EE 233 HW#6 Solution

(13.70) (a)

$$H(s) = \frac{\frac{1}{SC}}{\frac{1}{SC} + R + SC}$$
$$= \frac{1}{s^2LC + SCR + 1}$$
$$= \frac{\frac{1}{LC}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$
$$= \frac{100}{s^2 + s20 + 100}$$
$$= \frac{100}{(s+10)^2}$$
$$\mathcal{L}^{-1}\left\{\frac{100}{(s+10)^2}\right\} = te^{-10t}100$$
$$= 100te^{-10t}$$

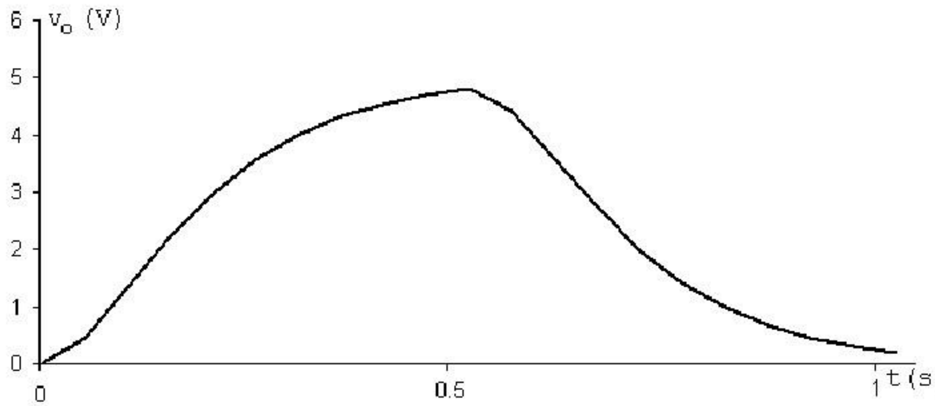
if $V_i = 5[u(t) - u(t - 0.5)]$ So
for $0 \le t \le 0.5$:

$$y(t) = \int_0^t 5(100)\lambda e^{-10\lambda} d\lambda$$
$$= 5e^{-10\lambda}(10\lambda + 1)|_{\lambda=0}^{\lambda=t}$$
$$= -5e^{-10t}(10t + 1) - (-5e^{-0}(10(0)) + 1)$$
$$= -5e^{-10t}(10t + 1) + 5$$

And for $0.5 \le t \le \infty$:

$$V_0(t) = \int_{t-0.5}^{t} 5(100)\lambda e^{-10\lambda} d\lambda$$
$$= -5e^{-10\lambda}(10\lambda + 1)\Big|_{\lambda = t - 0.5}^{\lambda = t}$$
$$= -5e^{-10t}(10t + 1) - (-5e^{-10t+5}(10t - 5 + 1))$$
$$= -5e^{-10t}(10t + 1 - e^{5}(10t - 4))$$

(b) The graph is shown here:



(13.80) input of 30, $u(t)$ is applied, the response is

$V_{out} = (50e^{-8000t} - 20e^{-5000t})u(t)$ when $V_{in} = 30\mu(t)$.

$$\begin{aligned} V_0(s) &= \frac{50}{s+8000} - \frac{20}{s+5000} \\ &= \frac{50(s+5000) - 20(s+8000)}{(s+5000)(s+8000)} \\ &= \frac{50s + 250000 - 20s - 160000}{(s+5000)(s+8000)} \\ &= \frac{30s + 90000}{(s+5000)(s+8000)} \\ &= \frac{30(s+3000)}{(s+5000)(s+8000)} \end{aligned}$$

So to find $H(s)$ we need to divide by $\mathcal{L}\{30u(t)\} = \frac{30}{s}$.

$$\begin{aligned} H(s) &= \frac{30(s+3000)}{\frac{30}{s}(s+5000)(s+8000)} \\ &= \frac{s(s+3000)}{(s+5000)(s+8000)} \end{aligned}$$

$$V_{in} = 120\cos(6000t)$$

We know that $s = j\omega$ and $\omega = 6000$, so

$$\begin{aligned} H(j6000) &= \frac{(j6000)(3000 + j6000)}{(5000 + j6000)(8000 + j6000)} = 0.206 + 0.472j \\ &= 0.52\angle 66.37° \end{aligned}$$

Thus, $y(t) = (120)(0.52\cos(6000t + 66.37))$

$$y(t) = 62.4\cos(6000t + 66.37)$$

(13.91) We can transform the current source into a voltage source. So the source voltage becomes $(50K)(0.5\delta(t))$ and the resistor in series with the source.

$$C_{eq} = \frac{(1\mu F)(250nF)}{1\mu F + 250nF} = 0.2\mu F$$

$$
\begin{aligned}
H(s) &= \frac{0.5\frac{1}{s(1\mu F)}}{\frac{1}{s(1\mu F)}+\frac{1}{s(250nF)}+50K} \\
&= \frac{0.5}{1+\frac{1\mu F}{250nF}+50000(1\mu F)s} \\
&= \frac{0.5}{5+.05s} \\
&= \frac{10}{100+s} \\
\mathcal{L}^{-1}\left\{\frac{10}{100+s}\right\} &= 10e^{-100t}u(t) \\
&= V_0(t)
\end{aligned}
$$

(b) We would expect the circuit to produce a decaying exponential with $\tau = RC$

$$\frac{1}{(50K\Omega)(0.2\mu F)} = 100$$

So yes, this makes sense.

(Problem1) (a)

$$
\begin{aligned}
H(s) &= \frac{\frac{1}{SC}}{\frac{1}{SC}+R} \\
&= \frac{1}{SCR+1} \\
&= \frac{\frac{1}{RC}}{S+\frac{1}{RC}} \\
&= \frac{250}{s+250} \\
\mathcal{L}^{-1}\left\{\frac{250}{s+250}\right\} &= 250e^{-250t}
\end{aligned}
$$

So for $0 \le t \le 4$ ms,

$$V_0(t) = \int_0^t 16(250)e^{-250\lambda}d\lambda$$

$$= -16e^{-250\lambda}\Big|_{\lambda=0}^{\lambda=t}$$

$$= 16(1-e^{-250t})$$

and for $4\text{ms} \le t \le \infty$:

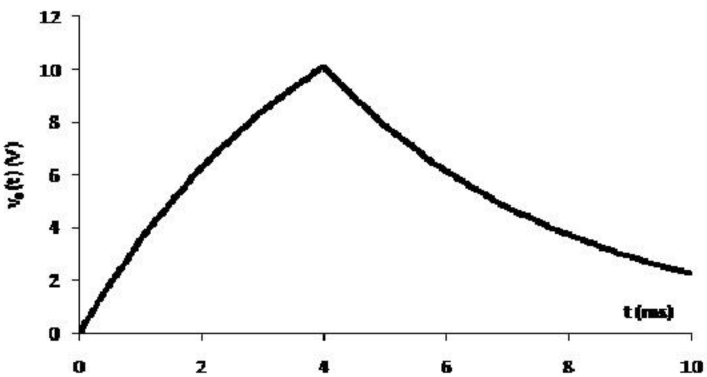
$$V_0(t) = \int_{t-0.004}^{t} 16(250)e^{-250\lambda}d\lambda$$

$$= -16e^{-250\lambda}\Big|_{\lambda = t - 0.004}^{\lambda = t}$$

$$= -16e^{-250t} + 16e^{-250(t-0.004)}$$

$$= 16e^{-250t}(e - 1)V$$

(b) The graph is shown here:



(Problem2)

P 14.3 [a] $H(s) = \frac{V_o}{V_i} = \frac{R}{sL + R + R_l} = \frac{(R/L)}{s + (R + R_l)/L}$

[b] $H(j\omega) = \frac{(R/L)}{\left(\frac{R+R_l}{L}\right) + j\omega}$

$|H(j\omega)| = \frac{(R/L)}{\sqrt{\left(\frac{R+R_l}{L}\right)^2 + \omega^2}}$

$|H(j\omega)|_{\text{max}}$ occurs when $\omega = 0$ because as w increse, then the denominator will get bigger and decrease the gain

[c] $|H(j\omega)|_{\text{max}} = \frac{R}{R + R_l}$

[d] $\frac{|H(j\omega)|_{\text{max}}}{\sqrt{2}} = |H(j\omega_c)|$

$|H(j\omega_c)| = \frac{R}{\sqrt{2}(R + R_l)} = \frac{R/L}{\sqrt{\left(\frac{R+R_l}{L}\right)^2 + \omega_c^2}}$

$\therefore \quad \omega_c^2 = \left(\frac{R + R_l}{L}\right)^2; \qquad \therefore \quad \omega_c = (R + R_l)/L$

[e] $\omega_c = \frac{127 + 75}{0.01} = 20{,}200 \text{ rad/s}$

$H(j\omega) = \frac{12{,}700}{20{,}200 + j\omega}$

$H(j0) = 0.6287$

$H(j20{,}200) = \frac{0.6287}{\sqrt{2}} \underline{/-45^\circ} = 0.4446\underline{/-45^\circ}$

$H(j6060) = \frac{12{,}700}{20{,}200 + j6060} = 0.6022\underline{/-16.70^\circ}$

$H(j60{,}600) = \frac{12{,}700}{20{,}200 + j60{,}600} = 0.1988\underline{/-71.57^\circ}$

(Problem3)

a) We know center frequency

$$w_0 = \sqrt{\frac{1}{LC}} = \frac{20krad}{s}$$

So we can get $L = 0.05H$

Use the definition of quality factor

$$Q = w_0 RC = 5$$

We get $R = 5K\Omega$

b) Lower cutoff frequency:

$$w_{c1} = w_0 \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right] = 18010rad/s$$

$$f_{c1} = \frac{w_{c1}}{2\pi} = 2866.4Hz = 2.866KHz$$

Upper cutoff frequency:

$$w_{c2} = w_0 \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right] = 22099.8rad/s$$

$$f_{c2} = \frac{w_{c2}}{2\pi} = 3517.3Hz = 3.517KHz$$

c) The bandwidth in this filter is

$$B = f_{c1} - f_{c2} = 3.517KHz - 2.866KHz = 0.651KHz = 651Hz$$