EE233 HW9 Solution

Dec. 2nd

Due Date: Dec. 9th

1. Consider the circuit below.

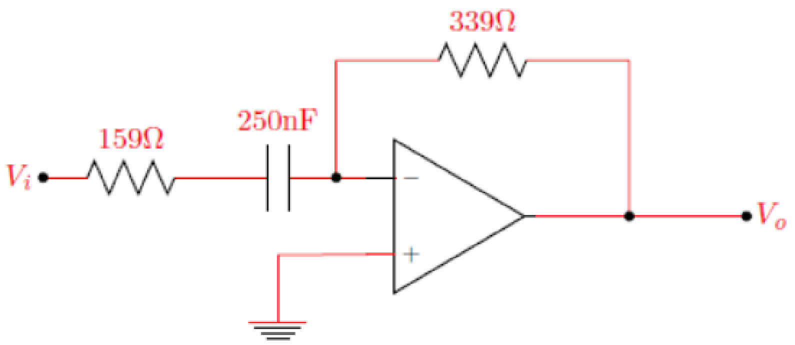
(a) Design a HPF with a passband gain of 8dB and cutoff frequency of 4kHz using 250nF capacitor. Draw the circuit diagram and label all components.

Solution:

$$\omega_c = \frac{1}{R_s C} \rightarrow R_s = \frac{1}{\omega_c C} = \frac{1}{(8000\pi)(250\times 10^{-9})} = 159\Omega$$

$$20\log(k) = 8 \rightarrow k = 2.51$$

$$R_f = kR_s = 399\Omega$$



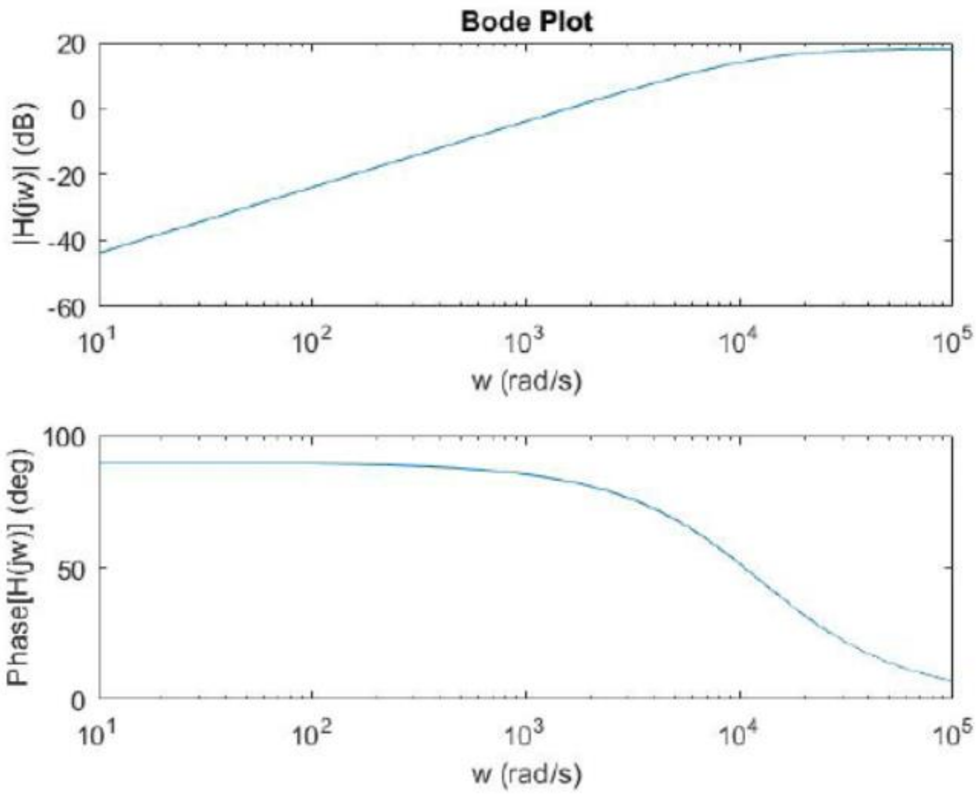
(b) If the value of the feedback resistor in the filter is changed but the value of the resistor in the forward path is unchanged, what characteristic of the filter is changed? Briefly justify your answer.

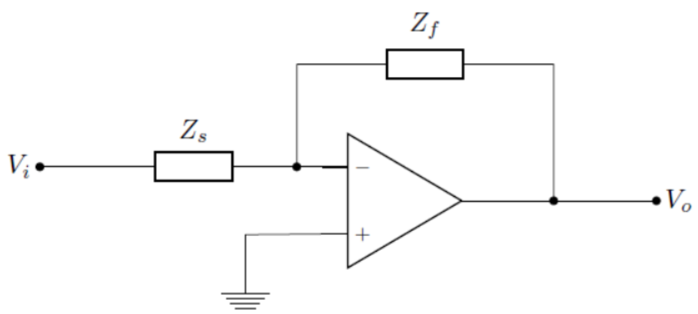
Solution:

The passband gain will change.

(c) Sketch the Bode magnitude and phase plots for the filter.

Solution:





2. Consider the circuit in problem 1.

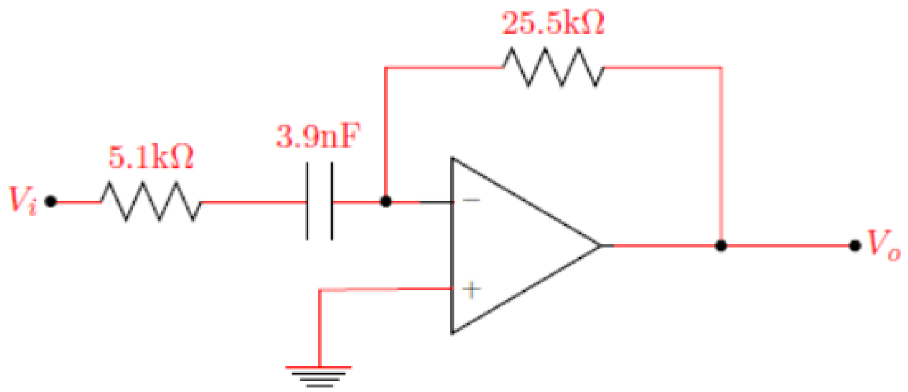
   (a) Design a HPF with a passband gain of 14dB and a cutoff frequency of 8kHz using a 3.9nF capacitor. Draw the circuit diagram and label all components.

   Solution:

$$\omega_c = \frac{1}{R_s C} \rightarrow R_s = \frac{1}{\omega_c C} = \frac{1}{(16000\pi)(3.9\times 10^{-9})} = 5100\Omega$$

$$20\log(k) = 14 \rightarrow k = 5$$

$$R_f = kR_s = 25.5\Omega$$



(b) Using only three components from Appendix H of the textbook (file posted on Canvas), design a HPF with a cutoff frequency and passband gain as close as possible to the specifications above in part (a). Draw the circuit diagram and label all component values.

Solution:

$$R_1 = 5101\Omega \rightarrow R_1 = 4700\Omega$$
$$R_2 = 25.5k\Omega \rightarrow R_2 = 27k\Omega$$

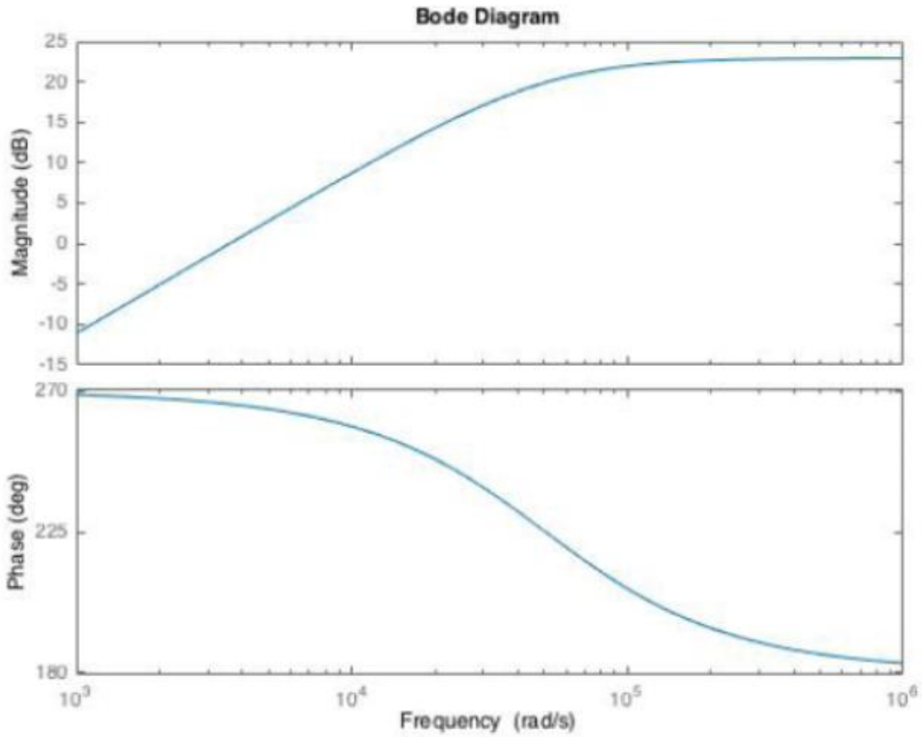
(c) Calculate the percent error in this new filter's cutoff frequency and passband gain when compared to the values specified in part (a).

Solution:

$$\omega_c = \frac{1}{R_1 C} = 54555 rad/s,\ f_c = 8683Hz$$
$$K = \frac{R_2}{R_1} = \frac{27k\Omega}{4.7k\Omega} = 5.74dB$$
$$Freqeucny\ error: 14.8\%$$

(d) Sketch the Bode magnitude and phase plots for both filter designs on the same set of axes.

Solution:



3. Using 10nF capacitors, design an active broadband first-order band-reject filter that has a lower cutoff frequency of 400Hz, an upper cutoff frequency of 4000Hz, and a passband gain of 0dB. Use prototype versions of the LPF and HPF in the design process (use scaling).

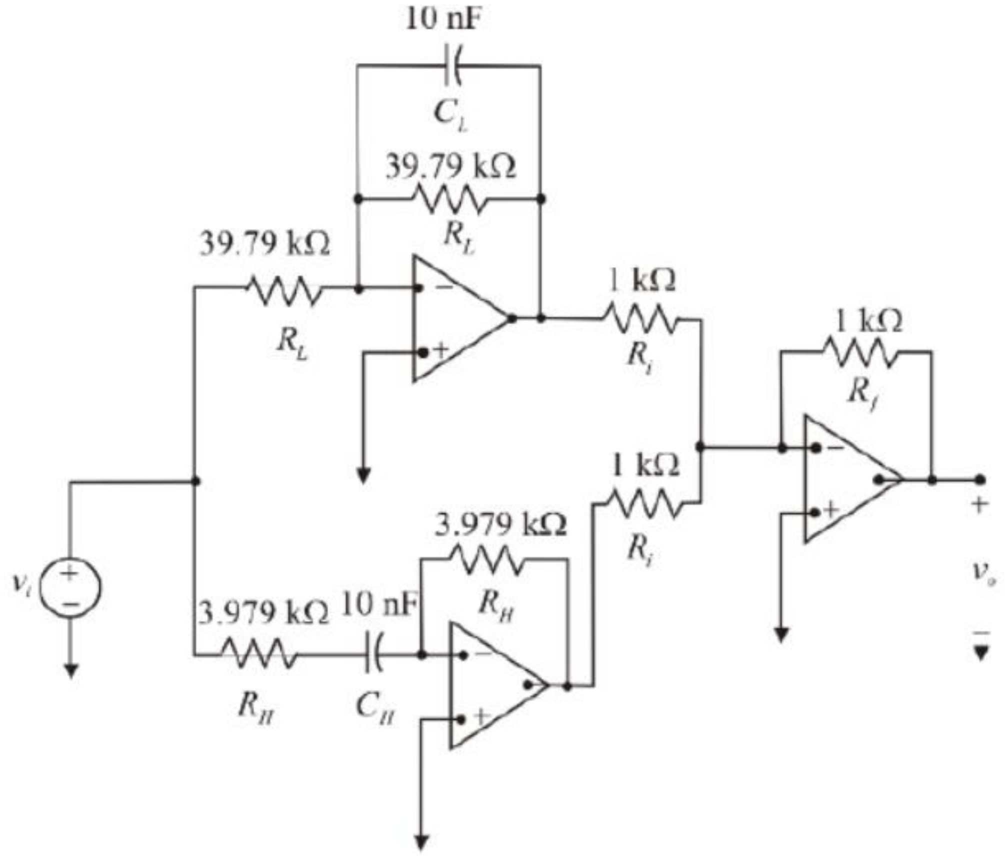
   (a) Draw the circuit diagram of the filter and label all the components.

   Solution:

$$\omega_{c1} = \frac{1}{R_L C_L} \rightarrow R_L = \frac{1}{(800\pi)(10\times10^{-9})} = 39.8k\Omega$$

$$\omega_{c2} = \frac{1}{R_H C_H} \rightarrow R_H = \frac{1}{(8000\pi)(10\times10^{-9})} = 3.98k\Omega$$

$$k = 1 \rightarrow R_s = R_f, so\ R_s = R_f = 1k\Omega$$



(b) Write the transfer function of the scaled filter.

Solution:

$$H(s) = \frac{s^2 + 800\pi s + 16\pi^2 \times 10^5}{(s + 400\pi)(s + 4000\pi)}$$

(c) Use the transfer function to find $H(j\omega_0)$, where ω_0 is the center frequency.

Solution:

$$H(j\omega_0) = \frac{16\pi^2 \times 10^5 - \omega_0{}^2 + j800\pi\omega_0}{16\pi^2 \times 10^5 - \omega_0{}^2 + j4400\pi\omega_0}$$

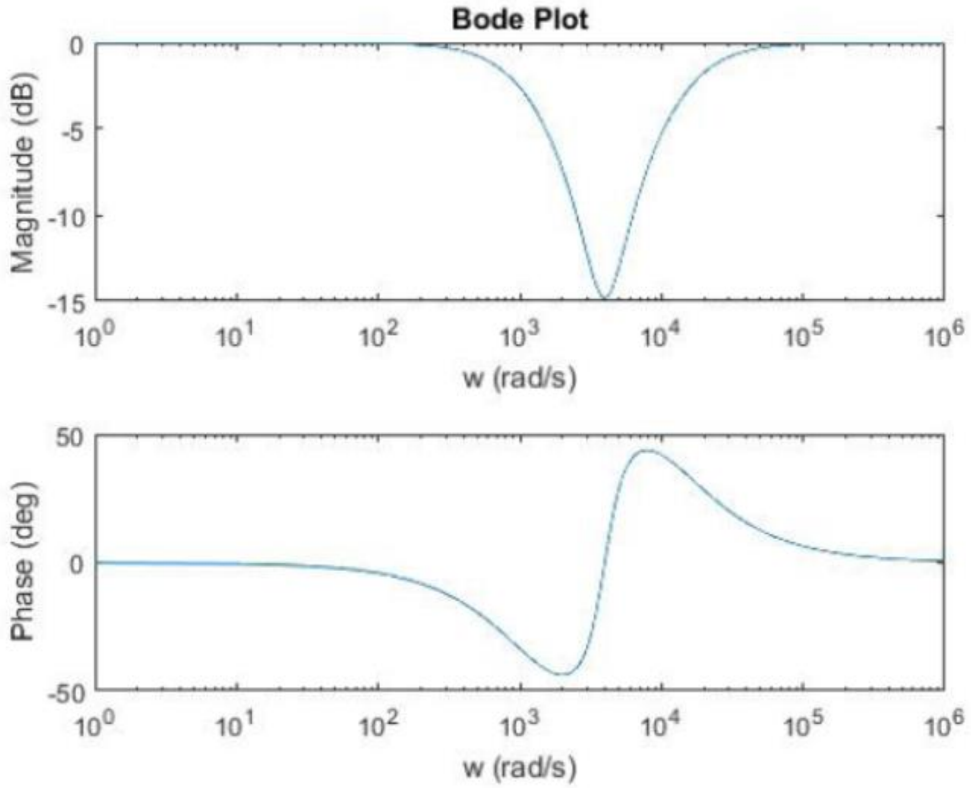
(d) What is the passband gain (in dB) of the filter at the center frequency?

Solution:

$$|H(j\omega_0)| = \frac{800\pi(16\pi^2 \times 10^5)}{4400\pi(16\pi^2 \times 10^5)} = 0.1818 = -14.8dB$$

(e) Sketch the Bode magnitude and phase plots for the filter.

Solution:



4. Design an active LPF with a passband gain of 4dB, a cutoff frequency of 1kHz, and a gain roll-off rate of -60dB/decade.

   (a) Calculate the resistor and capacitor values used in the circuit.

   Solution:

$$-60dB \to 3rd\ order\ \ filter \to \omega_{c3} = \sqrt{2^{1/3} - 1} = 0.5 rad/s$$

$$K_f = \frac{2000\pi}{0.5} = 4000\pi rad/s$$

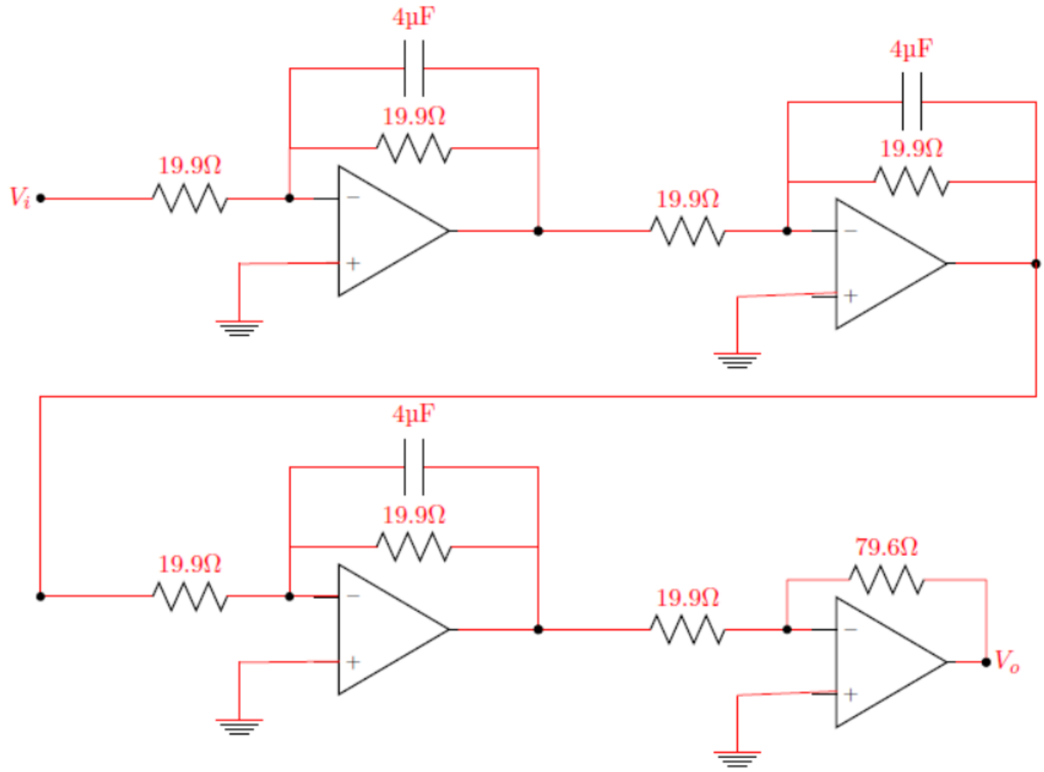
$$Choose\ C = 4\mu F\ \ in\ this\ case,$$

$$K_m = \frac{1}{4000\pi \times 4 \times 10^{-6}} = 19.9 \to R = 19.9\Omega$$

$$k = 4 \to R_f = 79.6\Omega\ \ when\ \ R = R_i = 19.9\Omega$$

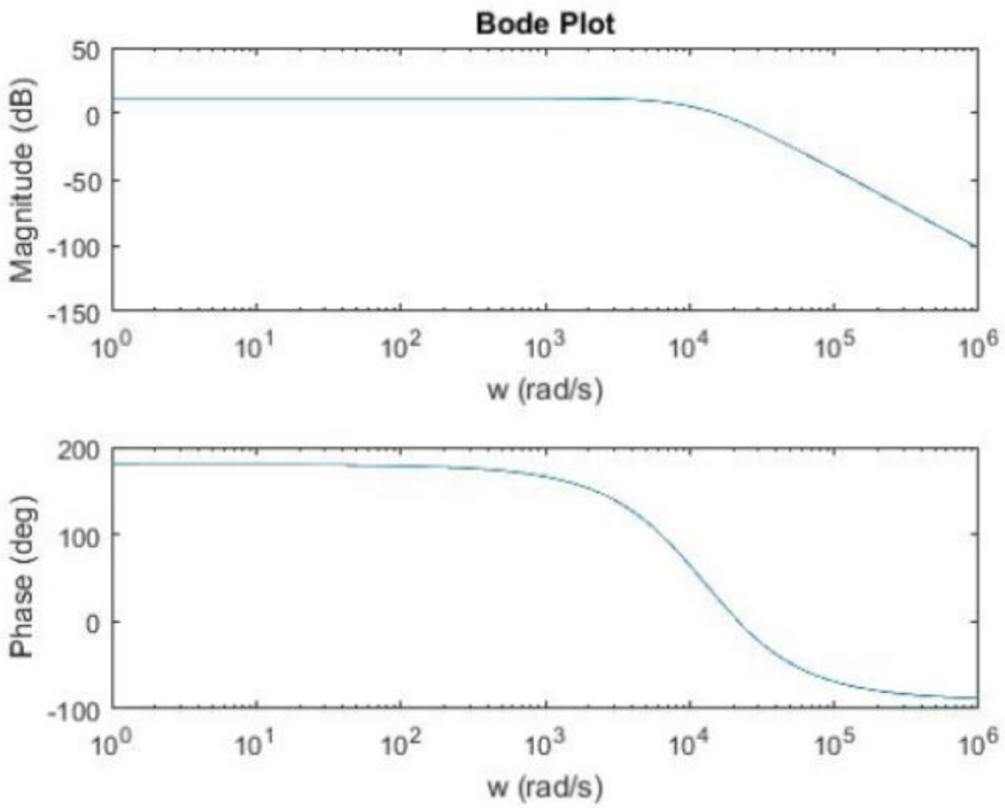
   (b) Draw the circuit diagram of the filter and label all the components.

   Solution:



(c) Sketch the Bode magnitude and phase plots for the filter.

Solution:



5. Your company has developed a new type of loud speaker system that uses very low power to reproduce sound. Like a home stereo system it users 3 independent speakers, the woofer, midrange, and tweeter, to reproduce the low, mid and high frequencies, respectively. The woofer is designed to reproduce frequencies up to 200Hz. The midrange delivers frequencies between 200Hz and 1400Hz. The tweeter is designed for frequencies above 1400Hz.

   You have been given the job of designing a filter system to separate the signal and deliver only the desired frequencies to each speaker. Others on your ream will take care of the rest (buffering of your input signal, level-adjustment to boost the filter output to the appropriate level, line conditioning to prevent RF interference, etc.) Also assume, for this exercise, that phase-distortion is not a problem. Each should filter should be designed separately.

   For the low frequency speaker uses an active low-pass filter with two cascaded first order filters to get sharper roll-off (a steeper cutoff in the frequency plot). The cutoff frequency should be 100Hz and should have unity gain in the passband. Make sure to take into account the shift in cutoff frequency that occurs when you cascade the two sections.

   For the midrange circuit implement a band-pass filter by cascading low-pass, high-pass, and inverting op-amp circuits. The resulting filter should pass frequencies from 100Hz to 1400Hz, and have a maximum gain of 10 (20dB).

   The tweeters your company developed require more precise frequency selection. Implement a 4th order high pass Butterworth filter with a cutoff frequency of 1400Hz.

   For each of the three filters you should provide:
   - A schematic of the op-amp circuit
   - Specifications for all components used
   - Transfer function H(s) for your circuit
   - Magnitude only Bode Plot of the showing the frequency response of your circuit

   Use reasonable values for each component. For ease of grading, use 1 μF capacitors when possible. Assume that you will use ideal op amps.

   Bonus: Simulate your circuits in Multisim and show the frequency response.

#7 (LPF)

```
In[3207]:= Clear[c0, cp, R0, Rp, fc, kf, km, wc]
           c0 = 1;  (* Unscaled C *)
           cp = 10^-6; (* Scaled C *)
           R0 = 1;  (* Unscaled R *)
           fc = 100; (* Desired Corner Frequency *)
           wcp = 2*π*fc; (* Desired Corner Frequencey rad/s *)
           n = 2;  (* Filter order *)
           wc0 = Sqrt[2^(1/n) - 1]  (* Scaled Corner frequency *)
           kf = wcp/wc0 // N (* Frequency scaling factor *)
           km = (1/kf) (c0/cp) // N (* Magnitude scaling factor *)
           Rp = km*R0  (* Scaled resistor value *)
```

Out[3213]= $\sqrt{-1+\sqrt{2}}$

Out[3214]= 976.265

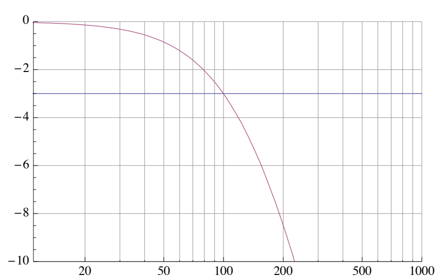
Out[3215]= 1024.31

Out[3216]= 1024.31

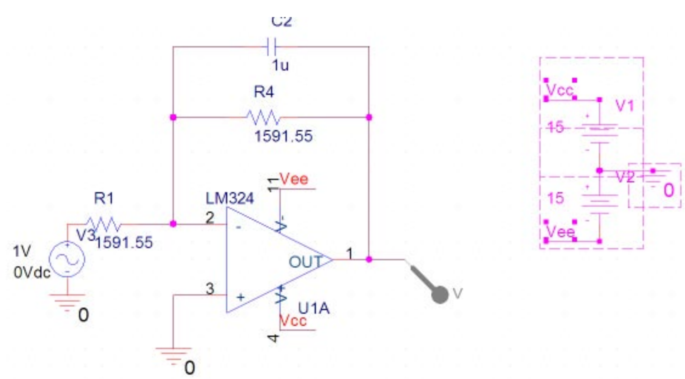
```
In[3217]:= Clear[R, R1, R2, s, c];
           R2 = Rp;  (* LPF resistor R2 *)
           R1 = R2;  (* LPF resistor R1 *)
           c = cp;   (* LPF Capacitor *)
           h1 = (1/(R1*c)) / (s + 1/(R2*c))  (* First order LPF transfer function *)
           s = I*2*π*f; (* s=j*w *)
           h = h1^2; (* Transfer function in dB *)
           hdb = 20 Log[10, Abs[h]];  (*  *)
           LogLinearPlot[{-3, hdb}, {f, 10^-6, 10^3},
            PlotRange -> {{11, 10^3}, {-10, 0}}, GridLines -> Automatic]
```

Out[3220]= $\frac{976.265}{976.265 + s}$

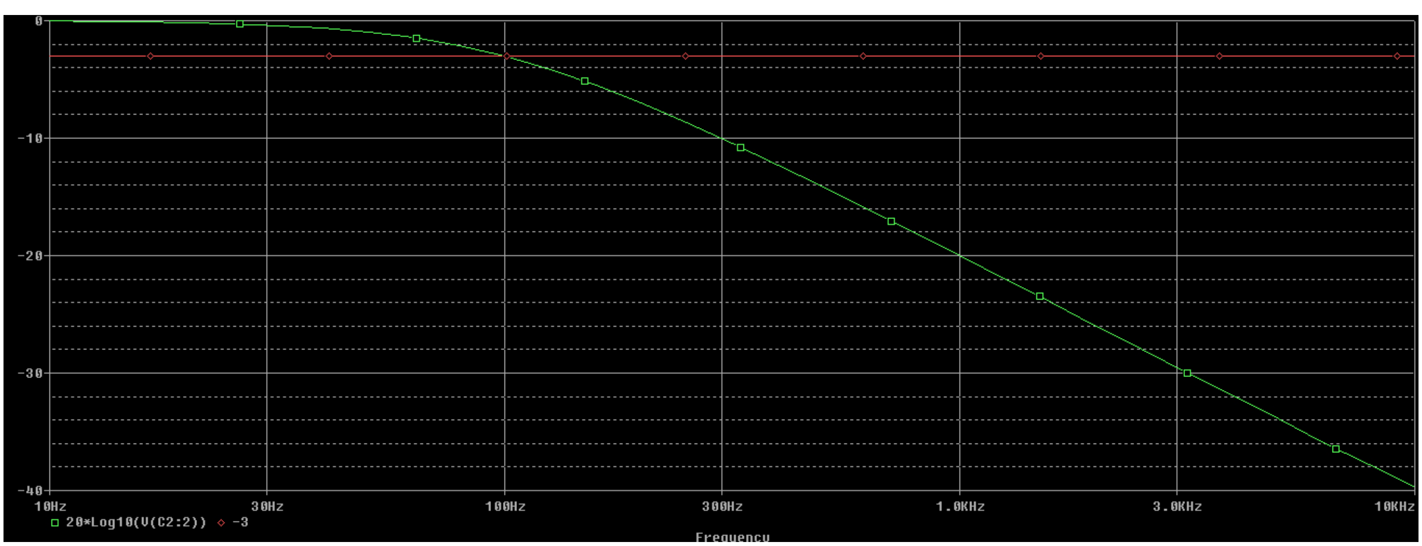
Out[3223]=



Circuit



PSpice Sim



#7 BPF

In[3682]:=
```
Clear[c0, cp, R0, Rp, fc, kf, km, wc]
c0 = 1;  (* Unscaled C *)
cp = 10^-6; (* Scaled C *)
R0 = 1;  (* Unscaled R *)
fc = 1400;  (* Desired Corner Frequency *)
wcp = 2 * π * fc;  (* Desired Corner Frequencey rad/s *)
n = 1;  (* Filter order *)
wc0 = Sqrt[2^(1/n) - 1]  // N(* Scaled Corner frequency *)
kf = wcp/wc0 (* Frequency scaling factor *)
km = 1/kf c0/cp  (* Magnitude scaling factor *)
Rp = km * R0  (* Scaled resistor value *)
```

Out[3686]= 1.

Out[3687]= 8796.46

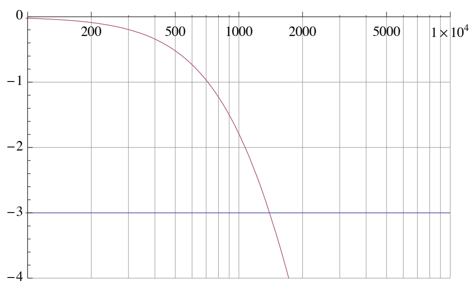
Out[3688]= 113.682

Out[3689]= 113.682

In[3690]:=
```
Clear[R, R1, R2, s, c, f];
R2 = Rp;   (* LPF resistor R2 *)
R1 = R2;   (* LPF resistor R1 *)
c = cp;   (* LPF Capacitor *)
hlp = (1/(R1*c)) / (s + 1/(R2*c))  (* First order LPF transfer function *)
s = I * 2 * π * f; (* s=j*w *)
hdb = 20 Log[10, Abs[hlp]];   (* Transfer function in dB *)
LogLinearPlot[{-3, hdb}, {f, 10^-6, 10^4},
 PlotRange → {{0.1 * 10^3, 10 * 10^3}, {0, -4}}, GridLines → Automatic]
```

Out[3692]= $\frac{8796.46}{8796.46 + s}$

Out[3693]=



In[3694]:=
```
Clear[c0, cp, R0, Rp, fc, kf, km, wc]
c0 = 1;  (* Unscaled C *)
cp = 10^-6; (* Scaled C *)
R0 = 1;  (* Unscaled R *)
fc = 100;  (* Desired Corner Frequency *)
wcp = 2 * π * fc;  (* Desired Corner Frequencey rad/s *)
n = 1;  (* Filter order *)
wc0 = Sqrt[2^(1/n) - 1]  // N(* Scaled Corner frequency *)
kf = wcp/wc0 (* Frequency scaling factor *)
km = 1/kf c0/cp  (* Magnitude scaling factor *)
Rp = km * R0  (* Scaled resistor value *)
```

Out[3698]= 1.

Out[3699]= 628.319

Out[3700]= 1591.55

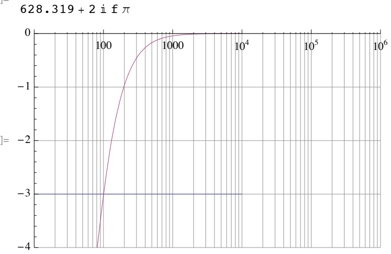
Out[3701]= 1591.55

In[3702]:=
```
Clear[Rh, R1h, R2h, s, ch, f];
R2h = Rp;   (* HPF resistor R2 *)
R1h = R2h;   (* HPF resistor R1 *)
ch = cp;   (* HPF Capacitor *)
h = s / (s + 1/(R1h*ch))   (* First order HPF transfer function *)
s = I * 2 * π * f; (* s=j*w *)
hhp = h (* Transfer function in dB *)
hdb = 20 Log[10, Abs[hhp]];  (*  *)
LogLinearPlot[{-3, hdb}, {f, 10, 10^4}, GridLines → Automatic, PlotRange → {{10, 10^6}, {0, -4}}]
```

Out[3703]= $\frac{s}{628.319 + s}$

Out[3704]= $\frac{2 i f \pi}{628.319 + 2 i f \pi}$

Out[3705]=



In[3754]:=
```
(* design of inverting constant-gain amplifier stage *)
Clear[G, Rf, Ri]
G = 10;
Rf = 10^4;
Ri = Rf/G
hinv = Rf/Ri
```

Out[3757]= 1000

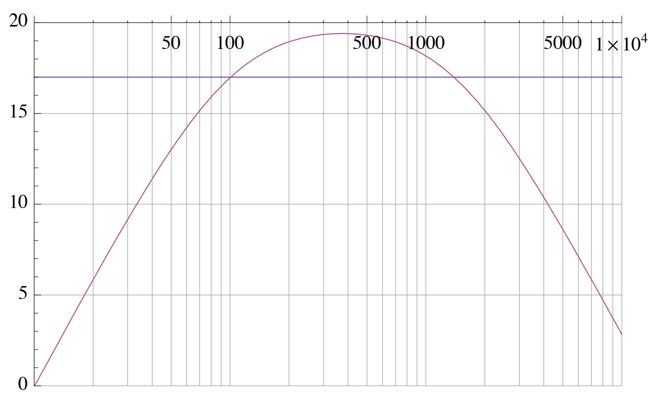
Out[3758]= 10

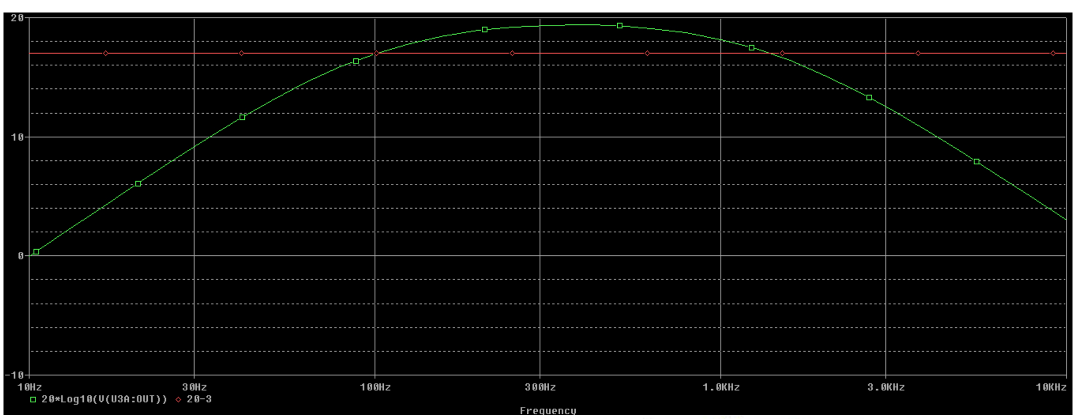
In[3751]:=
```
hbp = hlp * hhp * hinv
hbpdb = 20 Log[10, Abs[hbp]]
LogLinearPlot[{20 - 3, hbpdb}, {f, 10, 10^4},
 PlotRange → {{10, 10^4}, {20, 0}}, GridLines → Automatic]
```

Out[3751]= $$\frac{552\,698.\, i\, f}{(628.319 + 2\, i\, f\, \pi)\,(8796.46 + 2\, i\, f\, \pi)}$$

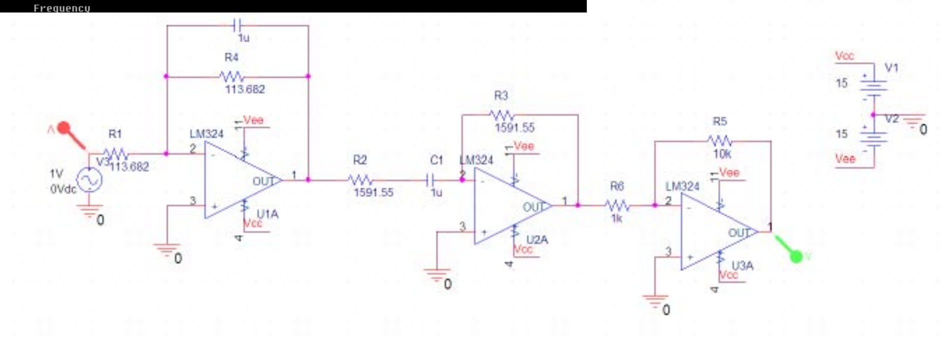
Out[3752]= $$\frac{20\,\mathrm{Log}\left[552\,698.\,\mathrm{Abs}\left[\frac{f}{(628.319+2\,i\,f\,\pi)\,(8796.46+2\,i\,f\,\pi)}\right]\right]}{\mathrm{Log}[10]}$$

Out[3753]=





Circuit and PSpice simulation results.



#7 HPF - 4th order Butterworth

In[4698]:=
```
Clear[c0, cp, R0, Rp, fc, kf, km, wc, n, wcp]
c10 = 2.61; (*Using values from example 15.9*)
c20 = 0.38;
c30 = 1.08;
c40 = 0.924;
fc = 1400;  (* Desired cutoff frequency *)
wcp = 2 * π * fc; (* Desired Corner Frequencey rad/s *)
n = 1;
wc0 = Sqrt[2^(1/n) - 1]  // N(* Scaled Corner frequency *);
kf = wcp/wc0 (* Frequency scaling factor *)
km = 1000;
c1 = c10/(km * kf)  (* Scale example values by kf, km*)
c2 = c20/(km * kf)
c3 = c30/(km * kf)
c4 = c40/(km * kf)
R = 1000 (* R is arbitrary *)
```

Out[4705]= 8796.46

Out[4707]= 2.9671×10^{-7}

Out[4708]= 4.31992×10^{-8}

Out[4709]= 1.22777×10^{-7}

Out[4710]= 1.05042×10^{-7}

Out[4711]= 1000

In[4712]:=
```
(* Deterimine Transfer function *)
Clear[s]
k1 = R * c1
k2 = R * c2
(* TF of first section *)
hb1 = s^2/(s^2 + (2/k1) s + 1/(k1*k2))

k3 = R * c3;
k4 = R * c4;
(* TF of second section *)
hb2 = s^2/(s^2 + (2/k3) s + (1/(k3*k4)))
(* Optional gain stage *)
hgain = 1;
(* TF of complete filter *)
hbutt = hb1 * hb2 * hgain
```

Out[4713]= 0.00029671

Out[4714]= 0.0000431992

Out[4715]= $\frac{s^2}{7.80174 \times 10^7 + 6740.58\, s + s^2}$

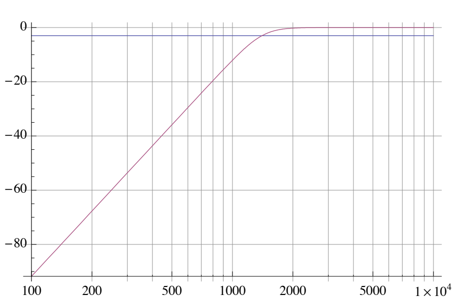
Out[4718]= $\frac{s^2}{7.7539 \times 10^7 + 16\,289.7\, s + s^2}$

Out[4720]= $\frac{s^4}{(7.80174 \times 10^7 + 6740.58\, s + s^2)\,(7.7539 \times 10^7 + 16\,289.7\, s + s^2)}$

In[4721]:=
```
s = I * 2 * π * f
hdb = 20 Log[10, Abs[hbutt]];  (* Transfer function in dB *)
LogLinearPlot[{-3, hdb}, {f, 100, 10^4}, GridLines → Automatic]
```

Out[4721]= 2 i f π

Out[4723]=



In[4768]:= (* As an alternative design using c=1uF *)

$$R1 = k1 / 10^{-6}$$

$$R2 = k2 / 10^{-6}$$

$$R3 = k3 / 10^{-6}$$

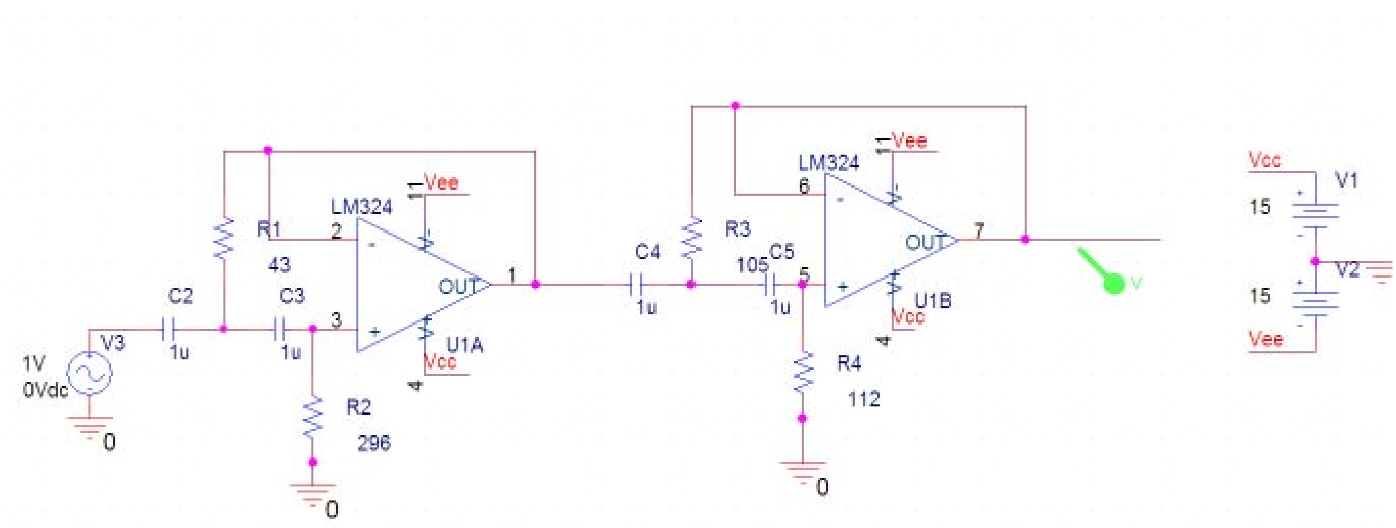
$$R4 = k4 / 10^{-6}$$

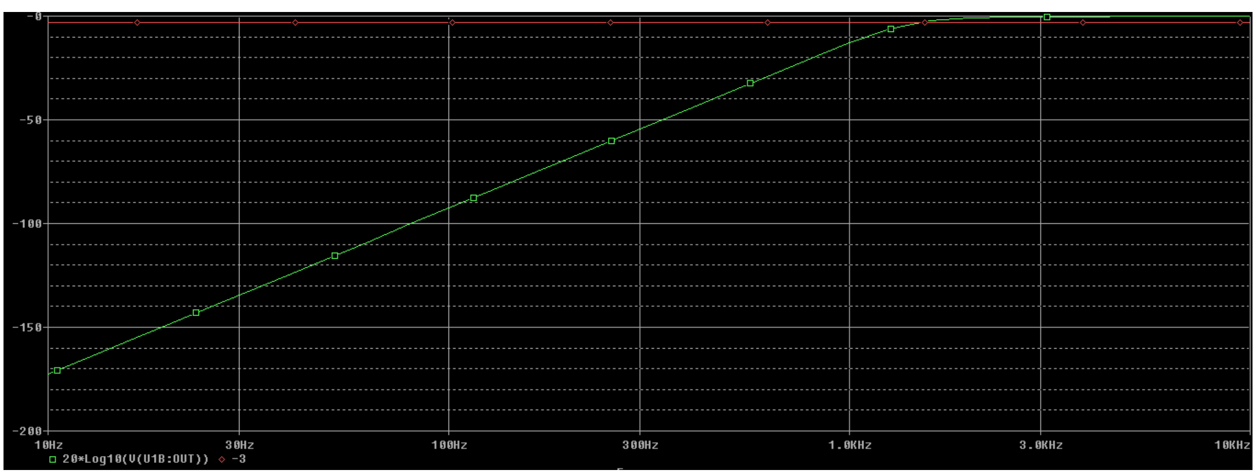
Out[4768]= 296.71

Out[4769]= 43.1992

Out[4770]= 122.777

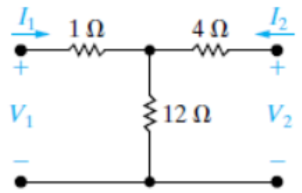
Out[4771]= 105.042





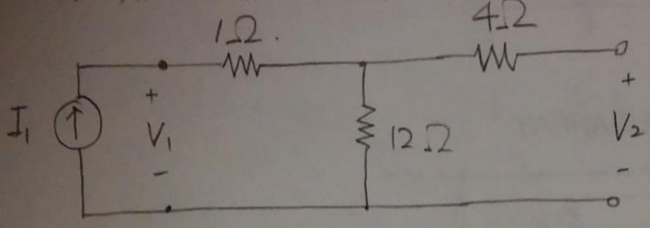
6. Textbook Problem 18.2: Find the 'z' parameters for the circuit shown in Fig. P.18.2.

Figure P18.2



Solution:

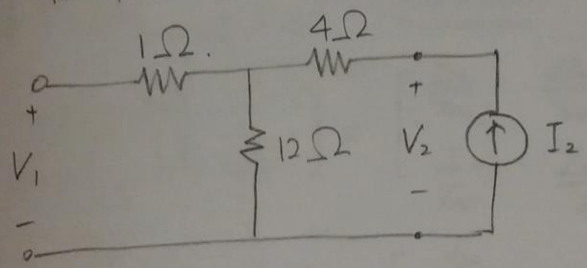
to find Z_{11} & Z_{21}



$$Z_{11} = \frac{V_1}{I_1} = 1 + 12 = 13\,\Omega$$

$$V_2 = I_1 \cdot 12 \rightarrow Z_{21} = \frac{V_2}{I_1} = 12\,\Omega$$

to find Z_{22} and Z_{12}



$$Z_{22} = \frac{V_2}{I_2} = 4 + 12 = 16\,\Omega$$

$$V_1 = I_2 \cdot 12 \rightarrow Z_{12} = \frac{V_1}{I_2} = 12\,\Omega$$

$\therefore$ $Z_{11} = 13\,\Omega$, $Z_{12} = 12\,\Omega$,

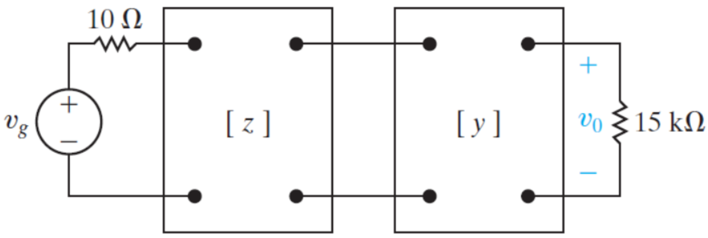
$Z_{21} = 12\,\Omega$, $Z_{22} = 16\,\Omega$

7. Textbook Problem 18.38: The z and y parameters for the resistive two-ports in Fig. P18.38 are given by:

$$z_{11} = \frac{35}{3}\ \Omega; \qquad y_{11} = 200\ \mu S;$$
$$z_{12} = -\frac{100}{3}\ \Omega; \qquad y_{12} = 40\ \mu S;$$
$$z_{21} = \frac{4}{3}\ k\Omega; \qquad y_{21} = -800\ \mu S$$
$$z_{22} = \frac{10}{3}\ k\Omega; \qquad y_{22} = 40\ \mu S;$$

Calculate v_o if v_g is 30mVDC.

Figure P18.38



Solution:

$$\Delta z = z_{11}z_{22} - z_{12}z_{21} = 83.33\times10^{3}$$

$$a' = \begin{bmatrix} a'_{11} & a'_{12} \\ a'_{21} & a'_{22} \end{bmatrix} = \begin{bmatrix} 0.0088 & 62.5 \\ 7.5\times10^{-4} & 2.5 \end{bmatrix}$$

$$\Delta y = y_{11}y_{22} - y_{12}y_{21} = 4\times10^{-8}$$

$$a'' = \begin{bmatrix} a''_{11} & a''_{12} \\ a''_{21} & a''_{22} \end{bmatrix} = \begin{bmatrix} 0.05 & 1250 \\ 5\times10^{-5} & 0.25 \end{bmatrix}$$

Using equations (18.74)-(18.77),

$$a = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 0.0036 & 26.5625 \\ 1.625\times10^{-4} & 1.5625 \end{bmatrix}$$

Using Table 18.2,

$$V_0 = \frac{V_g Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g} = 3.75V$$