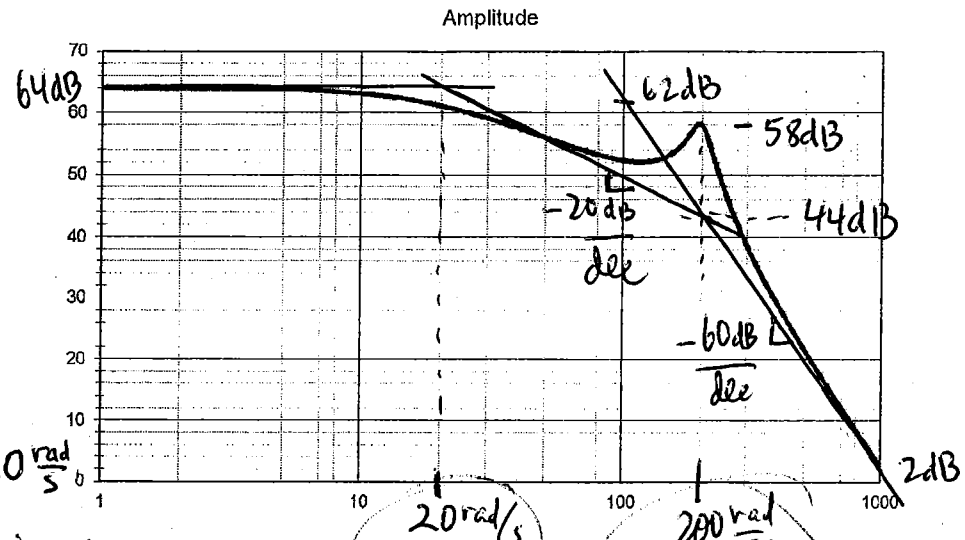
2. (a) What H(s) corresponds to the Bode magnitude plot (units dB vs rad/s) to the right? (10)



Pass band gain $\cong$ 64dB
$\cong$ 1600

Slope above 20 rad/s approaches 20 dB/dec, so single pole.

3dB drop occurs at about 20 $\frac{rad}{s}$.

At high frequency, slope is 60dB/decade, so 2 more poles. Sharp peak near 200 rad/s, so two complex poles (high Q)

Gain at corner is 58dB, which is 14dB above 44dB at corner (where asymptotes with slopes -20dB/dec & -60dB/dec meet), so $Q = 10^{14/20} = 5$

$$H(s) = \frac{1600\,(20)(200)^2}{(s+20)\left(s^2+\frac{200}{5}s+(200)^2\right)} = \frac{1.28\times10^9}{(s+20)(s^2+40s+4\times10^4)}$$

(b) What is H(s) for a Butterworth high pass filter with unity gain in the passband, a corner frequency of 5×10^4 rad/s and a gain of less than -60dB for $\omega < 10^4$ rad/s. (8)

$$A_{pb} - A_{sb} = 0 - (-60dB) = 60dB$$

$$\log_{10}\left(\frac{5\times10^4}{10^4}\right) = 0.7 \text{ decades}$$

$$n \geq \frac{60dB/0.7dec}{20dB/dec} = 4.29 \Rightarrow n = 5$$

5th order Butterworth

$$(s+1)(s^2+0.618s+1)(s^2+1.618s+1)$$

Scale corner frequency from $1 \rightarrow 5\times10^4$ rad/s $\quad s \rightarrow s/5\times10^4$

unity-gain HPF: $\Rightarrow$

$$\frac{(s/5\times10^4)^5}{\left(\frac{s}{5\times10^4}+1\right)\left(\left(\frac{s}{5\times10^4}\right)^2+0.618\left(\frac{s}{5\times10^4}\right)+1\right)\left(\left(\frac{s}{5\times10^4}\right)^2+1.618\left(\frac{s}{5\times10^4}\right)+1\right)} = H(s)$$

Can multiply top & bottom by $(5\times10^4)^5$ to simplify