EE233 HW7 Solution

Nov. 16th

Due Date: Nov. 23rd

1. Use a 500nF capacitor to design a low pass passive filter with a cutoff frequency of 50 krad/s.
   (a) Specify the cutoff frequency in hertz.

$$f_c = \frac{\omega_c}{2\pi} = \frac{50000}{2\pi} = 7957\,Hz$$

   (b) Specify the value of the filter resistor.

$$\frac{1}{RC} = 50000, so\, R = \frac{1}{(50\times10^3)(0.5\times10^{-6})} = 40\Omega$$

   (c) Assume the cutoff frequency cannot increase by more than 5%. What is the smallest value of load resistance that can be connected across the output terminal of filter?

With a load resistor added in parallel with the capacitor the transfer function becomes

$$H(s) = \frac{R_L\|(1/sC)}{R + R_L\|(1/sC)} = \frac{R_L/sC}{R[R_L + (1/sC)] + R_L/sC}$$

$$= \frac{R_L}{RR_LsC + R + R_L} = \frac{1/RC}{s + [(R+R_L)/RR_LC]}$$

$$|H(j\omega)| = \frac{(1/RC)}{\sqrt{\omega^2 + [(R+R_L)/RR_LC]^2}}$$

$|H(j\omega)|$ is maximum at $\omega = 0$.

$$|H(j\omega)|_{\max} = \frac{R_L}{R + R_L}$$

$$|H(j\omega_c)| = \frac{R_L}{\sqrt{2}(R + R_L)} = \frac{(1/RC)}{\sqrt{\omega_C^2 + [(R+R_L)/RR_LC]^2}}$$

$$\therefore\ \omega_c = \frac{R + R_L}{RR_LC} = \frac{1}{RC}(1 + (R/R_L))$$

$$\therefore\ \frac{R}{R_L} = 0.05 \qquad \therefore\ R_L = 20R = 800\,\Omega$$

   (d) If the resistor found in part c is connected across the output terminals, what is the magnitude of $H(j\omega)$ when ω=0?

$$H(j0)=\frac{R_L}{R+R_L}=\frac{800}{840}=0.9524$$

2. Design a passive RC high pass filter (like the figure below) with a cutoff frequency of 500Hz using 220pF capacitor.

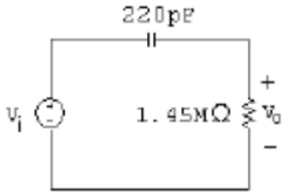
(a) What is the cutoff frequency in rad/s?

$$\omega_c = 2\pi(500) = 3141.59\ rad/s$$

(b) What is the value of the resistor?

$$\omega_c = \frac{1}{RC}\ so, R=\frac{1}{\omega_c C}=\frac{1}{(3141.59)(220\times10^{-12})}=1.45M\Omega$$

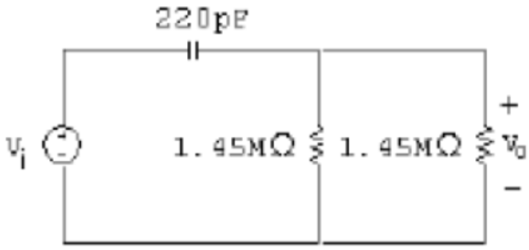
(c) Draw your circuit, labeling the component values and output voltage.



(d) What is the transfer function of the filter in part c?

$$H(s)=\frac{V_o(s)}{V_i(s)}=\frac{R}{R+1/sC}=\frac{s}{s+1/RC}=\frac{s}{s+3141.59}$$

(e) If the filter in part c is loaded with a resistor whose value is the same as the resistor in part b, what is transfer function of this loaded filter?



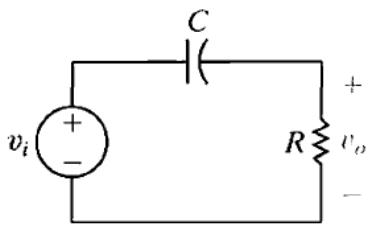
$$H(s)=\frac{V_o(s)}{V_i(s)}=\frac{R\parallel R_L}{R\parallel R_L+1/sC}=\frac{s}{s+\left(\frac{R+R_L}{R_L}\right)1/sC}=\frac{s}{s+2(3141.59)}$$

(f) What is the cutoff frequency of the loaded filter from part e?

$$\omega_c = 2(3141.59) = 6283.19\ rad/s$$

(g) What is the gain of the pass band of the loaded filter from part e?

$$H(\infty)=1$$



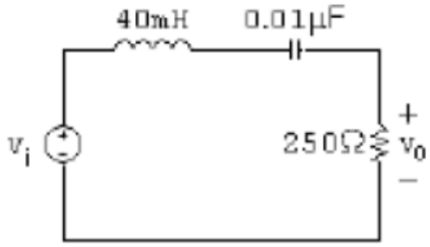
3. Design a series RLC bandpass filter structured (like figure below) with quality factor of 8 and a center frequency of 50 krad/s, using 0.01 μF .

(a) Draw your circuit, labeling the components values and output voltage.

$$\omega_o = \sqrt{\frac{1}{LC}} \text{ so } L = \frac{1}{\omega_o^2 C} = \frac{1}{(50000)^2(0.01\times10^{-6})} = 40mH$$

$$\beta = \frac{\omega_o}{Q} = \frac{50000}{8} = 6250\, rad/s$$

$$R = L\beta = (40\times10^{-3})(6250) = 250\Omega$$



(b) For the filters in part a, calculate the bandwidth and the values of two cutoff frequencies.

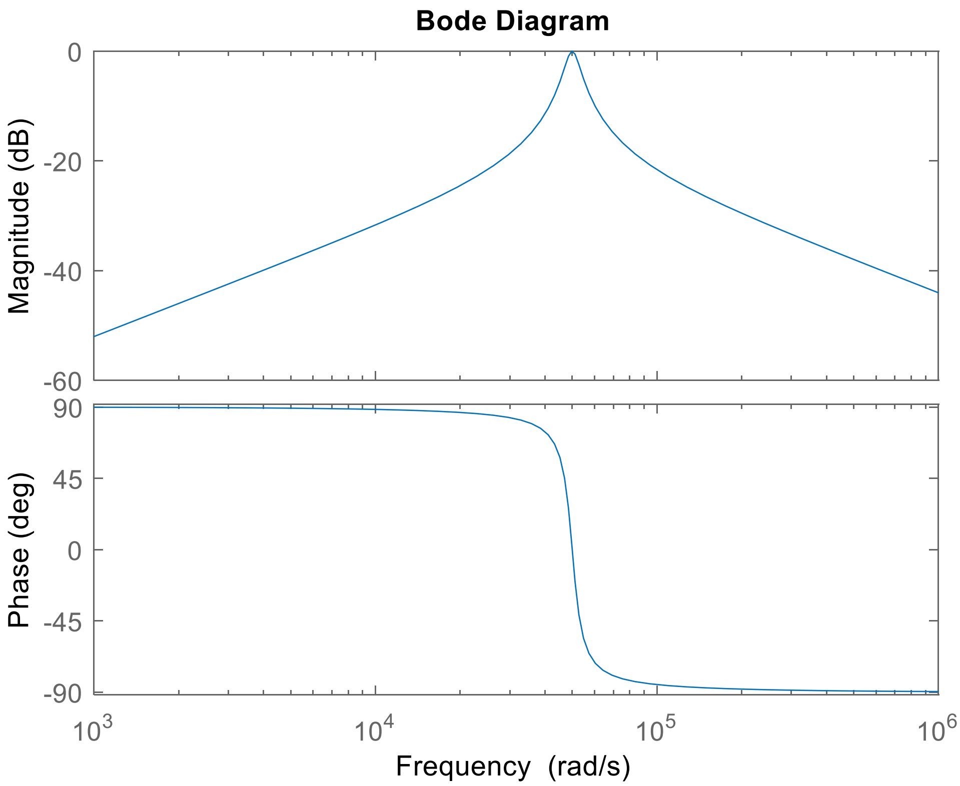
$$\omega_{c1,2} = \pm\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2} = \pm\frac{6250}{2} + \sqrt{\left(\frac{6250}{2}\right)^2 + 50000^2} = \pm 3125 + 50098$$

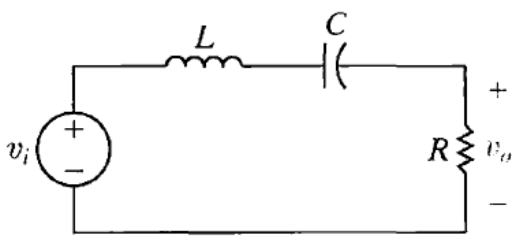
$$\omega_{c1} = 46973,\ \omega_{c2} = 53223$$

(c) Use the resistance, inductance, and capacitance values that you found in part a. Sketch the Bode magnitude plot (magnitude in dB vs $\log_{10}(\omega)$) both by hand (showing asymptotic behavior and value at corner frequencies) and with computer software such as Matlab. Then, compare the results. Also, plot the phase versus frequency and comment how it compares to expectations based on $H(j\omega)$.

Transfer function is:

$$\frac{6250\, s}{s^2 + 6250\, s + 2500000000}$$





4. Design an op-amp based low pass filter with a cutoff frequency of 2500Hz and passband gain of 5 using a 10nF capacitor.

   (a) Draw your circuit, labeling the component values and output voltage.

$$\frac{V_{in}-0}{R_1} = \frac{0-V_{out}}{\frac{\frac{1}{SC}R_2}{R_2+\frac{1}{SC}}}$$

$$\frac{\frac{\frac{1}{SC}R_2}{R_2+\frac{1}{SC}}}{R_1} = \frac{-V_{out}}{V_{in}}$$

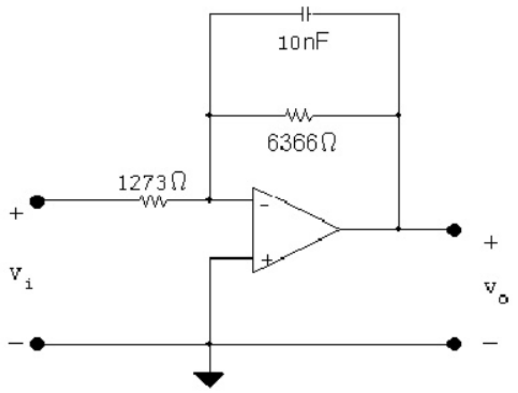
$$\frac{-V_{out}}{V_in} = \frac{\frac{R_2}{SCR_2+1}}{R_1}$$

$$= \left(\frac{R_2}{R_1}\right)\frac{1}{SCR_2+1}$$

$$= \left(\frac{R_2}{R_1}\right)\frac{\frac{1}{CR_2}}{S+\frac{1}{CR_2}}$$

So $\omega_c = \frac{1}{R_2C}$, $f_C = \frac{1}{2\pi R_2C}$, $R_2 = \frac{1}{2\pi R_2C} = 6.366k\Omega$

So for a gain of 5, $\frac{R_2}{R_1} = \frac{6.366k\Omega}{R_1} = 5,\ R_1 = 1.273k\Omega$



(b) If the value of the feedback resistor in the filter is changed but the value of the resistor in the forward path is unchanged, what characteristic of the filter is changed?
Since the resistor and capacitor in the feedback loop didn't change, the cutoff frequency of the filter remains the same. The forward path resistor changes the gain of the filter, the DC gain of the filter is changed inversely proportionally. That is, if we increase the forward path resistor, the gain of the filter decreases.

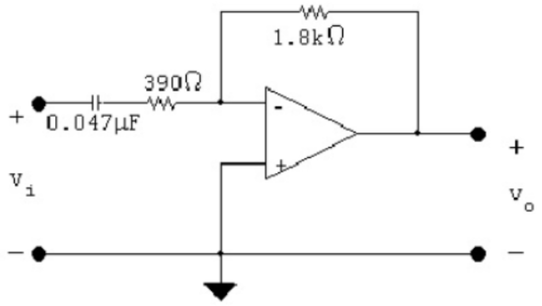
5. Using only three components from 'Common Standard Component Values' in the appendix of textbook, design an active high pass filter (i.e. using one op-amp) with a cutoff frequency and passband gain as close as possible to the specification that are: cutoff frequency of 8kHz and passband gain of 14dB.
   (a) Draw the circuit diagram and label all component values.

$$\frac{V_{in}-0}{\frac{1}{SC_1}+R_1} = \frac{0-V_{out}}{R_2}$$
$$\frac{R_2}{\frac{1}{SC_1}+R_1} = \frac{-V_{out}}{V_in}$$
$$\frac{SC_1R_2}{SC_1R_1+1} = \frac{\left(\frac{R_2}{R_1}\right)S}{S+\frac{1}{R_1C_1}}$$
$$= \left(\frac{R_2}{R_1}\right)\frac{S}{S+\frac{1}{R_1C_1}}$$

So, $\omega_c = \frac{1}{R_1C_1} = 2\pi(8k),\ R_1C_1 = 19.8\times10^{-6},\ 14dB = 5.01$

There are many possible combinations to satisfy the criteria. Let's choose $R_1 = 390\Omega$. Then, $C_1 = 0.047\mu F$. So the cutoff frequency is 8.682Hz. Then, $R_2 = 1950\Omega$. And the closest value is $R_2 = 1800\Omega$. These values give a passband gain of

$20\log\left(\frac{1800}{390}\right) = 13.3dB$.



(b) Calculate the percent error in this new filter's cutoff frequency and passband gain when compared to the given specification.

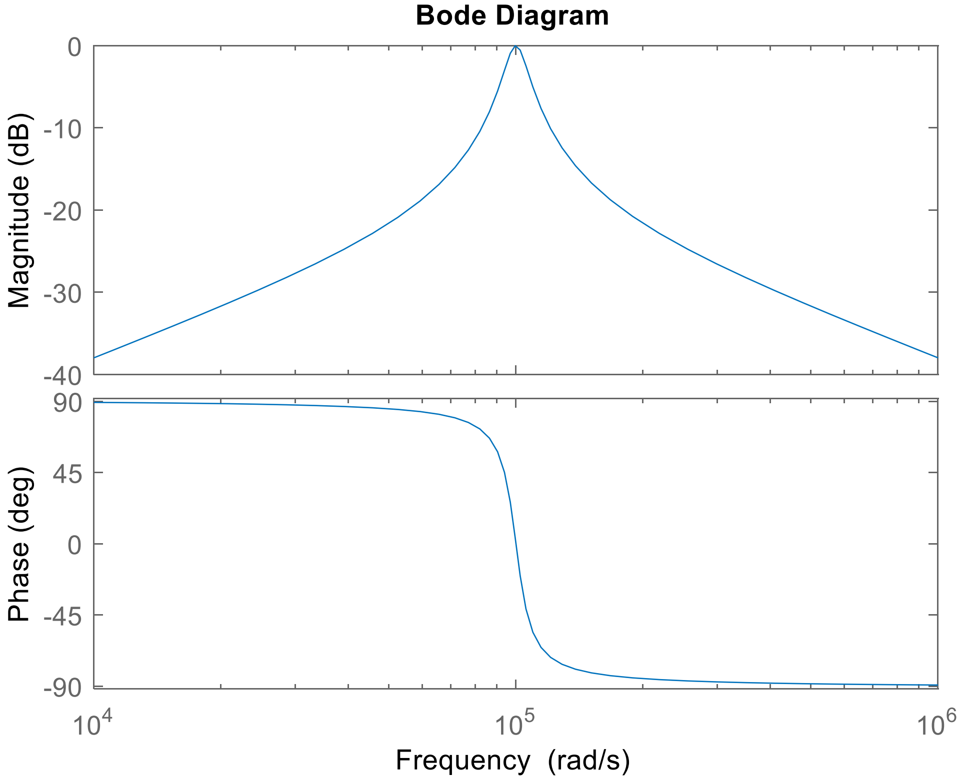
$$\frac{8683.8-8000}{8000}(100) = 8.5\%\text{ error in } f_c$$
$$\frac{13.3-14}{14}(100) = -5.1\%\text{ error in the passband.}$$

6. Let's consider the circuit below.

(a) Sketch the Bode magnitude plot (magnitude in dB vs $\log_{10}(\omega)$) both by hand (showing asymptotic behavior and value at corner frequencies) and with computer software such as Matlab. Then, compare the results. Also, plot the phase versus frequency and comment how it compares to expectations based on $H(j\omega)$.

The transfer function:

$$\frac{12500\,s}{s^2 + 12500\,s + 1.\times 10^{10}}$$



(b) Then, let's scale the bandpass filter in figure below so that the center frequency is 200kHz and the quality factor is still 8 using 2.5nF capacitor. Determine the values of the resistor, the inductor and the two cutoff frequencies of scaled filter.

For circuits of this form, the transfer function will be:

$$H(s) = \frac{\frac{s}{RC}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

Also, $\omega_o = \sqrt{\frac{1}{LC}}, \beta = \frac{1}{RC}, Q = \sqrt{\frac{R^2C}{L}}$

$$\omega_0 = \sqrt{\frac{1}{(10mH)(10nF)}} = 100,000 rad/s$$

$$\beta = \frac{1}{(8k\Omega)(10nF)} = 12,500 rad/s$$

$$K_f = \frac{\omega_0'}{\omega_o} = \frac{2\pi(200kHz)}{100krad/s} = 4\pi$$

$$K_m(small\ k) = \frac{1}{K_f}\frac{C}{C'} = \frac{1}{4\pi}\frac{10nF}{2.5nF} = \frac{1}{\pi}$$

$$R' = K_m R = 8k\Omega\left(\frac{1}{\pi}\right) = 2546.5\Omega$$

$$L' = \frac{K_m}{K_f}L = \frac{\frac{1}{\pi}}{4\pi}10mH = 253.3mG$$

$$\beta' = \frac{\omega_0'}{Q} = \frac{200k(2\pi)}{8} = 157,079.63$$

$$\omega_{c_1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2} = 118.6krad/s$$

$$\omega_{c_2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2} = 1337.7krad/s$$

To check: $\beta_0' = \omega_{c2}' - \omega_{c1}' = 157\,k\,rad\,/\,s = 4\pi\beta = K_m\beta$