Exam 2 — EE 233 Spring 2010

The test is closed book, with two sheets of notes and calculators allowed. Show all work. Be sure to state all assumptions made and check them when possible. The number of points per problem are indicated in parentheses. Total of 100 points in 3 problems on 3 pages (4th page has Laplace transform pairs).

1. The Laplace transform of an output voltage is:

\[ V(s) = \frac{5e^{-s}}{(s+1)^2} + \frac{2s+2}{s^2+6s+10} \]

(a) Find \( v(t) \) for \( t > 0^- \). (20)

\[ \mathcal{L}^{-1}\left\{ \frac{5e^{-s}}{(s+1)^2} \right\} = 5(t-1)e^{-t}u(t-1) \]

\[ s^2+6s+10 = (s+3)^2 + 1 = (s+3-j)(s+3+j), \]

\[ \frac{2s+2}{s^2+6s+10} = \frac{K}{s+3-j} + \frac{K^*}{s+3+j} \]

\[ K = \frac{(2s+2)(s+3-j)}{(s+3-j)(s+3+j)} \bigg|_{s=-3+j} = \frac{2s+2}{s+3+j} \bigg|_{s=-3+j} = \frac{2(-3+j)+2}{-3+j+3+j} = -3 \pm j \]

\[ \frac{2}{j^2} = 1 - \frac{2}{j} = 1+j \cdot 2 = \sqrt{1+4}\cdot\tan^{-1}(2) = \sqrt{5}\cdot63.4^\circ \]

\[ v(t) = 5(t-1)e^{-(t-1)}u(t-1) + 2\sqrt{5}e^{-3t}\cos(t+63.4^\circ) \]

(b) Use the Initial and Final Value Theorems to check your results. (10)

\[ \lim_{s \to 0} sV(s) = \lim_{s \to \infty} \frac{2s^2+2s}{s^2+6s+10} = 2, \quad \lim_{t \to 0} v(t) = 0 + 2\sqrt{5}\cos(63.4^\circ) \]

\[ \lim_{s \to 0} sV(s) = \lim_{s \to \infty} \frac{2s^2+2s}{s^2+6s+10} = 0, \quad \lim_{t \to \infty} v(t) = 0 (te^{-(t-1)} \to 0, e^{-3t} \to 0) \]
2. For the circuit to the right, note that the switch opens at \( t = 0 \),

\[
\begin{align*}
\text{(a) Draw the } s\text{-domain circuit valid for } t > 0. \quad (15) \\
\text{For } t < 0, \text{ switch closed for long time, so } L \text{ is short, } C \text{ is open} \\
\text{Find } I_c(s) \text{ (Laplace transform of capacitor current).} \quad (15)
\end{align*}
\]

Super Mesh (see above): \( 4s (I_1 - I_0) + 5I_1 + (5 + \frac{1}{2s})I_c = 0 \)

\( I_0 = \frac{2}{5} - \frac{1}{5} = \frac{1}{5} \), \( I_1 - I_c = 2I_c \), so \( I_1 = 3I_c \)

\( 4s (3I_c - \frac{1}{5}) + 5 (3I_c) + (5 + \frac{1}{2s})I_c = 0 \)

\( I_c (12s + 15 + 5 + \frac{1}{2s}) = 4 \Rightarrow I_c = \frac{4}{12s + 20 + \frac{1}{2s}} \)

Can also use node voltage:

\( V_1: \quad \frac{2}{5} + \frac{1}{5} + \frac{V_1}{4s} + \frac{V_1 - V_2}{5} = 0 \)

\( V_2: \quad \frac{V_2 - V_1}{5} + 2I_c + \frac{V_2}{5 + \frac{1}{2s}} = 0 \quad \text{where } I_c = \frac{V_2}{5 + \frac{1}{2s}} \)

solve 2x2 to get same answer
3. (a) Find the transfer function \( H(s) = \frac{V_o(s)}{V_i(s)} \) for the circuit to the right. Assume no stored energy in circuit at \( t = 0 \). (15)

\[
\begin{align*}
\frac{V_o(s)}{V_i(s)} &= H(s) = \frac{Z_{R \parallel C_2}}{sC_1 + Z_{C_1} + Z_{R \parallel C_2}} = \frac{1}{sC_1 + \frac{R}{SRC_2 + 1}} \\
&= \frac{sRC_1}{SRC_2 + 1 + sRC_1} = \frac{sRC_1}{SRC_1 + C_2 + 1} \\
&= \frac{0.02s}{0.025s + 1} = \frac{0.8s}{s + 40}
\end{align*}
\]

(b) Calculate the impulse response for this circuit. (10)

\[
h(t) = \mathcal{L}^{-1}\{H(s)\} = \mathcal{L}^{-1}\left\{\frac{0.8s}{s + 40}\right\} = 0.8 - \frac{32}{s + 40} \\
= 0.8\delta(t) - 32e^{-40t}u(t)
\]

(c) If \( v_i(t) = u(t) - u(t - 50) \), calculate and sketch \( v_o(t) \) versus time (seconds). (15)

Option 1: \( V_o(s) = V_i(s)H(s) = \left( \frac{1}{s} - \frac{e^{-50s}}{s} \right) \left( \frac{0.8s}{s + 40} \right) = \frac{0.8(1 - e^{-50s})}{s + 40} \)

Option 2: \( v_o(t) = v_i(t) \star h(t) \\
= \int_0^\infty v_i(\lambda - t)h(\lambda)\,d\lambda \\
= \int_0^{\infty} \left[ 0.8\delta(\lambda) - 32e^{-40t} \right] d\lambda = 0.8 + \frac{32}{40}e^{-40t} \]

End Of Exam