2 = 11.2 + j38.4A(rms)$$

$$I_3 = 500/12.5 + 500/(j50) = 40 - j10A(rms)$$
$$I_{g1} = I_1 + I_3 = 70 - j20A$$
$$S_{g1} = 250(70 + j20) = 17500 + j5000VA$$

Thus the V_{g1} source is delivering 17.5kW and 5000 magnetizing vars

$$I_{g2} = I_2 + I_3 = 51.2 + j28.4A(rms)$$
$$S_{g2} = 250(51.2 - j28.4) = 12800 - j7100VA$$

Thus the V_{g2} source is delivering 12.8kW and absorbing 7100 magnetizing vars.

(b)

$$\begin{aligned}
&\sum P_{dev} = 17.5 + 12.8 = 30.3kW \\
&\sum P_{abs} = 7500 + 2800 + \frac{(500)^2}{12.5} = 30.3kW = \sum P_{dev} \\
&\sum Q_{dev} = 9600 + 5000 = 14.6kVAR \\
&\sum Q_{abs} = 2500 + 7100 + \frac{(500)^2}{50} = 14.6kVAR = \sum Q_{dev}
\end{aligned}$$

2 Problem 2

(a)

$$\begin{aligned}
&I = \frac{465\angle 0^{circ}}{124 + j93} = 2.4 - j1.8 = 3\angle - 36.87^{\circ} A(rms) \\
&P = 3^2 \times 4 = 36W
\end{aligned}$$

(b)

$$\begin{aligned}
Y_L &= \frac{1}{120 + j90} = 5.33 - j4\ mS \\
\therefore\ X_c &= \frac{1}{-4 \times 10^{-3}} = -250\Omega
\end{aligned}$$

(c) $Z_L = \frac{1}{5.33 \times 10^{-3}} = 187.5\Omega$

(d) $I = \frac{465\angle 0^{\circ}}{191.5 + j3} = 2.4279\angle - 0.9^{\circ}$

$P = 2.4279^2 \times 4 = 23.58W$

(e) $Percentage = \frac{23.58}{36}(100) = 65.5\%$
Thus the power loss after the capacitor is added is 65.5% of the power loss before the capacitor is added

3 Problem 3

(a) according to Norton's theorem, we can regard the circuit in the box as a combination of an ideal stable current source and a resistor whose impedance is Z_{th}, so we will have

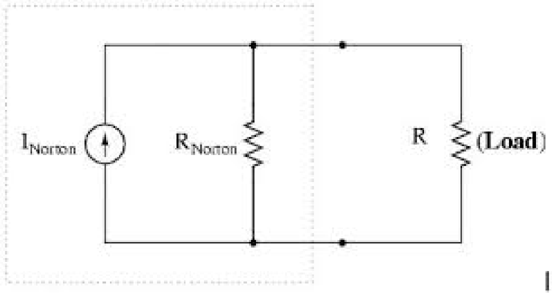


Figure 2: Norton's equivalent circuit

$$\frac{115.2 - j86.4 - 240}{Z_{th}} + \frac{115.2 - j86.4}{90 - j30} = 0$$

$$\therefore Z_{th} = \frac{240 - 115.2 + j86.4}{1.44 - j0.48} = 60 + j80\Omega$$

$$\therefore Z_L = 60 - j80\Omega$$

(b)

$$I = \frac{240\angle 0^\circ}{120\angle 0^\circ} = 2\angle 0^\circ A(rms)$$

$$P = 2^2 \times 60 = 240W$$

(c) Let $R = 15 + 15 + 15 + 15 = 60\Omega$

then $\frac{1}{2\pi(60)C} = 80$ so $C = 33.16uF$
Let $C = 22uF||10uF||1uF = 33uF$

4 Problem 10.4

(a) peak value of instantaneous power is

$$Z_c = -j\frac{1}{\omega c} = -j\frac{1}{(\frac{5}{9})(5000)} = -360j$$

$$Z_{eq} = \frac{(480)(-360j)}{(480 - 360j)} = 172.8 - 230.4j = 288\angle 53.13^\circ$$

$$I = \frac{240\angle 0^\circ}{288\angle 53.13^\circ}$$

$$\begin{aligned} P_{ave} &= (V_m I_m)\cos(\theta_v - \theta_i) \\ &= \frac{240 \cdot 0.833}{2}\cos(-53.13^\circ) \\ &= 60W \end{aligned}$$

$$
\begin{aligned}
Q &= \frac{240 \cdot 0.833}{2} \sin(53.3^\circ) \\
&= -80VAR \\
P_{inst} &= 60 + 60\cos(10000t) + 80sin(10000t)
\end{aligned}
$$

To find min and max, find where $dp_{inst}/dt = 0$

$$
\frac{dp_{inst}}{dt} = -60(10000)sin(10000t) + 80(10000)\cos(10000t)
$$

if $dp_{inst}/dt = 0$, then

$$
\begin{aligned}
6 \times 10^5 \sin(10000t) &= 80 \times 10000 \cdot \cos(1000t) \\
\frac{\sin(10000t)}{\cos(1000t)} &= \frac{8 \times 10^5}{6 \times 10^5} \\
\tan(10000t) &= \frac{4}{3} \\
10000t &= \tan^{-} 1(\frac{-4}{3}) = 53.3^\circ \\
P_{inst}(53.13^\circ) &= 60 + 60\cos(53.3^\circ) + 80\sin(53.13^\circ) \\
P_{max} &= P_{inst}(53.13^\circ) = 160\ W
\end{aligned}
$$

(b) Then P_{min} must occur $180^\circ(\pi)$ away from max

$$
\begin{aligned}
P_{min} &= 60 + 60\cos(53.53^\circ + 180^\circ) + 80\sin(53.13^\circ + 180^\circ) \\
&= -40\ W
\end{aligned}
$$

(c) from (a) we know it is 60W
(d) -80VAR
(e) because $\theta_v - \theta_i < 0$, circuit is capacitive, so we would say circuit generates magnetizing VARs.

5 Problem 10.13

$$
\begin{aligned}
P_{ave} &= 1280W = I_{eff}^2 R \\
R &= \frac{1280W}{|I_{eff}|^2}
\end{aligned}
$$

To find I_{eff} we need to calculate the RMS value of the current which has a specific function with time t.

$$
\begin{aligned}
I_{rms} &= \sqrt{\frac{1}{0.1s}(\int_0^{0.08s}(\frac{20A}{0.08s})^2 t^2 dt + \int_{0.08s}^{0.1s}(100-1000t)^2 dt)} \\
&= \sqrt{625000\int_0^{0.08} t^2 dt + 100000\int_{0.08}^{0.1}(1-10t)^2 dt} \\
&= \sqrt{625000 \times \frac{0.08^3}{3} + 100000 \times \frac{0.2^3}{30}} \\
&= 11.547A
\end{aligned}
$$

Then we have $R = \frac{1280}{11.547^2} = 9.6\Omega$

6 Problem 10.47

(10.50) (a) Use nodal analysis to find V_Φ. Then we can easily find V_{oc}.

$$\begin{aligned}
\frac{V_\Phi - 100}{5} + \frac{V_\Phi}{5j} - \frac{V_\Phi}{10} &= 0 \\
2(V_\Phi - 100) - 2jV_\Phi - vp &= 0 \\
2V_\Phi - 200 - 2jV_\Phi - V_\Phi &= 0 \\
V_\Phi - 2jV_\Phi &= 200 \\
V_\Phi(1-2j) &= 200 \\
V_\Phi &= \frac{200}{1-2j}\frac{(1+2j)}{(1+2j)} \\
&= \frac{2--+400j}{5} \\
&= 40 + 80j
\end{aligned}$$

So

$$\begin{aligned}
V_{TH} = V_\Phi + 0.1V_\Phi(-5j) &= V_{opencircuit} \\
&= (40+80j) - 5j\left(\frac{40}{10} + \frac{80}{10}\right) \\
&= 40 + 80j - 20j + 40 \\
V_{TH} &= (80+60j)V_{rms}
\end{aligned}$$

Now to find I short circuit (I_{SC}), first find the new V_Φ.

$$\begin{aligned}
\frac{V_\Phi - 100}{5\Omega} + \frac{V_\Phi}{5j} - \frac{V_\Phi}{5j} &= 0 \\
\frac{V_\Phi - 100}{5\Omega} &= 0 \\
\frac{V_\Phi}{5\Omega}20 \\
V_\Phi &= 100 \\
I_{sc} &= 0.1V_\Phi - \frac{V_\Phi}{-5j} \\
&= 10 - \frac{100}{-5j} \\
I_{sc} &= 10 + 20jArms \\
\therefore Z_{TH} &= \frac{80+60j}{10+20j} = 4 - 2j \\
R &= |Z_{TH}| \\
&= \sqrt{4^2 + 2^2} \\
&= 4.47\Omega
\end{aligned}$$

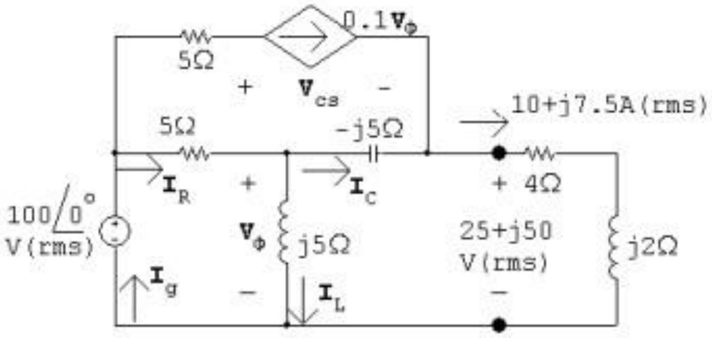
(b) Average power delivered to the load

$$\begin{aligned} I_{rms} &= \frac{80+60j}{4.47+4-2j} \\ &= \frac{80+60j}{8.47-2j} \\ &= 7.36+8.82j \\ |I_{rms}| &= 11.49 \\ \therefore P &= (11.49Arms)^2 4.47 \\ &= 590.13W \end{aligned}$$

(c) Z_0 would be $4+2j$ (conjugate of Z_{TH}).

$$\begin{aligned} I_{rms} &= \frac{80+60j}{4-2j+4+2j} \\ &= \frac{80+60j}{8} \\ &= 10+7.5j \\ |I_{rms}| &= 12.5 \\ P &= 12.5^2(4) \\ &= 625W \end{aligned}$$

(d) So we need to find the total average power developed in the circuit, which is equal to the average power absorbed by the circuit. In systems with only one source, we know that the source is delivering power. In circuits with more than one source, it is possible for one source to absorb power, so we need to check if each source is absorbing of delivering power.



$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} + \frac{\mathbf{V}_o - (25 + j50)}{-j5} = 0$$

$\mathbf{V}_\phi = 50 + j25\,\mathrm{V}$ (rms)

$0.1\mathbf{V}_\phi = 5 + j2.5\,\mathrm{V}$ (rms)

$5 + j2.5 + \mathbf{I}_C = 10 + j7.5$

$\mathbf{I}_C = 5 + j5\,\mathrm{A}$ (rms)

$$\mathbf{I}_L = \frac{\mathbf{V}_\phi}{j5} = 5 - j10\,\mathrm{A}\ \text{(rms)}$$

$\mathbf{I}_R = \mathbf{I}_C + \mathbf{I}_L = 10 - j5\,\mathrm{A}$ (rms)

$\mathbf{I}_g = \mathbf{I}_R + 0.1\mathbf{V}_\phi = 15 - j2.5\,\mathrm{A}$ (rms)

$S_g = -100\mathbf{I}_g^* = -1500 - j250\,\mathrm{VA}$

$100 = 5(5 + j2.5) + \mathbf{V}_{\mathrm{cs}} + 25 + j50 \quad \therefore \quad \mathbf{V}_{\mathrm{cs}} = 50 - j62.5\,\mathrm{V}$ (rms)

$S_{\mathrm{cs}} = (50 - j62.5)(5 - j2.5) = 93.75 - j437.5\,\mathrm{VA}$

Thus,

$\sum P_{\mathrm{dev}} = 1500$

$$\%\text{ delivered to } R_o = \frac{625}{1500}(100) = 41.67\%$$