Exam 2 Problem 1 Solution

(a) Method 1

$$f(t)=\begin{cases}\frac{2}{5}t, & 0\le t\le 5\\ -\frac{2}{5}t+4, & 5\le t\le 10\\ 0, & otherwise\end{cases}$$

So,

$$\begin{aligned}f(t)&=\frac{2}{5}t[u(t)-u(t-5)]+(-\frac{2}{5}t+4)[u(t-5)-u(t-10)]\\&=\frac{2}{5}tu(t)-\frac{4}{5}tu(t-5)+\frac{2}{5}tu(t-10)+4u(t-5)-4u(t-10)\\&=\frac{2}{5}tu(t)-\frac{4}{5}(t-5)u(t-5)+\frac{2}{5}(t-10)u(t-10)\end{aligned}$$

Then, using the Laplace transform table,

$$F(s)=\frac{2}{5s^2}-\frac{4e^{-5s}}{5s^2}+\frac{2e^{-10s}}{5s^2}$$

Method 2: using the definition of Laplace transform directly

$$F(s)=\int_0^5\frac{2}{5}te^{-st}dt+\int_5^{10}(-\frac{2}{5}t+4)e^{-st}dt=\frac{2}{5s^2}-\frac{4e^{-5s}}{5s^2}+\frac{2e^{-10s}}{5s^2}$$

In order to solve this integration, we have to use the integration by parts.

(b) Version A: $F(s)=\frac{2s^2+5s+9}{s^2+2s+1}=2+\frac{1}{s+1}+\frac{6}{(s+1)^2}$ using Partial Fraction Expansion

Then, we can use inverse Laplace transform table,

$f(t)=2\delta(t)+[e^{-t}+6te^{-t}]u(t)$

Version B: $F(s)=\frac{2s^2+6s+7}{s^2+2s+1}=2+\frac{2}{s+1}+\frac{3}{(s+1)^2}$ using Partial Fraction Expansion

Then, we can use inverse Laplace transform table,

$f(t)=2\delta(t)+[2e^{-t}+3te^{-t}]u(t)$