$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = \int_{0^-}^{\infty} \underline{e^{-st}}\, \underline{\frac{df}{dt}\,dt} = \int_{0^-}^{\infty} \underline{u}\, \underline{dv}$$

$$= uv\Big|_{0^-}^{\infty} - \int_{0}^{\infty} v\,du = e^{-st} f(t)\Big|_{0^-}^{\infty} - \int_{0^-}^{\infty} f(t)(-se^{-st})\,dt$$

$$= 0 - e^{0} f(0^-) + s\int_{0^-}^{\infty} f(t)e^{-st}\,dt$$

$$= -f(0^-) + sF(s) = sF(s) - f(0^-)$$

$$\mathcal{L}\left\{\frac{d^2f}{dt}\right\} = s\left[sF(s) - f(0^-)\right] - \frac{df}{dt}\Big|_{0^-}$$

$$= s^2F(s) - sf(0^-) - \frac{df}{dt}\Big|_{0^-}$$

$$\mathcal{L}\left\{\frac{d^{(n)}f}{dt}\right\} = s^n F(s) - s^{n-1} f(0^-) - \dots - \left.\frac{d^{(n-1)} f}{dt^{(n-1)}}\right|_{0^-}$$

$$\mathcal{L}\left\{\int_{0^-}^{t} f(\tau)\,d\tau\right\} = \int_{0^-}^{\infty} \overbrace{\left[\int_{0^-}^{t} f(\tau)\,d\tau\right]}^{u} \overbrace{e^{-st}\,dt}^{dv}$$

$$= uv\Big|_{0^-}^{\infty} - \int_{0^-}^{\infty} v\,du$$

$$= \left[\int_{0^-}^{t} f(\tau)\,d\tau \left(\frac{e^{-st}}{-s}\right)\right]_{0^-}^{\infty} - \underbrace{\int_{0^-}^{\infty} f(t)\frac{e^{-st}}{-s}\,dt}_{+\frac{F(s)}{s}}$$

$$(0 - 0)$$

$\int_{0^-}^{\infty} f(\tau)\,d\tau$ well-behaved

## Translation in time $(a > 0)$

$$\mathcal{L}\{f(t-a)\,u(t-a)\} = \int_{0^-}^{\infty} u(t-a) f(t-a) e^{-st}\,dt$$

$$t' = t - a \qquad t = t' + a$$

$$\mathcal{L}\{\ \} = \int_{a^-}^{\infty} u(t-a) f(t-a) e^{-st}\,dt$$

$$= \int_{0^-}^{\infty} u(t') f(t') e^{-(t'+a)s}\,dt'$$

$$e^{-sa} \underbrace{\int_{0^-}^{\infty} u(t') f(t') e^{-st'}\,dt'}_{F(s)}$$

Translation in frequency (s)

$$\mathcal{L}\{e^{-at} f(t)\} = F(s+a)$$

$$\mathcal{L}\{f(at)\} = \frac{1}{|a|} F\left(\frac{s}{a}\right)$$

Scale models

# Derivative in s-domain

$$\mathcal{L}\left\{ t\, f(t) \right\} = -\frac{dF(s)}{ds}$$

$$\mathcal{L}\left\{ t^n f(t) \right\} = (-1)^n \frac{d^{(n)} F(s)}{ds^{(n)}}$$

$$\mathcal{L}\left\{ \frac{f(t)}{t} \right\} = \int_s^\infty F(u)\, du$$

$$\mathcal{L}\left\{t^2 e^{-at}\right\}$$

$$f(t) = e^{-at} \Rightarrow F(s) = \frac{1}{s+a}$$

$$g(t) = t^2 e^{-at} = (-1)^2 \frac{d^2}{ds^2}\left(\frac{1}{s+a}\right)$$

$$G(s) = \frac{d}{ds}\left(-\frac{1}{(s+a)^2}\right)$$

$$G(s) = \frac{2}{(s+a)^3}$$

$$h(t) = \underline{\frac{d}{dt}\left[e^{-at} \sinh(\beta t)\right]}$$

$$= \frac{d}{dt}\left[e^{-at}\left(\frac{e^{\beta t} - e^{-\beta t}}{2}\right)\right]$$

Let $f(t) = \sinh(\beta t)$

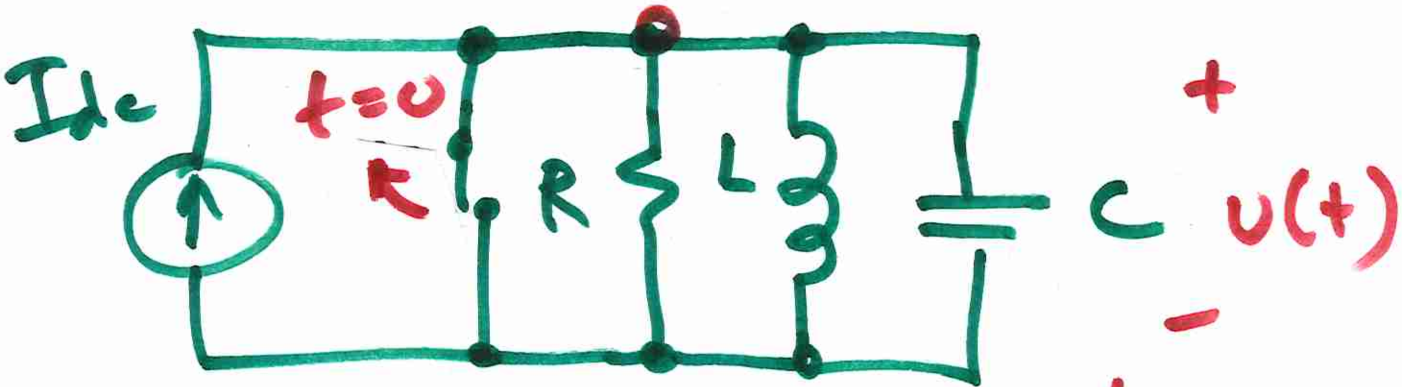
$$F(s) = \frac{1}{2}\left(\frac{1}{s-\beta} - \frac{1}{s+\beta}\right)$$

$$= \frac{1}{2}\left(\frac{\cancel{s}+\beta - (\cancel{s}-\beta)}{s^2-\beta^2}\right) = \frac{\beta}{s^2-\beta^2}$$

$$g(t) = e^{-at} f(t) \implies G(s) = \frac{\beta}{(s+a)^2-\beta^2}$$

$$H(s) = sG(s) - g(0^-) = \underline{\frac{\beta s}{(s+a)^2-\beta^2}} - \cancel{1 \sinh(0)}\ 0$$

# Applying Laplace Transforms




$$\sum I = 0 \Rightarrow \frac{v(t)}{R} + \frac{1}{L}\int_{-\infty}^{t} v(\tau)\,d\tau + C\frac{dv(t)}{dt} = I_{dc}u(t)$$

$$\frac{1}{L}\left[\int_{-\infty}^{0^-} v(\tau)\,d\tau + \int_{0^-}^{t} v(\tau)\,d\tau\right]$$

$$i_L(0^-) = 0$$

⋮

$$\mathcal{L}\left\{\frac{v}{R} + \frac{1}{L}\int_{0^-}^{t} v(\tau)\,d\tau + C\frac{dv(t)}{dt} = I_{dc}\,u(t)\right\}$$

$$\frac{V(s)}{R} + \xrightarrow{0}\cancel{\frac{i_L(0^-)}{s}} + \frac{1}{L}\frac{V(s)}{s} + C\left[sV(s) - \xrightarrow{0}\cancel{v(0^-)}\right] = \frac{I_{DC}}{s}$$

$$V(s) * \left\{\frac{1}{R} + \frac{1}{sL} + sC\right\} = \frac{I_{DC}}{s}$$

$$V(s) = \frac{I_{DC}}{s\left[\frac{1}{R} + \frac{1}{sL} + sC\right]} = \frac{I_{DC}}{s^2C + \frac{s}{R} + \frac{1}{L}}$$

$$V(s) = \frac{I_{DC}/C}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

$$= \frac{I_{DC}/C}{\left(s + \frac{1}{2RC}\right)^2 \underbrace{- \frac{1}{4(RC)^2} + \frac{1}{LC}}_{-\beta^2}}$$

$$= \frac{I_{DC}/C}{(s + a)^2 - \beta^2}$$

$$\mathcal{L}^{-1}\{V(s)\}$$

$$v(t) = \frac{I_{DC}}{\beta C}\left[e^{-at} \sinh \beta t\right]$$

$$\beta^2 = \frac{1}{4(RC)^2} - \frac{1}{LC}$$