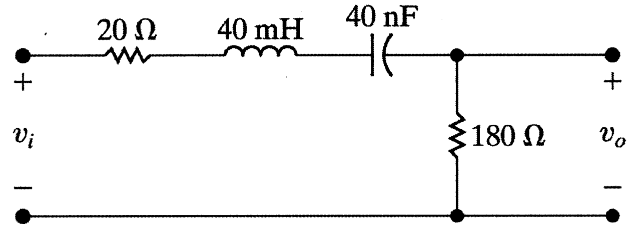
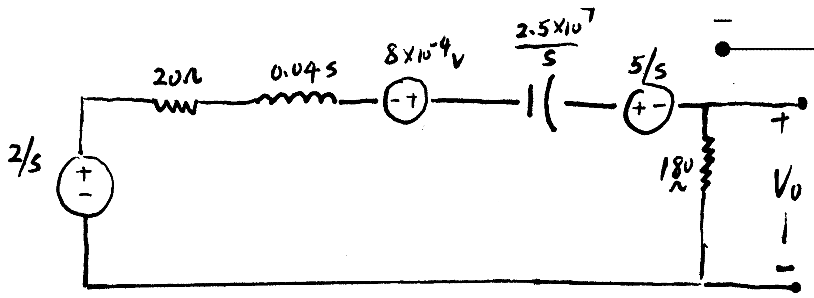
2. In circuit to the right, $v_i(t) = 2\,u(t)$ V, $i_L(0^-) = 20$ mA, and $v_C(0^-) = 5$ V.



(a) Draw the s-domain circuit valid for t > 0. (6)



$$Z_L = sL = 0.04s \qquad Z_C = \frac{1}{sC} = \frac{2.5\times10^7}{s}$$

$$i_L(0^-)\cdot L = 0.02 \times 0.04 = 8\times10^{-4}\,V \qquad \frac{V_C(0^-)}{s} = \frac{5}{s}$$

$$\mathcal{L}\{v_i\} = \mathcal{L}\{2u(t)\} = \frac{2}{s}$$

(b) Find $V_0(s)$. (6)

$$I(s) = \frac{\frac{2}{s} + 8\times10^{-4} - \frac{5}{s}}{20 + 0.04s + 2.5\times10^7/s + 180}$$

$$= \frac{8\times10^{-4} - 3/s}{200 + 0.04s + 2.5\times10^7/s}$$

$$V_0(s) = I(s)\cdot R_0 = \frac{8\times10^{-4} - 3/s}{200 + 0.04s + 2.5\times10^7/s}\cdot 180$$

$$= \frac{0.144s - 540}{0.04s^2 + 200s + 2.5\times10^7}$$

$$= \frac{3.6s - 13500}{s^2 + 5000s + 6.25\times10^8}$$

(c) Check your answer with the initial and final value theorems (compare to the behavior of the time-domain circuit). (6)

Initial value theorem

$$V_0(0^+) = i_L(0^-)\cdot R = 0.02 \times 180 = 3.6V$$

$$\lim_{s\to\infty} sV_0(s) = \lim_{s\to\infty} \frac{3.6s^2 - 13500s}{s^2 + 5000s + 6.25\times10^8} = 3.6$$

Therefore, $\lim_{t\to0} V_0(t) = \lim_{s\to\infty} sV_0(s)$

IVT checks

Final value theorem (FVT)

$$\lim_{t\to\infty} V_0(t) = 0$$

This is because capacitor is open-circuit at $t\to\infty$

$$\lim_{s\to0} sV_0(s) = \lim_{s\to0} \frac{3.6s^2 - 13500s}{s^2 + 5000s + 6.25\times10^8} = 0$$

Therefore, $\lim_{t\to\infty} V_0(t) = \lim_{s\to0} sV_0(s)$

FVT check