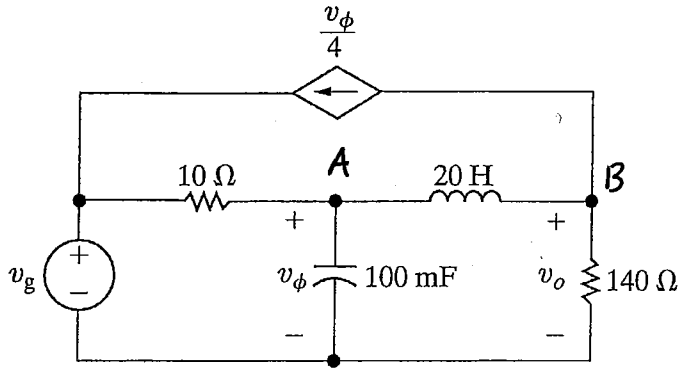
3. In circuit to the right, there's no stored energy at time $t = 0^-$.



a) Calculate the transfer function $H(s) = V_o(s)/V_g(s)$, (8)

In s-domain, $Z_C = \frac{1}{sC} = \frac{10}{s}$, $Z_L = sL = 20s$

At node A: $\frac{V_\phi - V_g}{10} + \frac{V_\phi}{10/s} + \frac{V_\phi - V_o}{20s} = 0$

At node B: $\frac{V_\phi}{4} + \frac{V_o - V_\phi}{20s} + \frac{V_o}{140} = 0 \Rightarrow V_\phi(35s - 7) + V_o(7 + s) = 0$

$V_\phi\left[\frac{1}{10} + \frac{s}{10} + \frac{1}{20s}\right] - V_o\left[\frac{1}{20s}\right] = \frac{V_g}{10}$

$V_\phi = \frac{7+s}{7-35s} V_o = \frac{(7+s)}{(1-5s)7} V_o$

$\left[\frac{(7+s)}{1-5s}\right)\left[2s + 2s^2 + 1\right] - 7\Big] V_o = 14s\, V_g$

$\frac{V_o}{V_g} = \frac{14s(1-5s)}{(7+s)(2s^2+2s+1) - 7(1-5s)} = \frac{14s(1-5s)}{2s^3 + (14+2)s^2 + (14+1+35)s}$

$= \frac{-7(5s-1)}{s^2 + 8s + 25}$

b) What would be the impulse response $h(t)$? (8)

roots: $\frac{-8 \pm \sqrt{8^2 - 4(25)}}{2} = -4 \pm j3$

$h(t) = \mathcal{L}^{-1}\{H(s)\}$

$H(s) = \frac{K}{s+4-j3} + \frac{K^*}{s+4+j3}$

$K = \left.\frac{-35s + 7}{s+4+j3}\right|_{s=-4+j3} = \frac{140 - j105 + 7}{j6} = \frac{49 - j35}{j2}$

$= \frac{7}{2}(-5 - j7)$

$= 30.1 \angle 50.2$

$= 30.1 \angle -129.8$

$h(t) = 2|K| \cos(3t - 129.8) e^{-4t} u(t)$

$= 60.2 \cos(3t - 129.8) e^{-4t} u(t)$

$= -60.2 \cos(3t + 50.2) e^{-4t} u(t)$

$= e^{-4t}[-35\cos(3t) + 49\sin(3t)]$

End of Exam