(12.56)

$$\begin{aligned}
I_L(s)/K &= \frac{V(s)/K}{sL+R+\frac{1}{sC}} \\
&= \frac{\frac{1}{(s+a)^2}}{sL+R+\frac{1}{sC}} \\
&= \frac{1}{(sL+R+\frac{1}{sC})}\frac{1}{(s+a)^2} \\
&= \frac{\frac{s}{L}}{(s^2+s\frac{R}{L}+\frac{1}{LC})}\frac{1}{(s+a)^2} \\
&= \frac{100s}{(s^2+s(1500)+10^6)}\frac{1}{(s+100)^2} \\
&= \frac{100s}{(s+750-661j)(s+750+661j)}\frac{1}{(s+100)^2} \\
&= \frac{K_1}{(s+750-661j)}+\frac{K_2}{(s+750+661j)}+\frac{K_3}{(s+100)^2}+\frac{K_4}{s+100}
\end{aligned}$$

Now, use partial fraction decomposition to find K_1, K_2, K_3, and K_4.

$$\frac{100s}{(s+750-661j)(s+750+661j)}\frac{1}{(s+100)^2} = \frac{K_1}{(s+750-661j)}+\frac{K_2}{(s+750+661j)}+\frac{K_3}{(s+100)^2}+\frac{K_4}{s+100}$$

To find K_1, multiply both sides of the equation (above) by $(s+750-661j)$

$$\frac{100s}{(s+750+661j)}\frac{1}{(s+100)^2} = K_1+\frac{(K_2)(s+750-661j)}{(s+750+661j)}+\frac{(K_3)(s+750-661j)}{(s+100)^2}+\frac{(K_4)(s+750-661j)}{s+100}$$

Now evaluate at $s=-750+661j$ to get K_1.

$$K_1 = -6.69\times10^{-5}+5.706\times10^{-5}j$$

$K_2 = K_1^*$, so

$$K_2 = -6.69\times10^{-5}-5.706\times10^{-5}j$$

To find K_3, multiply both sides of the equation by $(s+100)^2$ and evaluate it at $s=-100$.

$$\begin{aligned}
\frac{100(-100)}{(-100)^2+1500(-100)+10^6} &= K_3 \\
11.6279\times10^{-3} &= K_3
\end{aligned}$$

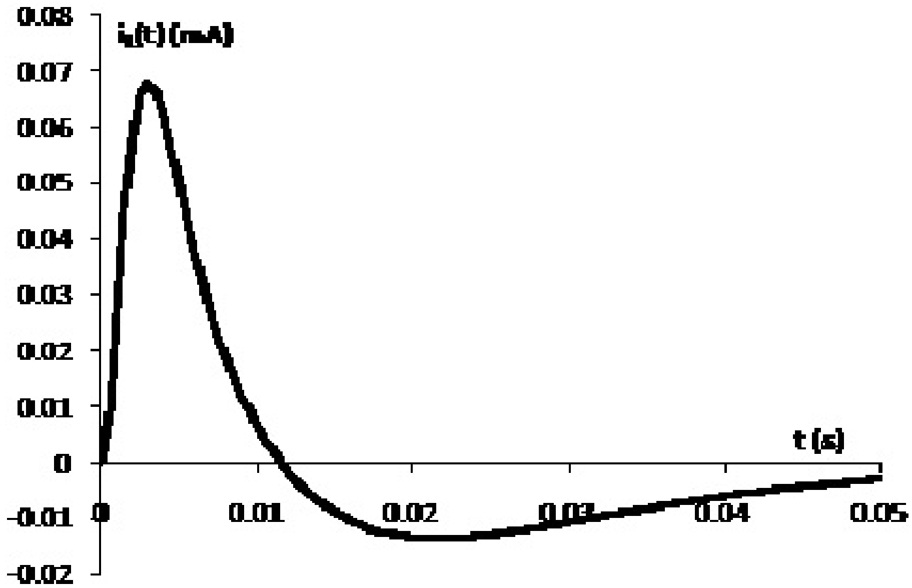
To find K_4, multiply both sides of equation by $(s+100)^2$, take the derivative, then evaluate at $s=100$.

$$\begin{aligned}
\left[\frac{(s^2+1500s+10^6)(100)-(100s)(2s+1500+10^6)}{(s^2+1500s+10^6)^2}\right]_{s=-100} &= K_4 \\
113.86\times10^{-6} &= K_4
\end{aligned}$$

So we have

$$\begin{aligned}
F(s) &= \frac{-6.69\times10^{-5}+5.706\times10^{-5}j}{(s+750-661j)}+\frac{-6.69\times10^{-5}-5.706\times10^{-5}j}{(s+750+661j)}+\frac{11.6279\times10^{-3}}{(s+100)^2}+\frac{113.86\times10^{-6}}{s+100} \\
i_L(t)/K &= [2(87.9)\times10^{-6}e^{-750t}\cos(661t+139.3^\circ)-0.11636te^{-100t}+115.12e^{-100t}]u(t) \\
i_L(t) &= K[2(87.9)\times10^{-6}e^{-750t}\cos(661t+139.3^\circ)-0.11636te^{-100t}+115.12e^{-100t}]u(t)
\end{aligned}$$

This equation is showin in the following figure. We will find the maximum graphically. From the equation of $i_L(t)$, we can see that as t approaches infinity, $i_L(t)$ goes to zero, so we can be sure the maximum value in the graph is the one we want.



The maximum value of the inductor current is 0.068K mA. Therefore,

$$K = \frac{40}{0.068} = 588.$$

(3.) (a.) Using polynomial long division and partial fraction decomposition,

$$\begin{aligned} F(s) &= \frac{s^3 + 5s^2 + 7s + 12}{s^2 + 3s + 2} \\ &= s + 2 + \frac{-s + 8}{s^2 + 3s + 2} \\ &= s + 2 + \frac{-s + 8}{(s + 2)(s + 1)} \\ &= s + 2 + \frac{K_1}{s + 1} + \frac{K_2}{s + 2} \end{aligned}$$

Now, we'll use partial fraction decomposition to find K_1 and K_2.

$$\frac{-s + 8}{(s + 2)(s + 1)} = \frac{K_1}{s + 1} + \frac{K_2}{s + 2}$$

To find K_1, multiply both sides by $(s + 1)$ to get:

$$\frac{-s + 8}{s + 2} = K_1 + \frac{(K_2)(s + 1)}{s + 2}$$

Evaluate at $s = -1$ to find K_1.

$$K_1 = 9$$

Do similar computation to find K_2, multiply both sides by $s + 2$, and evaluate at $s = -2$.

$$K_2 = -10.$$

And

$$F(s) = s + 2 + \frac{9}{s + 1} - \frac{10}{s + 2}.$$

Now, use the Laplace Transform table to find the inverse transform of $F(s)$,

$$\begin{aligned} f(t) &= \mathcal{L}^{-1}\{F(s)\} \\ &= \mathcal{L}^{-1}\{s + 2 + \frac{9}{s+1} - \frac{10}{s+2}\} \\ &= \frac{d\delta(t)}{ds} + 2\delta(t) + (9e^{-t} - 10e^{-2t})u(t) \end{aligned}$$

(b.)

$$\begin{aligned} F(s) &= \frac{2s^2 - 4s + 2}{(s+2)^3} \\ &= \frac{2(s-1)^2}{(s+2)^3} \\ &= \frac{K_1}{(s+2)^3} + \frac{K_2}{(s+2)^2} + \frac{K_3}{s+2} \end{aligned}$$

Solve for K_1, K_2, and K_3 in the following equation using partial fraction decomposition.

$$\frac{2(s-1)^2}{(s+2)^3} = \frac{K_1}{(s+2)^3} + \frac{K_2}{(s+2)^2} + \frac{K_3}{s+2}$$

To find K_1, multiply both sides of above equation by $(s+2)^3$, evaluate at $s = -2$.

$$\begin{aligned} [2(s-1)^2]_{s=-2} &= [K_1 + K_2(s+2) + K_3(s+2)^2]_{s=-2} \\ K_1 &= 18 \end{aligned}$$

To find K_2, multiply both sides of the equation by $(s+2)^3$, take derivative of both sides with respect to s, then evaluate at $s = -2$.

$$\begin{aligned} \frac{d}{ds}[2(s-1)^2]_{s=-2} &= \frac{d}{ds}[K_1 + K_2(s+2) + K_3(s+2)^2]_{s=-2} \\ [4(s-1)]_{s=-2} &= [K_2 + 2K_3(s+2)]_{s=-2} \\ K_2 &= -12 \end{aligned}$$

Find K_3, plug in K_2 to the derivative computed in previous step.

$$\begin{aligned} [4(s-1)]_{s=1} &= [-12 + 2K_3(s+2)]_{s=1} \\ K_3 &= 2 \end{aligned}$$

So we have

$$F(s) = \frac{18}{(s+2)^3} - \frac{12}{(s+2)^2} + \frac{2}{s+2}$$

Now, using the Laplace Transform table,

$$f(t) = [18t^2e^{-2t} - 12te^{-2t} + 2e^{-2t}]u(t)$$

(c.)

$$\begin{aligned} F(s) &= \frac{4s+2}{(s+1)(s+2)^2} \\ &= \frac{K_1}{s+1} + \frac{K_2}{(s+2)^2} + \frac{K_3}{s+2} \end{aligned}$$

Solve for K_1, K_2, and K_3 in the following equation using partial fraction decomposition.

$$\frac{4s+2}{(s+1)(s+2)^2} = \frac{K_1}{s+1} + \frac{K_2}{(s+2)^2} + \frac{K_3}{s+2}$$

First, solve for K_1. Multiply both sides of the above equation by $(s+1)$ to get:

$$\frac{4s+2}{(s+2)^2} = K_1 + \frac{(K_2)(s+1)}{(s+2)^2} + \frac{(K_3)(s+1)}{s+2}$$

Now evaluate at $s=-1$ to get $K_1 = -2$.
To solve for K_2, multiply both sides by $(s+2)^2$ to get:

$$\frac{4s+2}{(s+1)} = \frac{K_1(s+2)^2}{s+1} + K_2 + K_3(s+2)$$

Evaluating at $s=-2$, we see that $K_2 = 6$.
To solve for K_3, multiply both sides by $(s+2)^2$ to get:

$$\frac{4s+2}{(s+1)} = \frac{K_1(s+2)^2}{s+1} + K_2 + K_3(s+2)$$

Now take the derivative with respect to s of both sides,

$$\frac{(s+1)(4)-(4s+2)}{(s+1)^2} = \frac{(s+1)(2K_1(s+2)) - (K_1(s+2)^2)}{(s+1)^2} + K_3$$

Now evaluate at $s=-2$ to get: $K_3 = 2$. So

$$F(s) = \frac{-2}{s+1} + \frac{6}{(s+2)^2} + \frac{2}{s+2}$$

Using the Laplace Transform table,

$$f(t) = [-2e^{-t} + 6te^{-2t} + 2e^{-2t}]u(t)$$

(d.)

$$\begin{aligned} F(s) &= \frac{s+3}{2s^2+6s+5} \\ &= \frac{1}{2}\frac{s+3}{(s+\frac{3}{2}+\frac{j}{2})(s+\frac{3}{2}-\frac{j}{2})} \\ &= \frac{K_1}{s+\frac{3}{2}-\frac{j}{2}} + \frac{K_2}{s+\frac{3}{2}+\frac{j}{2}} \end{aligned}$$

So, do partial fraction decomposition to find K_1 and K_2.

$$\frac{1}{2}\frac{s+3}{(s+\frac{3}{2}+\frac{j}{2})(s+\frac{3}{2}-\frac{j}{2})} = \frac{K_1}{s+\frac{3}{2}-\frac{j}{2}} + \frac{K_2}{s+\frac{3}{2}+\frac{j}{2}}$$

To find K_1, multiply both sides of the above equation by $(s+\frac{3}{2}-\frac{j}{2})$, and evaluate at $s=-\frac{3}{2}+\frac{j}{2}$.

$$\begin{aligned} \frac{1}{2}\left[\frac{s+3}{s+\frac{3}{2}-\frac{j}{2}}\right]_{s=-\frac{3}{2}+\frac{j}{2}} &= \left[K_1 + \frac{(K_2)(s+\frac{3}{2}-\frac{j}{2})}{s+\frac{3}{2}+\frac{j}{2}}\right]_{s=-\frac{3}{2}+\frac{j}{2}} \\ \frac{1}{2}\frac{\frac{3}{2}+\frac{j}{2}}{j} &= K_1 \\ \frac{1}{4}-\frac{3j}{4} &= K_1 \end{aligned}$$

Since K_1 is a complex number, we know $K_2 = K_1^*$. So $K_2 = \frac{1}{4} + \frac{3j}{4}$. So we have the following,

$$F(s) = \frac{\frac{1}{4} - \frac{3j}{4}}{s + \frac{3}{2} - \frac{j}{2}} + \frac{\frac{1}{4} + \frac{3j}{4}}{s + \frac{3}{2} + \frac{j}{2}}$$

Now, using the Laplace Transform tables, we are able to compute $f(t)$ once we know $|K_1|$ and θ.

$$\begin{aligned} |K_1| &= \frac{\sqrt{10}}{4} \approx 0.791 \\ \theta &= \tan^{-1}(-3) = -71.57^\circ \\ f(t) &= 1.58e^{-3t/2} \cos\left(\frac{t}{2} + -71.57^\circ\right) u(t) \end{aligned}$$

(e.)

$$\begin{aligned} F(s) &= \frac{3s+2}{(s^2+2s+5)^2} \\ &= \frac{3s+2}{(s+1-2j)^2(s+1+2j)^2} \\ &= \frac{K_1}{(s+1-2j)^2} + \frac{K_2}{s+1-2j} + \frac{K_1^*}{(s+1+2j)^2} + \frac{K_2^*}{s+1+2j} \end{aligned}$$

So, we only need to find K_1 and K_2. Do this using partial fraction decomposition.

$$\frac{3s+2}{(s+1-2j)^2(s+1+2j)^2} = \frac{K_1}{(s+1-2j)^2} + \frac{K_2}{s+1-2j} + \frac{K_1^*}{(s+1+2j)^2} + \frac{K_2^*}{s+1+2j}$$

To find K_1, multiply both sides of the above equation by $(s+1-2j)^2$, and evaluate at $s = -1+2j$ to get:

$$\begin{aligned} K_1 &= \frac{-1+6j}{(4j)^2} \\ &= \frac{-1+6j}{-16} \\ &= \frac{1}{16} - \frac{3j}{8} \end{aligned}$$

To find K_2, multiply both sides of the equation by $(s+1-2j)^2$, take the derivative with respect to s, and evaluate at $s = -1+2j$.

$$\begin{aligned} \frac{d}{ds}\left[\frac{3s+2}{(s+1+2j)^2}\right]_{s=-1+2j} &= \frac{d}{ds}\left[K_1 + (K_2)(s+1-2j) + \frac{(K_1^*)(s+1-2j)^2}{(s+1+2j)^2} + \frac{(K_2^*)(s+1-2j)^2}{s+1+2j}\right]_{s=-1+2j} \\ K_2 &= \frac{(-16)(3) - (-1+6j)(8j)}{(4j)^4} \\ &= \frac{j}{32} \end{aligned}$$

Therefore,

$$F(s) = \frac{\frac{1}{16} - \frac{3j}{8}}{(s+1-2j)^2} + \frac{\frac{j}{32}}{s+1-2j} + \frac{\frac{1}{16} + \frac{3j}{8}}{(s+1+2j)^2} + \frac{-\frac{j}{32}}{s+1+2j}$$

And now, once we determine $|K_1|$, $|K_2|$, θ_1(angle for K_1, and θ_2(angle for K_2, we can use the

Laplace Transform table to find $f(t)$.

$$\begin{aligned}|K_1| &= 0.380173\\ |K_2| &= 1/32\\ \theta_1 &= -80.5377^\circ\\ \theta_2 &= 90^\circ\\ f(t) &= 2t(0.380173)e^{-t}\cos(2t-80.5377^\circ)u(t) + \frac{1}{16}e^{-t}\cos(2t+90^\circ)u(t)\\ &= 0.761te^{-t}\cos(2t-80.5377^\circ)u(t) + \frac{1}{16}e^{-t}\cos(2t+90^\circ)u(t)\end{aligned}$$

(4.) Initial value Theorem:

$$\lim_{s\to\infty} sF(s) = \lim_{t\to 0^+} f(t)$$

Final value Theorem:

$$\lim_{s\to 0} sF(s) = \lim_{t\to\infty} f(t)$$

(a.) Initial Value Theorem not applicable since impulse equations used.
Final:

$$\lim_{s\to 0^+} \frac{s^4+5s^3+7s^2+12s}{s^2+3s+2} = 0$$

$$\lim_{t\to\infty}\left[\frac{d\delta(t)}{ds} + 2\delta(t) + (9e^{-t} - 10e^{-2t})u(t)\right] = 0$$

(b.) Initial:

$$\lim_{s\to\infty} \frac{2s^3-4s^2+2s}{(s+2)^3} = \lim_{s\to\infty} \frac{2-\frac{4}{s}+\frac{2}{s^2}}{1+\frac{6}{s}+\frac{12}{s^2}+\frac{8}{s^2}} = 2$$

$$\lim_{t\to 0^+}\left[(18t^2e^{-2t} - 12te^{-2t} + 2e^{-2t})u(t)\right] = 0 - 0 + 2 = 2$$

Final:

$$\lim_{s\to 0} \frac{2s^3-4s^2+2s}{(s+2)^3} = 0/8 = 0$$

$$\lim_{t\to\infty}\left[(18t^2e^{-2t} - 12te^{-2t} + 2e^{-2t})u(t)\right] = 0$$

(c.) Initial:

$$\lim_{s\to\infty} \frac{4s^2+2s}{(s+1)(s+2)^2} = 0$$

$$\lim_{t\to 0^+}[-2e^{-t} + 6te^{-2t} + 2e^{-2t}]u(t) = -2 + 0 + 2 = 0$$

Final:

$$\lim_{s\to 0} \frac{4s^2+2s}{(s+1)(s+2)^2} = 0$$

$$\lim_{t\to\infty}[-2e^{-t} + 6te^{-2t} + 2e^{-2t}]u(t) = 0$$

(d.) Initial:

$$\lim_{s\to\infty} \frac{s^2+3s}{2s^2+6s+5} = 1/2$$

$$\lim_{t\to 0^+} \left[1.58e^{-3t/2}\cos\left(\frac{t}{2} - 71.57^\circ\right)u(t)\right] = 1/2$$

Final:

$$\lim_{s\to 0} \frac{s^2+3s}{2s^2+6s+5} = 0$$

$$\lim_{t\to\infty} \left[1.58e^{-3t/2}\cos\left(\frac{t}{2} - 71.57^\circ\right)u(t)\right] = 0$$

(e.) Initial:

$$\lim_{s\to\infty} \frac{3s^2+2s}{(s^2+2s+5)^2} = 0$$

$$\lim_{t\to 0^+} \left[0.761te^{-t}\cos(2t - 80.5377^\circ)u(t) + \frac{1}{16}e^{-t}\cos(2t+90^\circ)u(t)\right] = 0$$

Final:

$$\lim_{s\to 0} \frac{3s^2+2s}{(s^2+2s+5)^2} = 0$$

$$\lim_{t\to\infty} \left[0.761te^{-t}\cos(2t - 80.5377^\circ)u(t) + \frac{1}{16}e^{-t}\cos(2t+90^\circ)u(t)\right] = 0$$