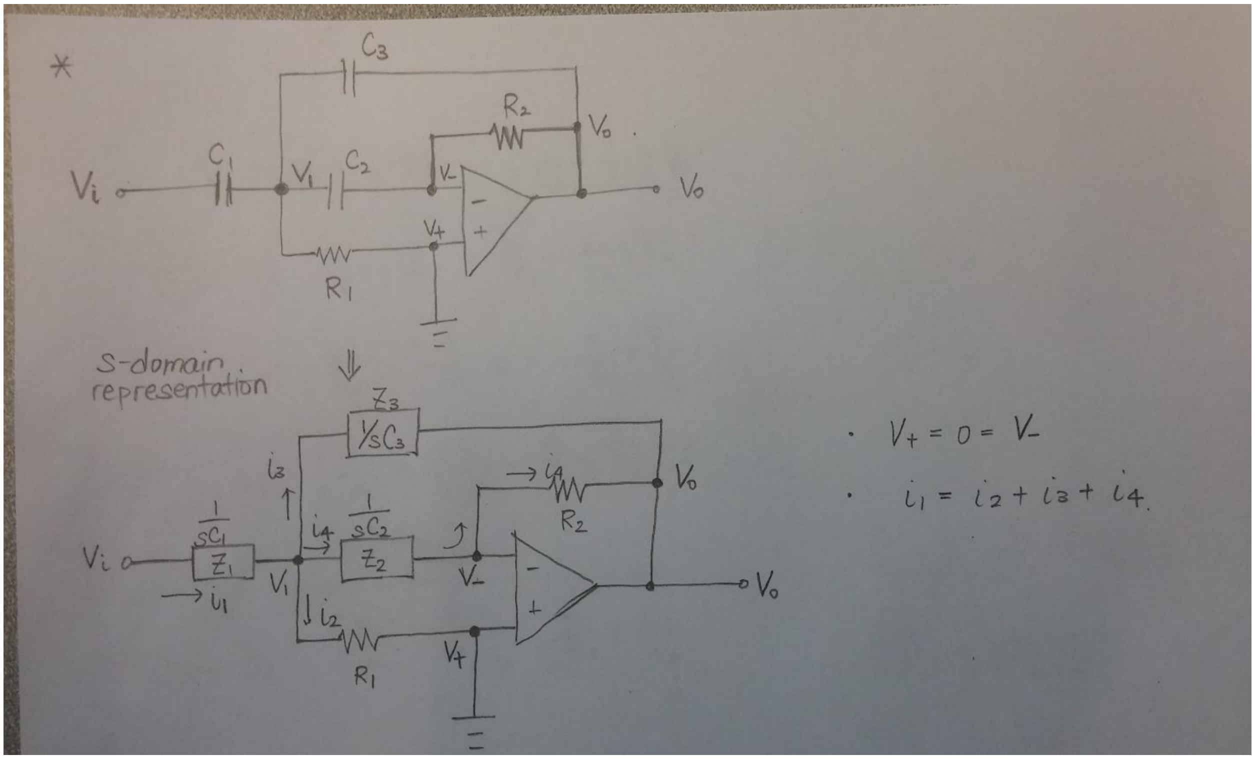
Solution of Exam 3 Problem 1

1. Solution



$$i_1 = \frac{V_i - V_1}{Z_1},\ i_2 = \frac{V_1 - V_+}{R_1} = \frac{V_1}{R_1},\ i_3 = \frac{V_1 - V_0}{Z_3},\ i_4 = \frac{V_1 - V_-}{Z_2} = \frac{V_1}{Z_2} = \frac{V_- - V_0}{R_2} = \frac{-V_0}{R_2}$$

Then, since $i_1 = i_2 + i_3 + i_4$,

$$\frac{V_i - V_1}{Z_1} = \frac{V_1}{R_1} + \frac{V_1 - V_0}{Z_3} + \frac{-V_0}{R_2} \quad (1)$$

$$and\ \frac{V_1}{Z_2} = \frac{-V_0}{R_2} \quad (2)$$

First simplify (1). So, we get $V_i = \left(1 + \frac{Z_1}{R_1} + \frac{Z_1}{Z_3}\right)V_1 - \left(\frac{Z_1}{Z_3} + \frac{Z_1}{R_2}\right)V_0 \ \ (3)$.

Then, we plug (2) in (3). So, we get $V_i = -\left(\frac{Z_2}{R_2} + \frac{Z_1 Z_2}{R_1 R_2} + \frac{Z_1 Z_2}{R_2 Z_3} + \frac{Z_1}{Z_3} + \frac{Z_1}{R_2}\right)V_0$.

Then, we present Z_1, Z_2 ,and Z_3 in terms of s , C_1 , C_2 ,and C_3 . The relation is shown in the picture above.

Hence, we get $\frac{V_0(s)}{V_i(s)} = \frac{-R_1R_2C_1C_2s^2}{R_1R_2C_2C_3s^2 + (R_1C_1 + R_1C_2 + R_1C_3)s + 1}$.

Finally, it is a high pass filter because it rejects the low frequency as we set s is zero. And it passes the high frequency as we set s is infinity.