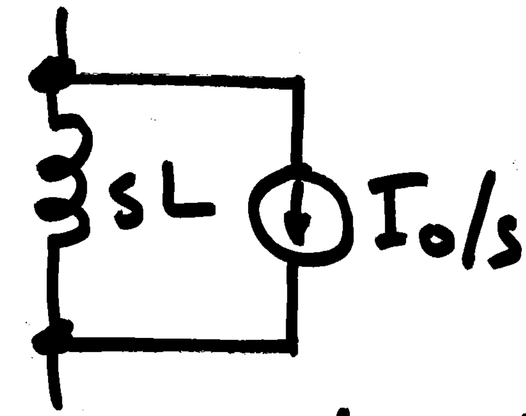
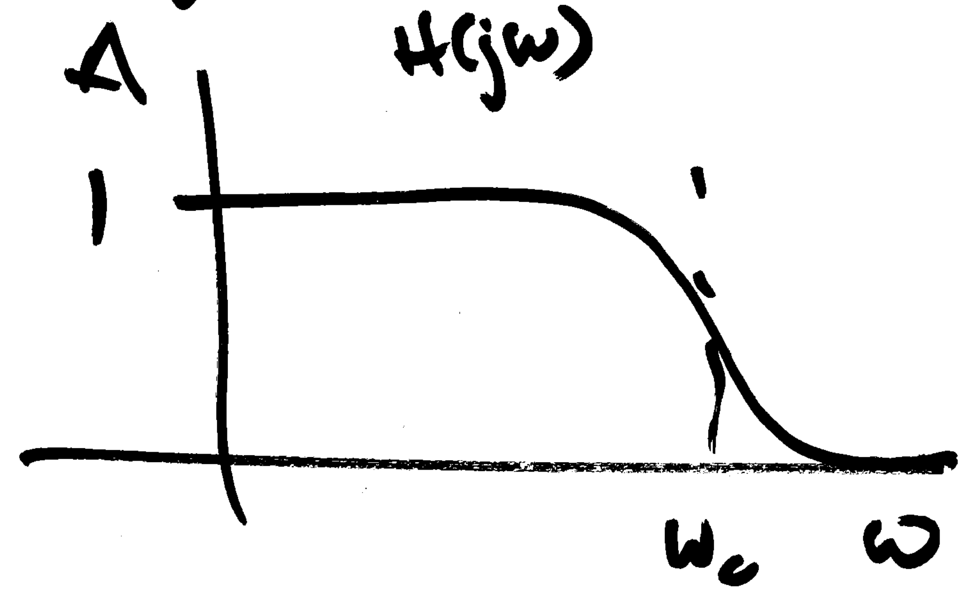
$H(s) \Rightarrow$ Sinusoidal Steady-State

<u>$H(j\omega)$</u>

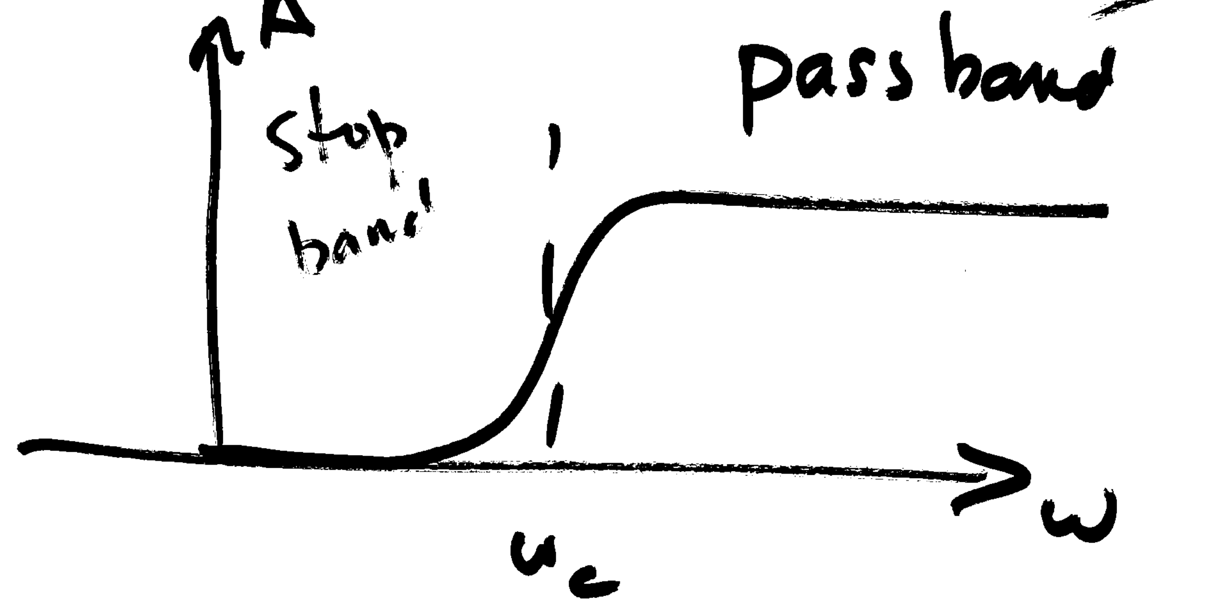


Design of frequency-selective circuits

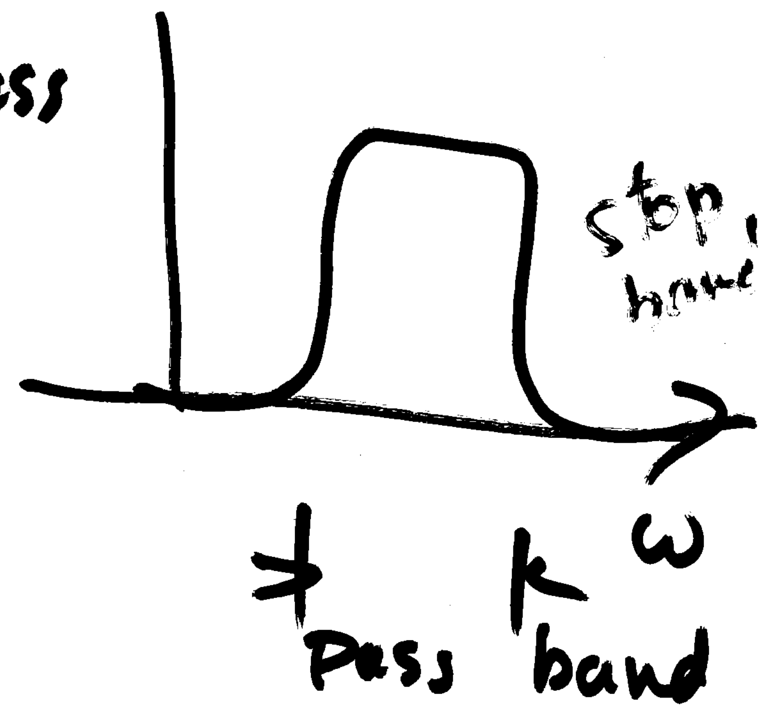
Low pass filter



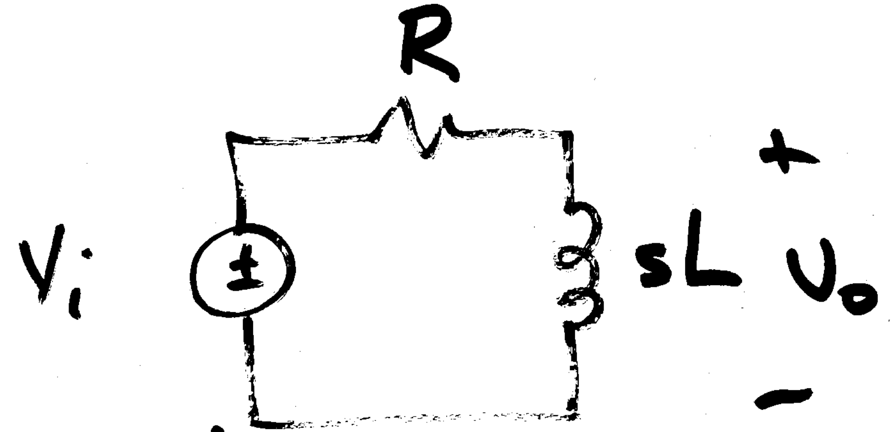
High Pass Filter

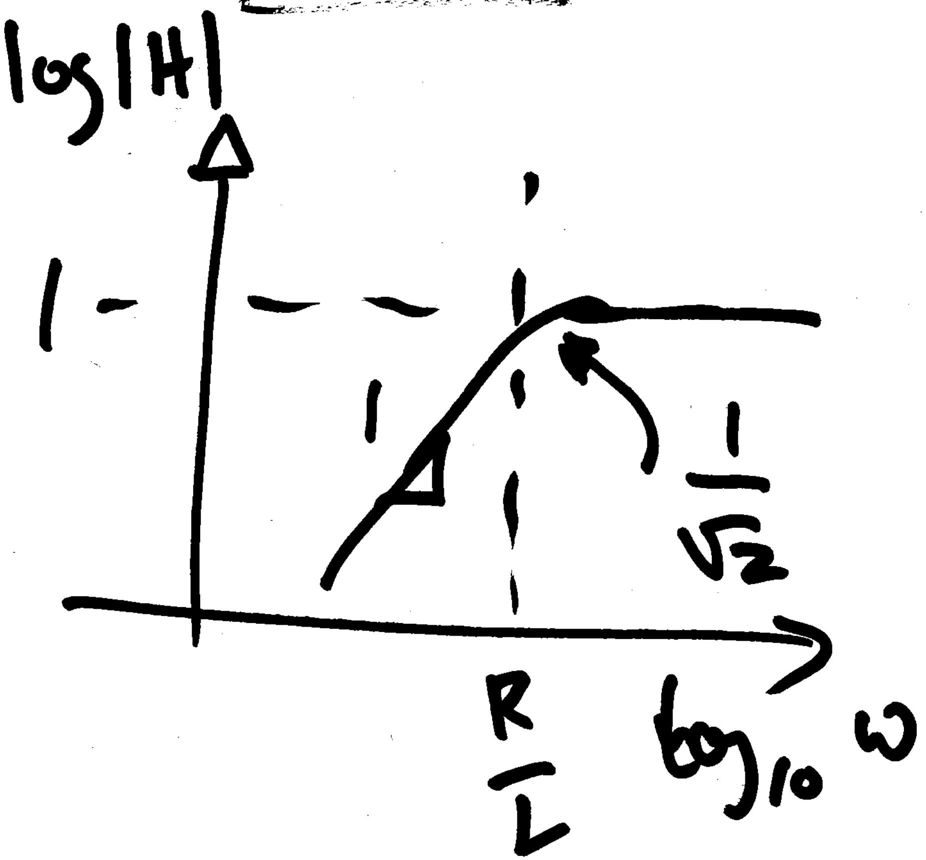


Band Pass



Band Reject





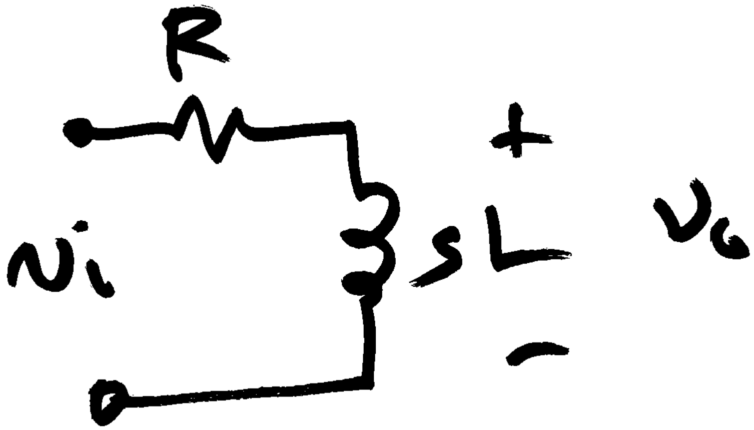
$$V_o(s) = \frac{sL}{R+sL} V_i(s)$$

$$\frac{V_o}{V_i} = \frac{sL}{R+sL}$$

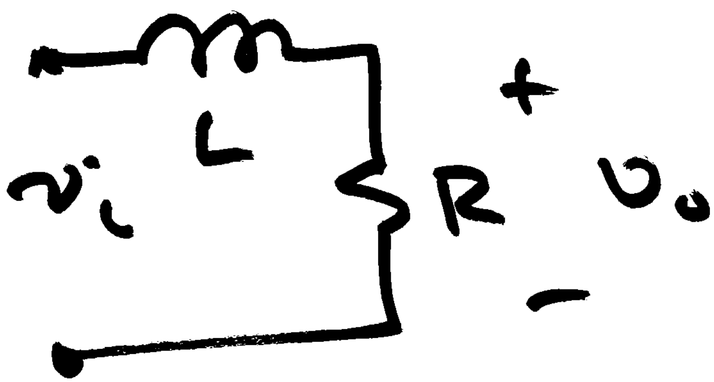
$$H(j\omega) = \frac{j\omega L}{R+j\omega L}$$

$$R = \omega_c L$$

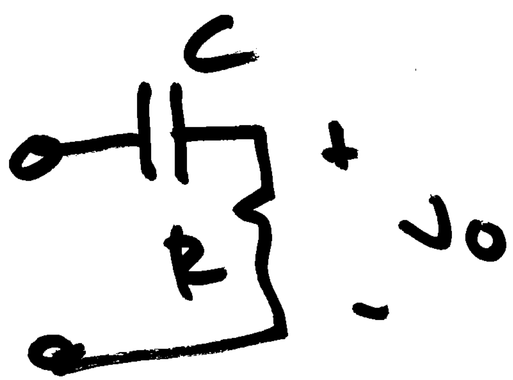
$$\omega_c = \frac{R}{L}$$



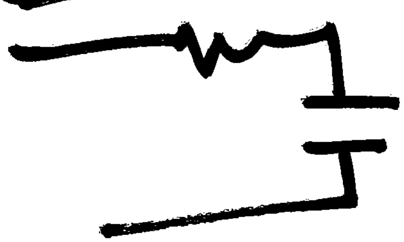
HPF



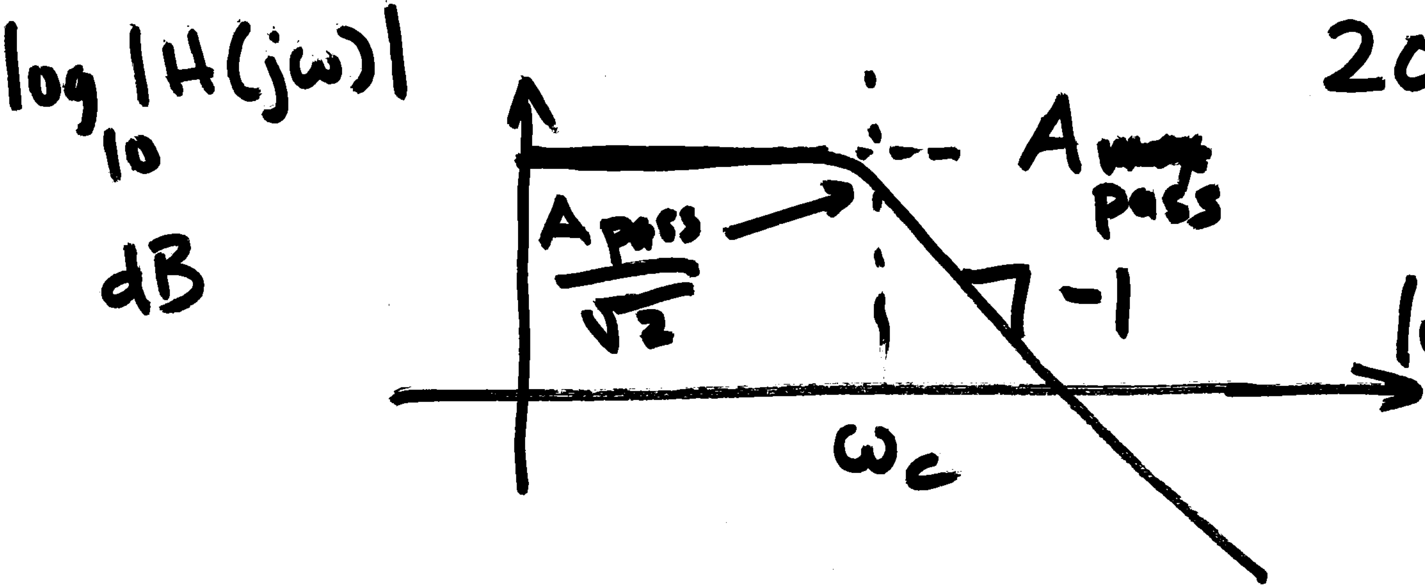
LPF



HPF

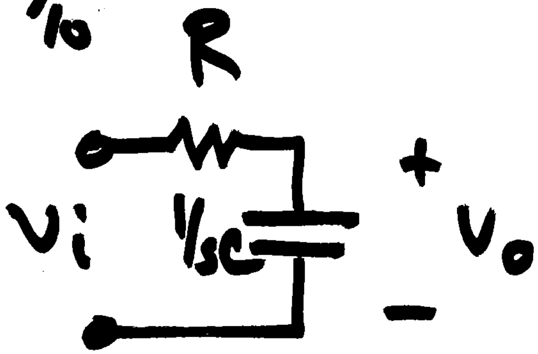


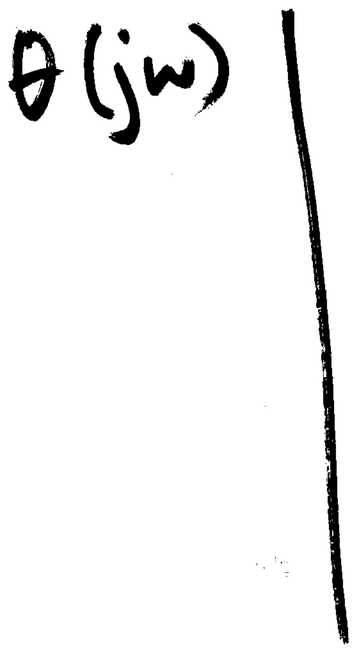
LPF



$20\log_{10}|A| \Rightarrow dB$

$$\frac{1}{j\omega + \omega_c}$$



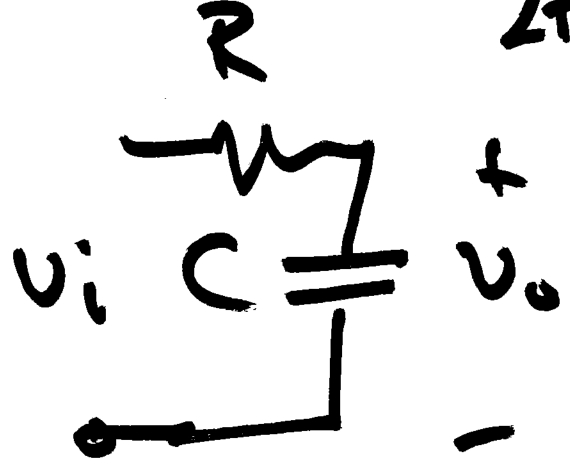


$$V_o = \frac{1/sC}{R + 1/sC} V_i$$

$$= \frac{1}{sRC + 1} = \frac{1}{RC}\left(\frac{1}{s + \frac{1}{RC}}\right)$$

$$= \frac{1}{RC}\left[\frac{1}{\frac{1}{RC} + j\omega}\right]$$

$$\frac{1}{RC} = \omega_c$$



$$\omega_c = 500\ \text{MHz} = \frac{1}{RC}$$

$$2\pi \times 5 \times 10^8\ \text{Hz}$$

$$3.14 \times 10^9\ \frac{\text{rad}}{\text{s}} = \frac{1}{RC}$$

$$C = 22\ \text{pF}$$

$$R = \frac{1}{3\times10^9 * C} = \frac{1}{\omega_c C}$$

$$= \frac{1}{(3\times10^9)(2\times10^{-11})}$$

$$= \frac{1}{6\times10^{-2}} = \frac{100}{6} = 17\,\Omega$$

$\omega << \frac{1}{RC}$ $\quad \frac{V_o}{V_i} \cong \cancel{\frac{1}{RC}} \frac{1}{RC}\left(\frac{1}{\frac{1}{RC}+j\omega}\right)$

$\cong \frac{1}{RC}\left(\frac{1}{1/RC}\right) = 1$

$\omega >> \frac{1}{RC} = \omega_c$ $\quad \frac{V_o}{V_i} \cong \frac{1}{RC}\left[\frac{1}{j\omega}\right] = \frac{1}{j\omega RC}$

$= \frac{1}{j\omega/\omega_c} = -\frac{j\omega_c}{\omega}$

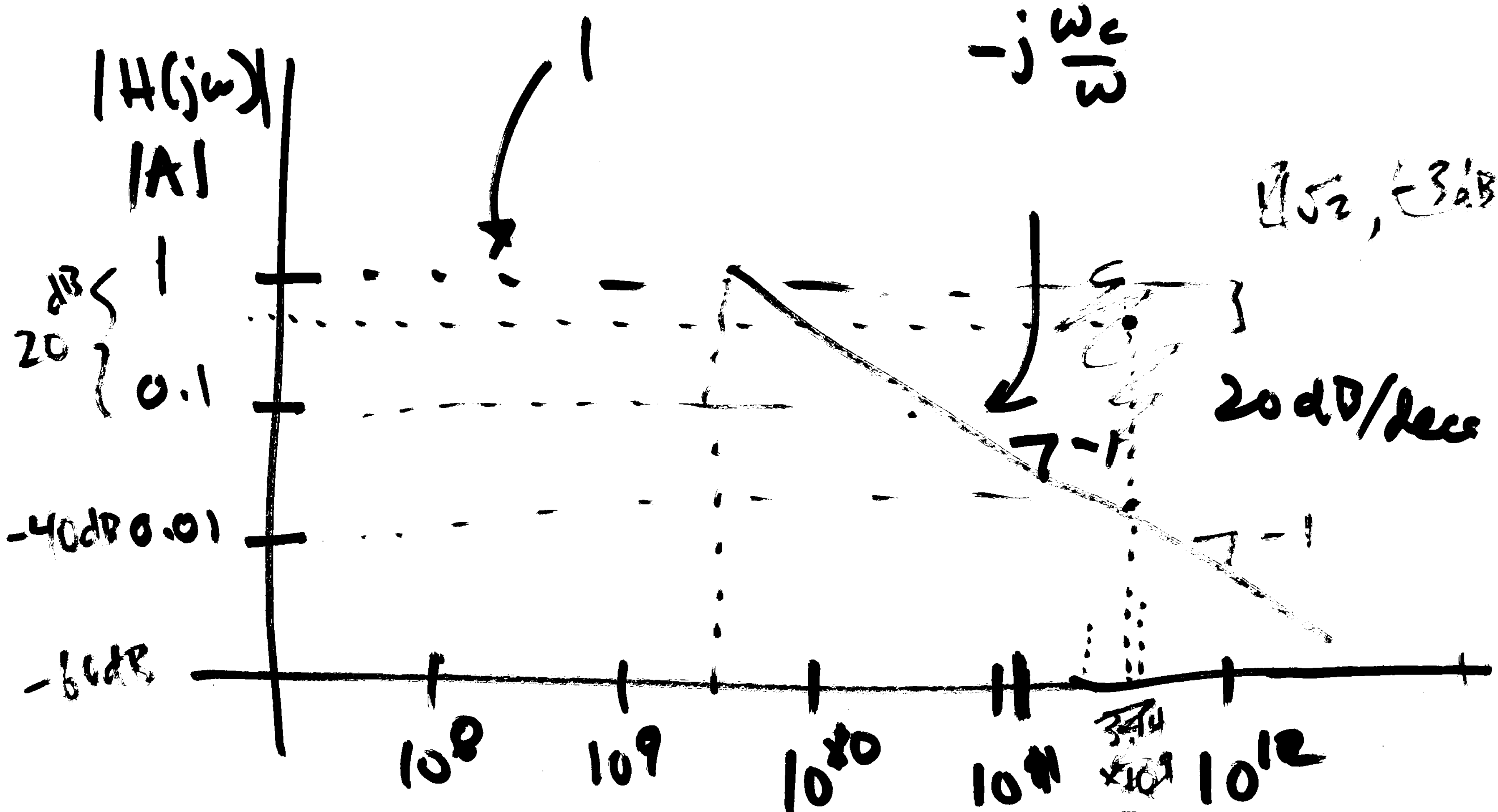
$= -j\omega_c(\omega)^{-1}$

~~tore~~

$\frac{1}{\omega}$ slope -1

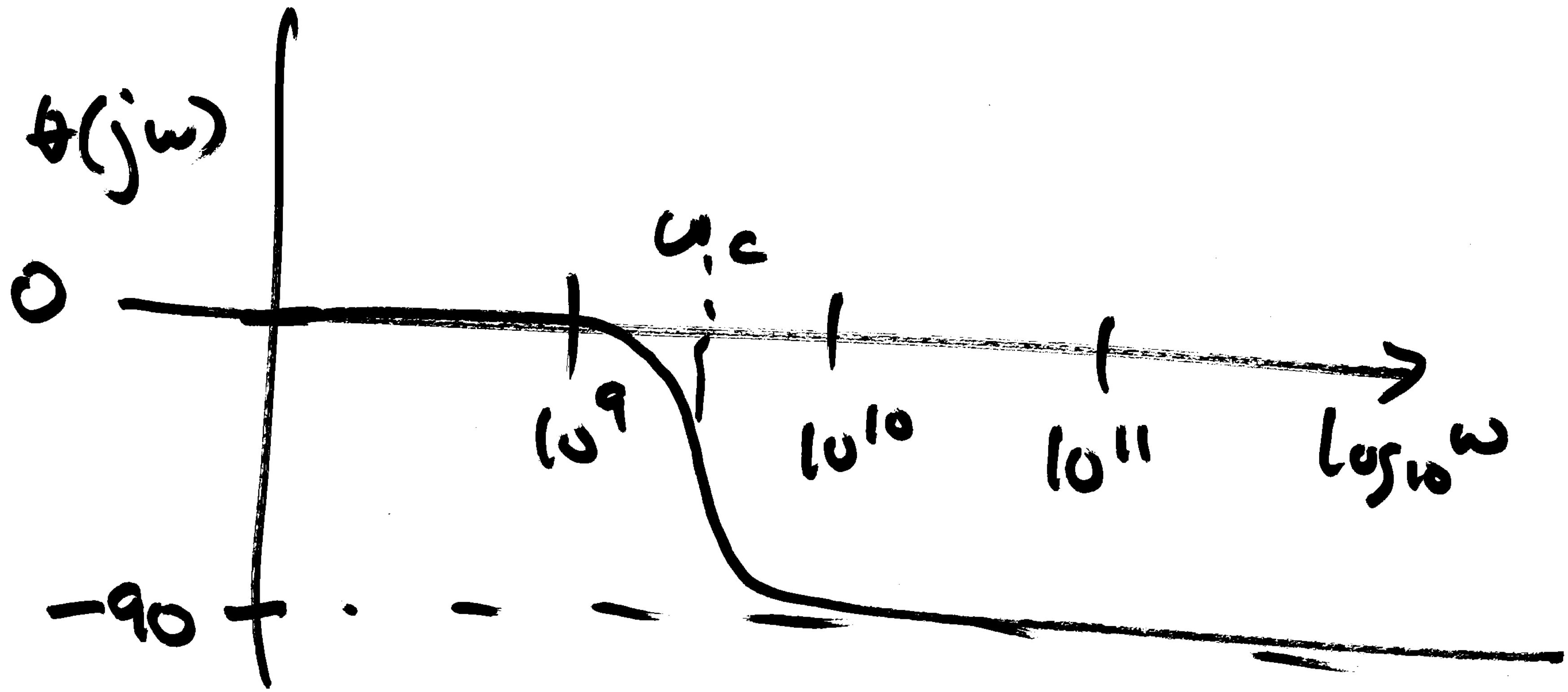
(log-log)

20 dB/decade

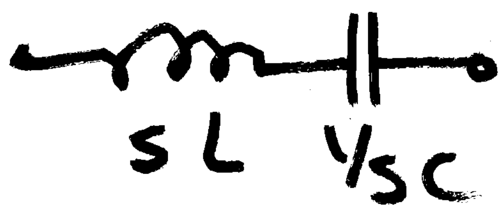


$$A(j\omega) = \frac{\omega_c}{\omega_c + j\omega}$$

$$\angle A \cong \begin{cases} 0 & \omega \ll \omega_c \\ -90 & \omega \gg \omega_c \end{cases}$$



$\phi(j\omega)$
0
-90
10^9
ω_c
10^{10}
10^{11}
$\log_{10}\omega$



$$Z = sL + \frac{1}{sC}$$

$$= \frac{s^2CL + 1}{sC}$$

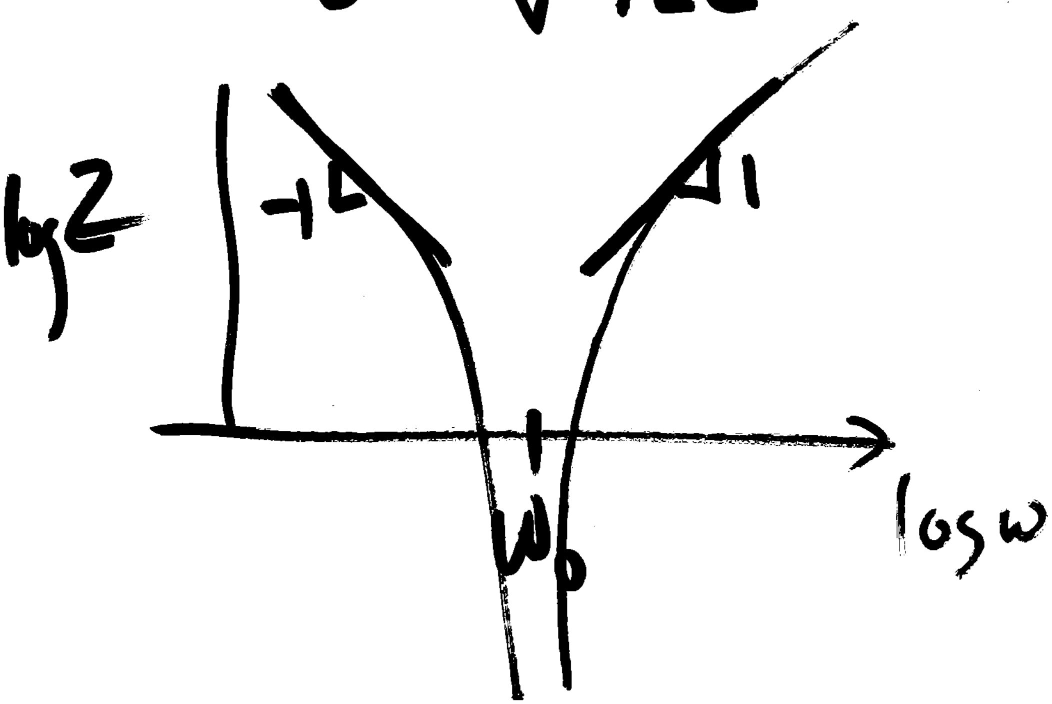
$$Z(j\omega) = \frac{-\omega^2 LC + 1}{j\omega C}$$

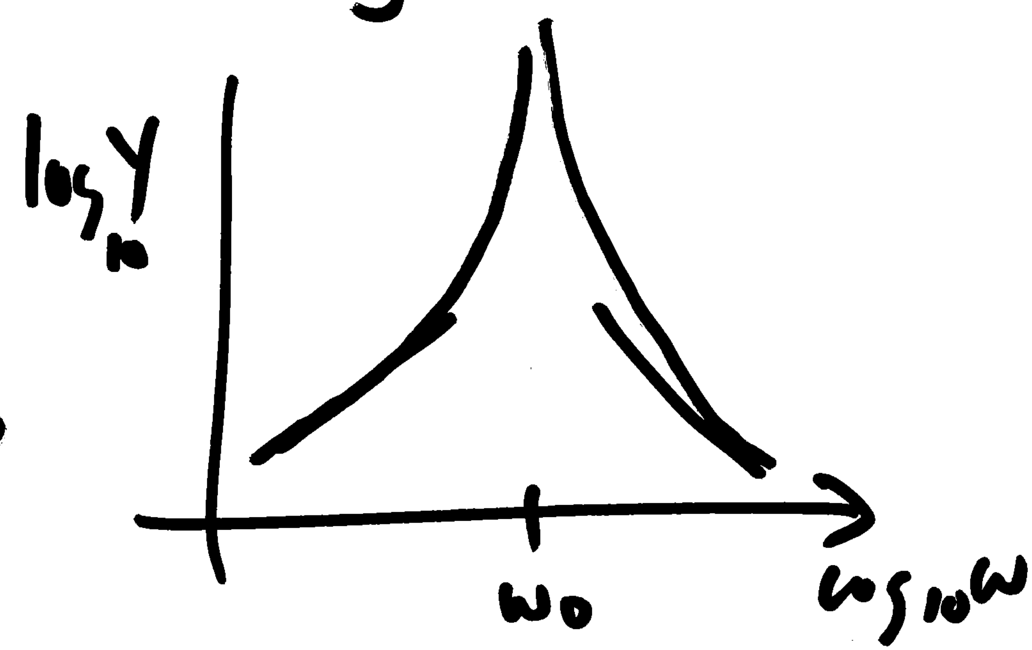
$$= \frac{1 - \omega^2 LC}{j\omega C}$$

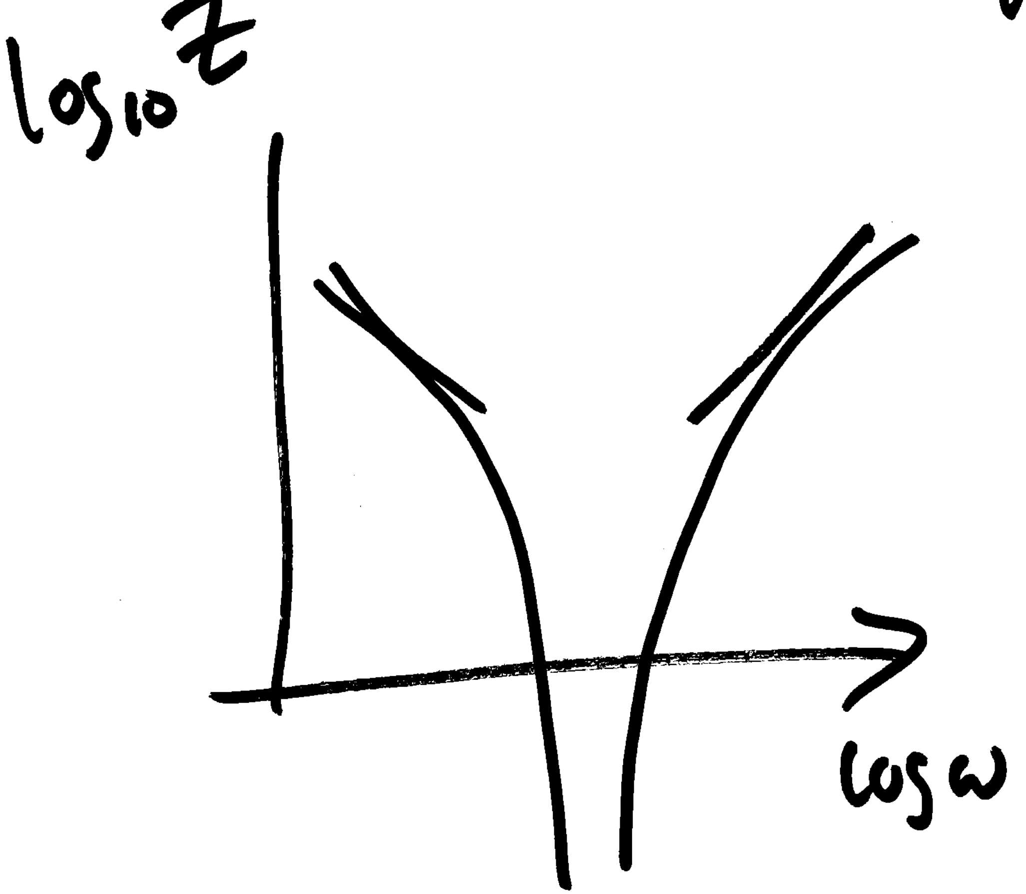
$Z \to 0$

$\omega^2 \to \frac{1}{LC}$

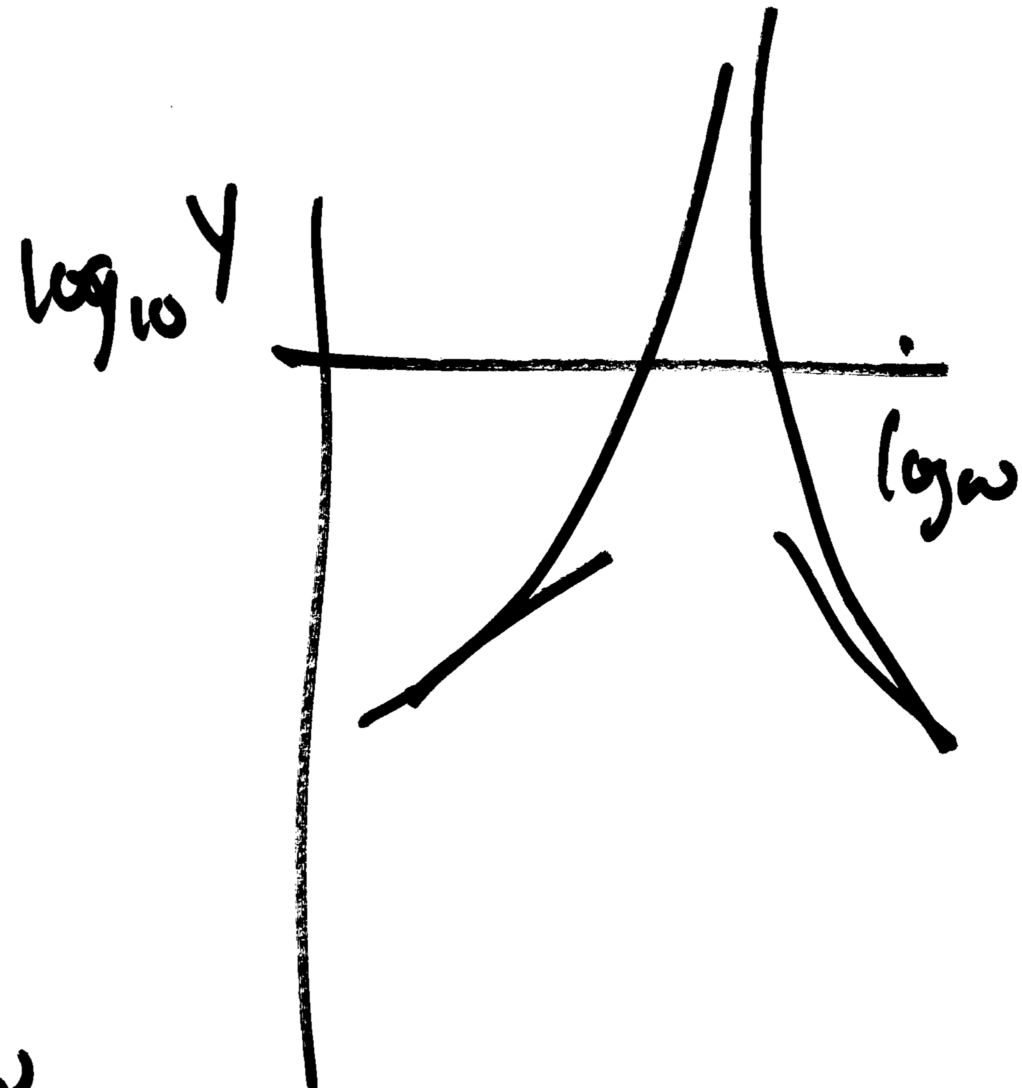
$\omega_0 \to \sqrt{1/LC}$



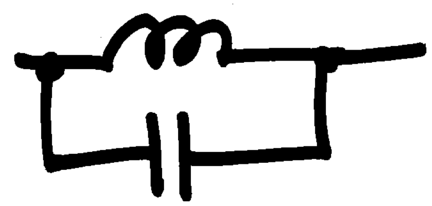




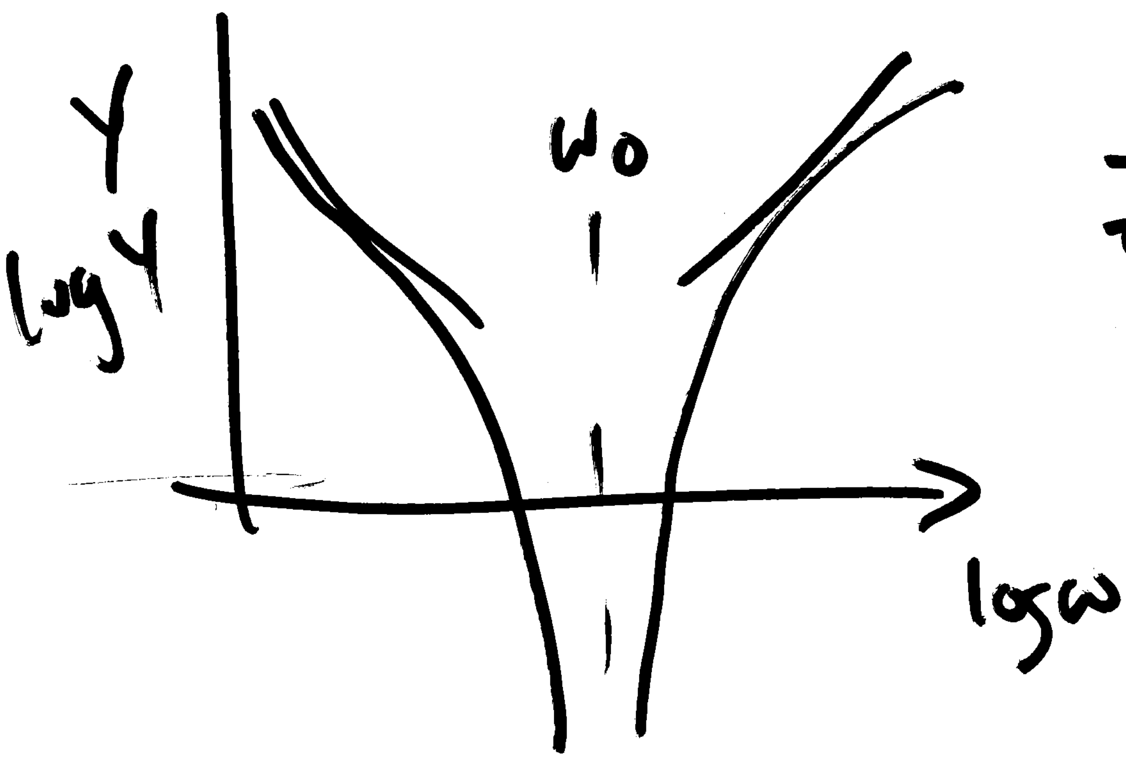
log₁₀ Z
log ω

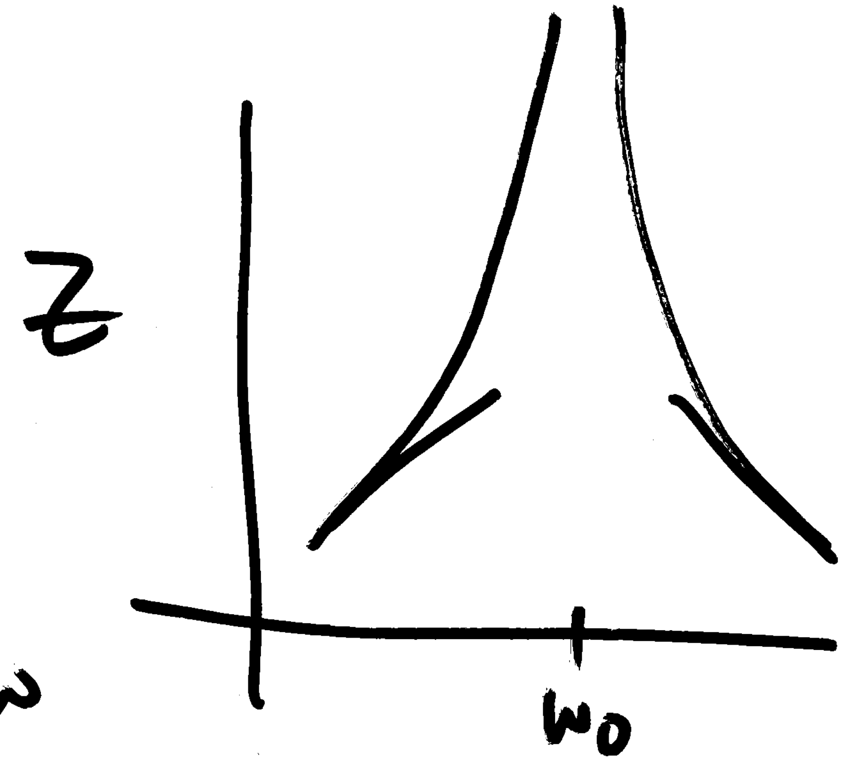


log₁₀ Y
log ω



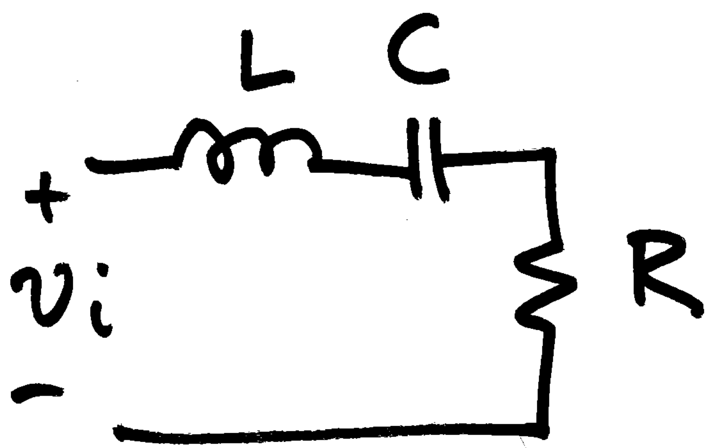
$$Y = sC + \frac{1}{sL} = \frac{1-\omega^2 LC}{j\omega L}$$





$\omega_c = R/L \qquad \omega_c = RC$

Band pass Filter $\quad$ R, L, C

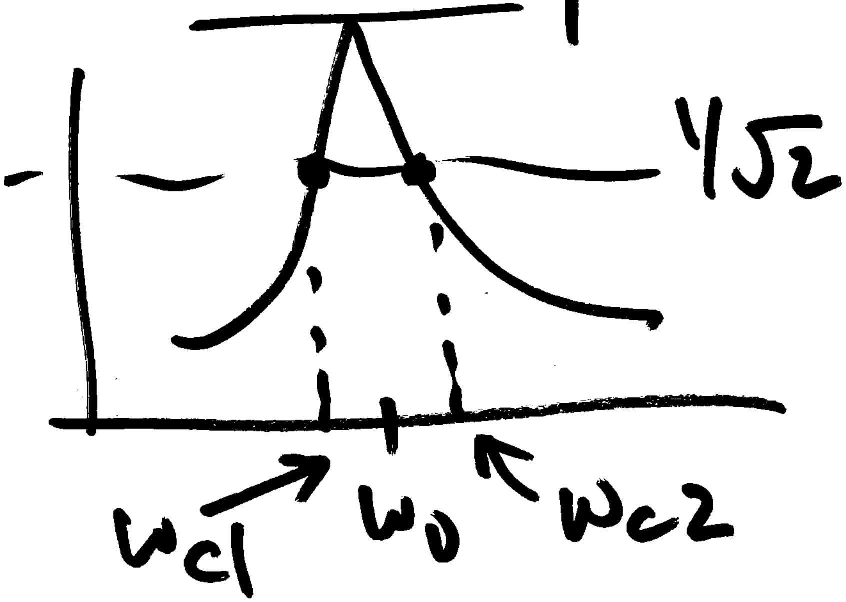


$\Rightarrow$

$$V_o(s) = \frac{R\,V_i(s)}{sL + 1/sC + R} = \frac{s(R/L)\,V_i(s)}{s^2 + s(R/L) + (1/LC)}$$

$$H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{j\omega(R/L)}{-\omega^2 + j\omega(R/L) + (1/LC)}$$

Maximum of $H(j\omega)$



$$1/LC - \omega^2 = 0$$

$$\omega = \omega_0 = \sqrt{1/LC}$$

center frequency

$$|H(j\omega)| = \frac{\omega R/L}{\sqrt{\left(\frac{\omega R}{L}\right)^2 + \left[\left(\frac{1}{LC}\right) - \omega^2\right]^2}}$$

$$= \frac{1}{\sqrt{1 + \Bigg[\underbrace{\frac{\left(\frac{1}{LC}\right) - \omega^2}{\omega R/L}}_{\pm 1}\Bigg]^2}} = \frac{1}{\sqrt{2}}$$

$$\frac{1}{LC} - \omega^2 = \pm\frac{\omega R}{L}$$

$$\omega^2 \pm \frac{\omega R}{L} - \frac{1}{LC} = 0$$

$$\omega = \frac{\pm\frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 + 4\left(\frac{1}{LC}\right)}}{2}$$

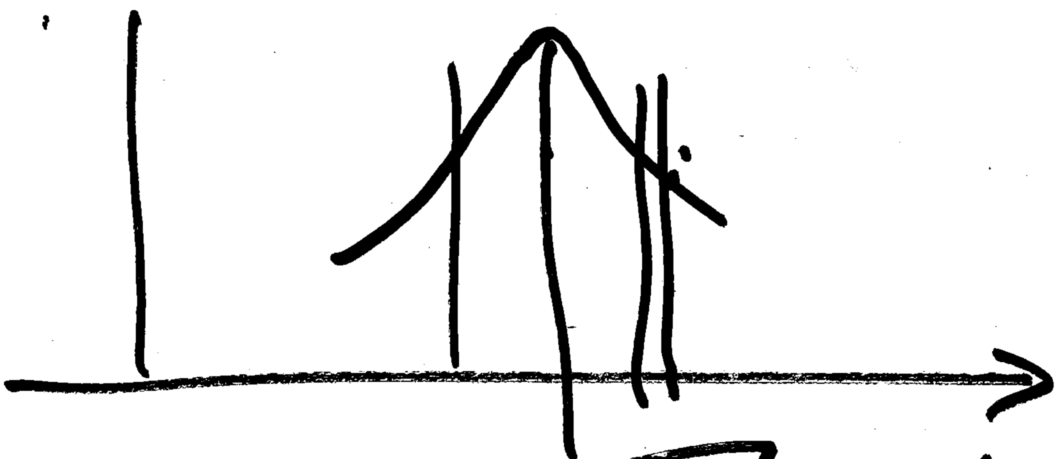
$$\omega_c = \pm\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

$$\omega_{c1} = -\frac{R}{2L} + \sqrt{}, \quad \omega_{c2} = +\frac{R}{2L} + \sqrt{}$$

$$\sqrt{\omega_{c1}\omega_{c2}} = \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right) - \left(\frac{R}{2L}\right)^2}$$

$$\omega_0 = \boxed{\cancel{\sqrt{\frac{1}{LC}}}}$$

$$\frac{\omega_0}{\omega_{c1}} = \frac{\omega_{c2}}{\omega_0}$$

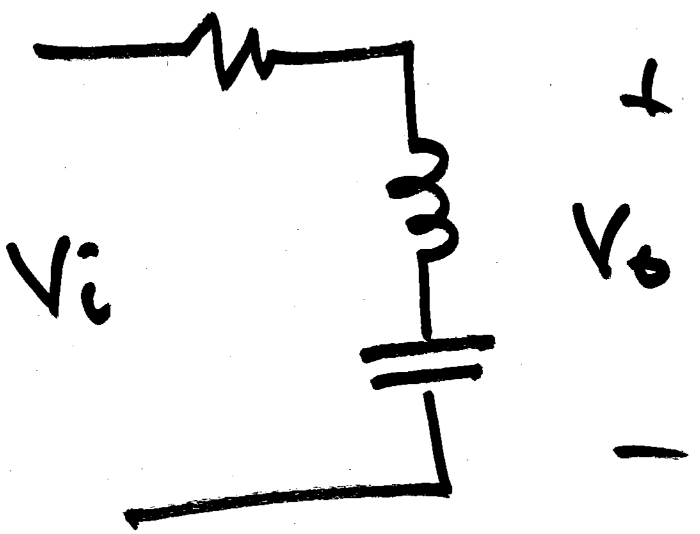


$\log \omega$

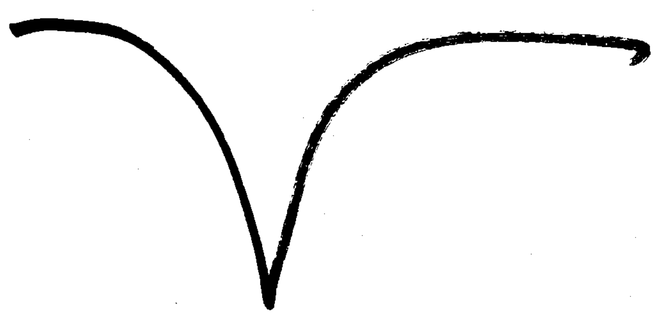
$$\beta = \text{Bandwidth}$$
$$= BW$$
$$= \omega_{c2} - \omega_{c1}$$
$$= R/L$$

$$\omega_0 = \sqrt{1/LC}$$

$$Q = \frac{\omega_0}{\beta} = \frac{\sqrt{1/LC}}{R/L} = \sqrt{\frac{L}{R^2C}} = \frac{1}{R}\sqrt{L/C}$$



$$H(s) = \frac{s^2 + (1/LC)}{s^2 + s\left(\frac{R}{L}\right) + \left(\frac{1}{LC}\right)}$$

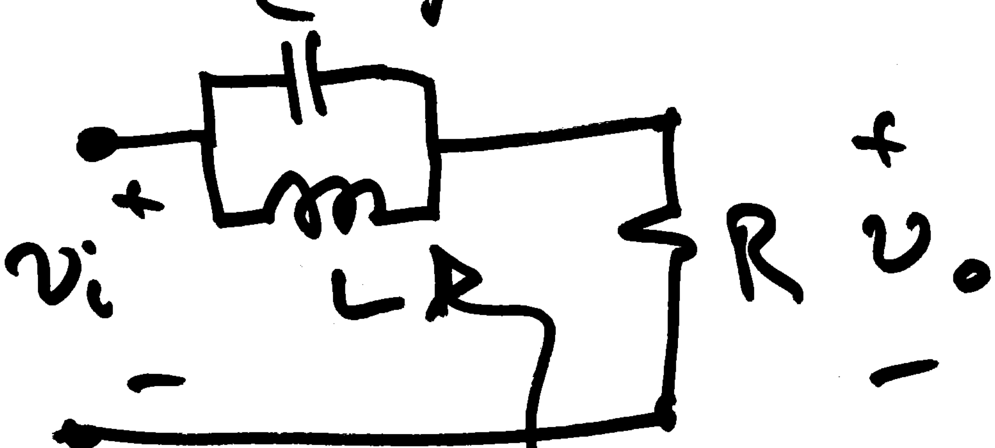


Notch or

Band - reject

Filter

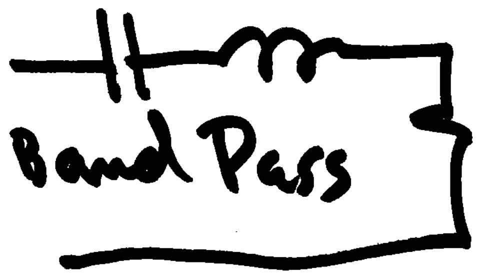
Band Reject Filter



$$H(s) = \frac{V_o(s)}{V_i(s)}$$

$$= \frac{R}{R + \frac{sL/\cancel{C}}{s^2L + 1/\cancel{sC}}}$$

$$= \frac{R(s^2LC+1)}{R(s^2LC+1) + sL}$$



$$Z_{L\|C} = \frac{\cancel{s}L/\cancel{s}C}{sL + 1/sC} =$$

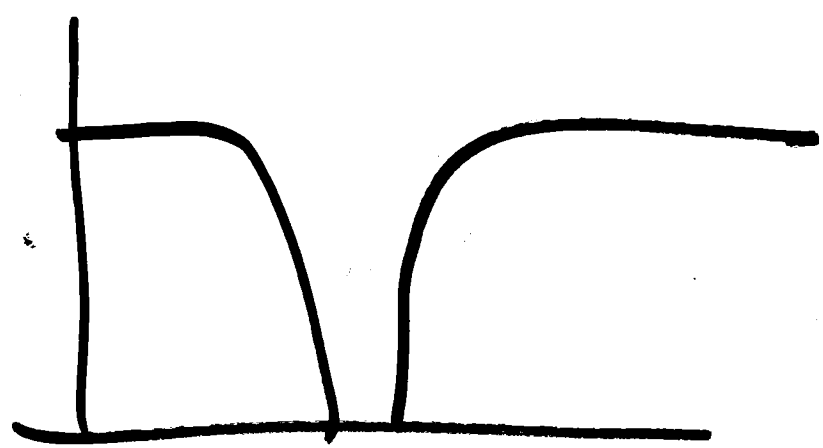
$$H(s) = \frac{R(s^2LC+1)}{R(s^2LC+1) + sL} = \frac{s^2RLC + R}{s^2RLC + R + sL}$$

$$= \frac{s^2 + 1/LC}{s^2 + s/RC + 1/LC}$$

$$H(j\omega) = \frac{-\omega^2 + 1/LC}{-\omega^2 + j\omega/RC + 1/LC}$$

$$= \frac{1/LC - \omega^2}{1/LC - \omega^2 + j\omega/RC}$$

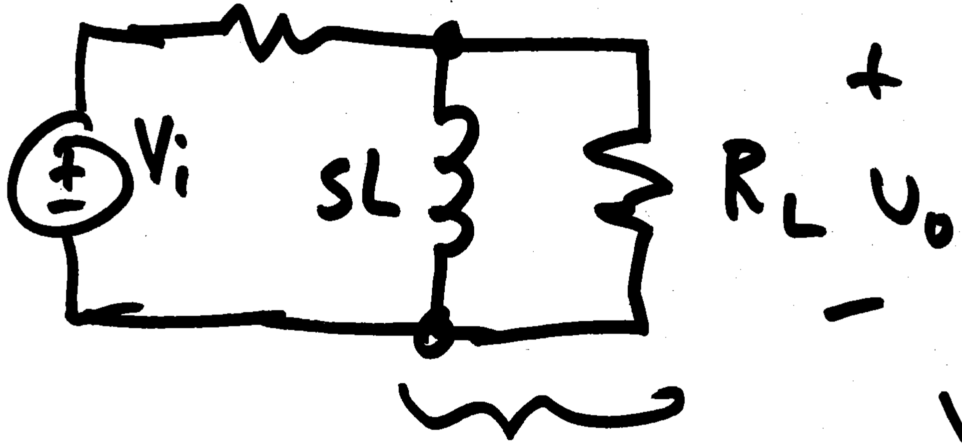
$$\omega = \omega_0 = \sqrt{1/LC} \Rightarrow H(j\omega) = 0$$



$$Q = \omega_0 RC$$

$$= \frac{RC}{\sqrt{LC}} = R\sqrt{\frac{C}{L}}$$

Loaded Passive Filter



$$Z = \left(\frac{R_L sL}{R_L + sL} \right)$$

$$\frac{V_o}{V_i} = \frac{\cancel{R_L}\, Z}{Z + R} = \frac{\dfrac{R_L s L}{R_L + sL}}{R + \dfrac{R_L s L}{R_L + sL}}$$

$$= \frac{R_L s L}{R(R_L + sL) + R_L s L} = \frac{s(R_L L)}{s(RL + R_L L) + R R_L}$$

$$= \frac{s\left(\frac{R_L \cancel{L}}{R\cancel{L} + R_L \cancel{L}}\right)}{s + \frac{RR_L}{RL + R_L L}} = \frac{s\left(R_L / (R+R_L)\right)}{s + \frac{R}{L}\left(\frac{R_L}{R+R_L}\right)}$$

$$K = R_L / (R+R_L)$$

$K = 1$ unloaded

$K < 1$ loaded

$$H(s) = \frac{Ks}{s + K\frac{R}{L}}$$

peak gain $= K$

$$H(j\omega) = \frac{j\omega K}{j\omega + K\frac{R}{L}}$$

ω_c: $K\frac{R}{L} = \omega_c$

$$H(s) = K\frac{s^m + a_1 s^{m-1} + \cdots \; a_{m} s^0}{s^n + b_1 s^{n-1} + \cdots \; b_n s^0}$$

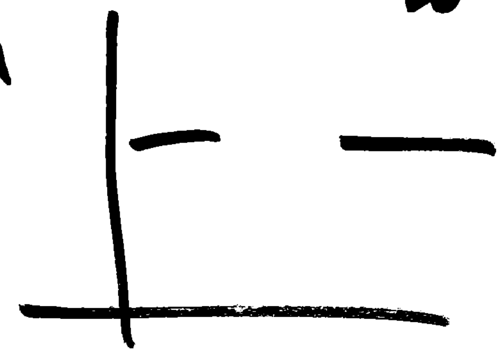
$$\lim_{s \to \infty} H(s) = K s^{m-n}$$

$$\lim_{\substack{s \to 0 \\ \omega \to 0}} H(j\omega) = K\frac{a_m}{b_n}$$

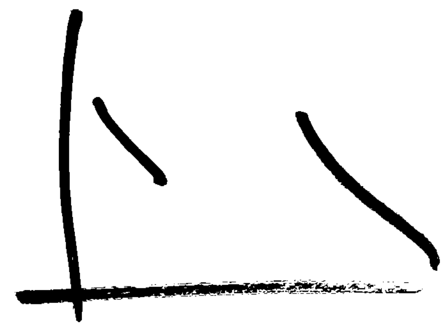
non-zero

$m > n$

$m = n$



$m < n$



$$H(s) = \frac{K(s+z_1)(s+z_2)\ \cdots}{(s+p_1)(s+p_2}$$

1 Real pole

$s = -a$

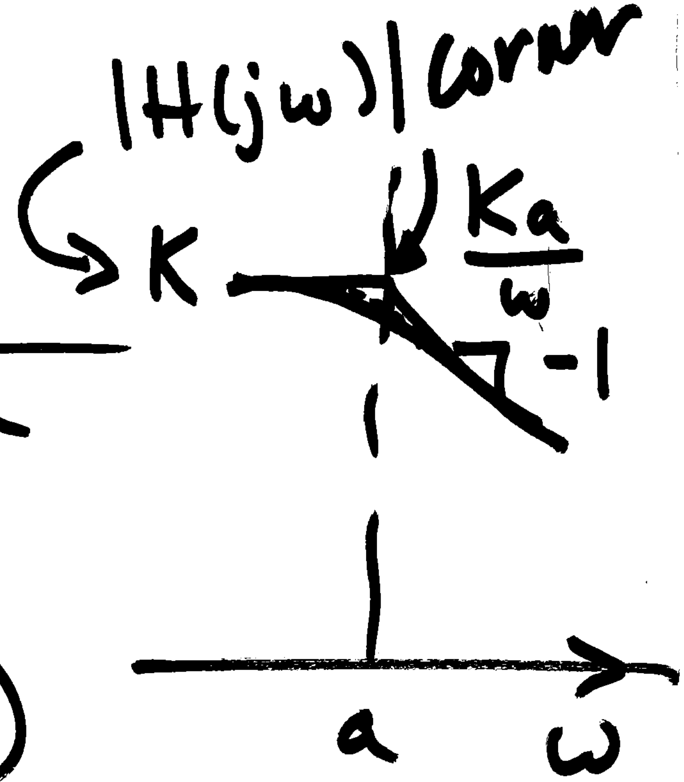
$$H(s) = \frac{Ka}{s+a}$$

$$H(j\omega) = \frac{Ka}{j\omega + a}$$

$$a = \omega$$

$$H(ja) = K\left(\frac{a}{a(1+j)}\right)$$

$$|H(ja)| = K/\sqrt{2} = 0.707\,K$$



dB often used for magnitude of gain

$$\left|\frac{V_o}{V_i}\right| \equiv \text{gain}$$

$|H(j\omega)| = 1$ 0dB

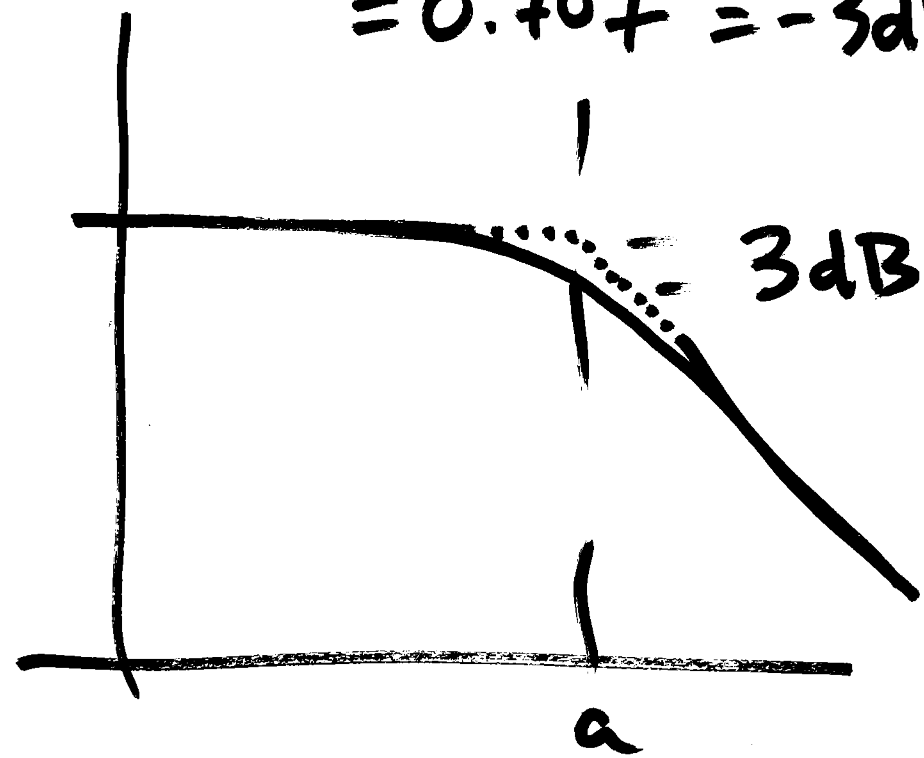
$= 0.1$ -20dB

$dB = 20\log_{10}|H(j\omega)|$

$= 10$ +20dB

$= 0.707 = -3dB$

$$\frac{Ka}{a+j\omega}$$



$\frac{1}{2} \Rightarrow -6\,\text{dB}$

$\frac{1}{\sqrt{10}} \quad -10\,\text{dB}$

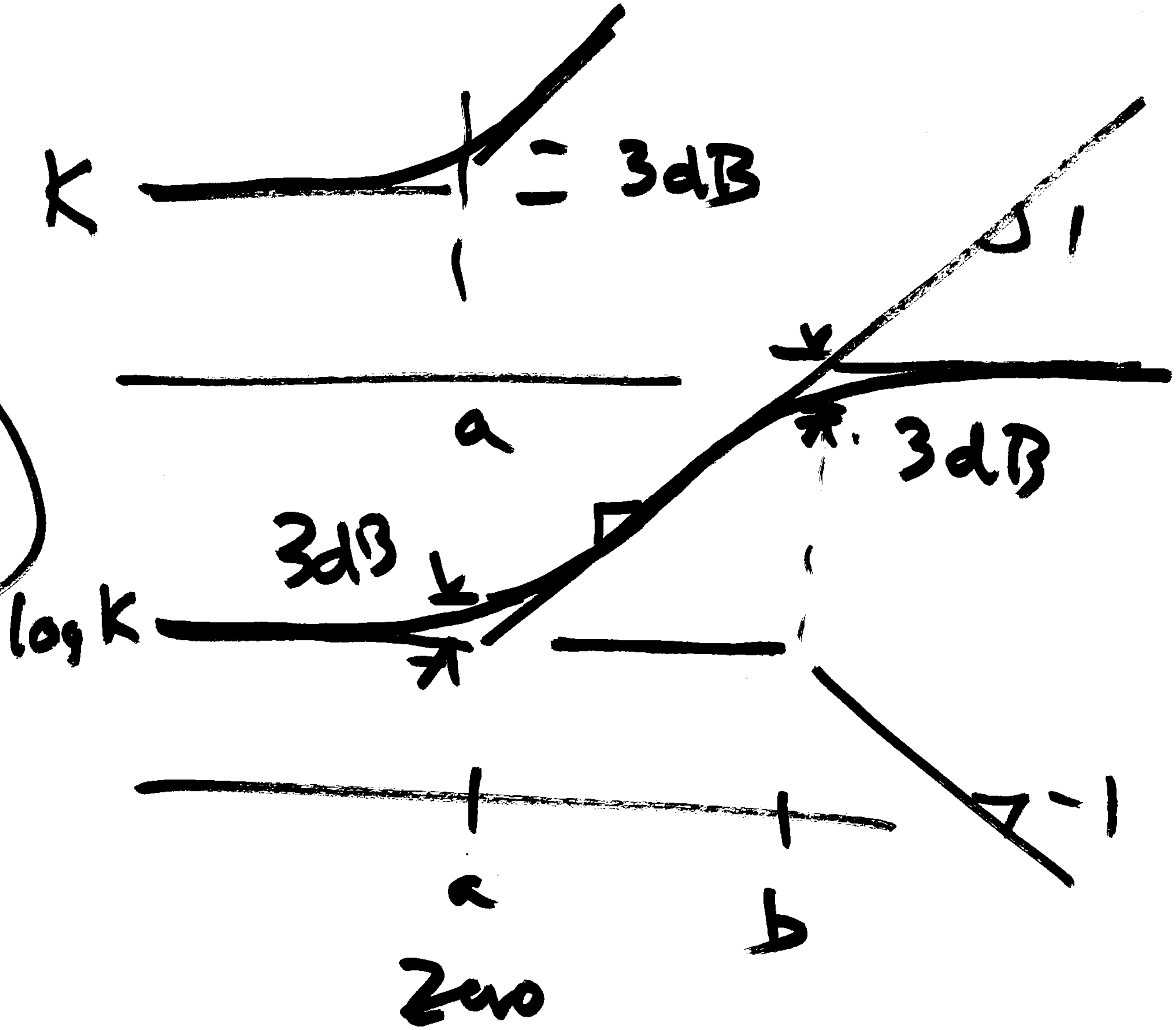
$5 \quad +20-6 = 14\,\text{dB}$

$8 \quad 6+6+6 = 18\,\text{dB}$

$= 2^3 \quad 3 \times 6 \quad ''$

$$K \frac{s+a}{a}$$

$$K\left(\frac{s+a}{a}\right)\left(\frac{b}{s+b}\right)$$

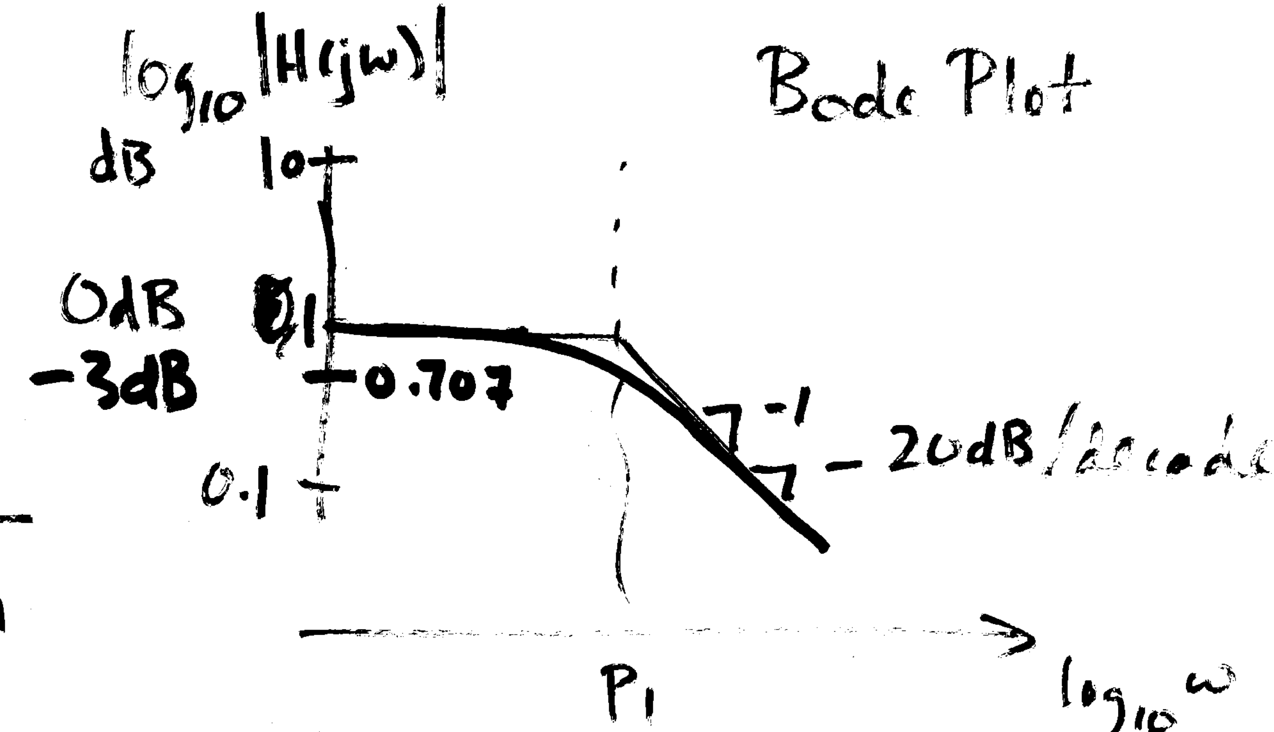


$$\frac{a}{s+a}$$

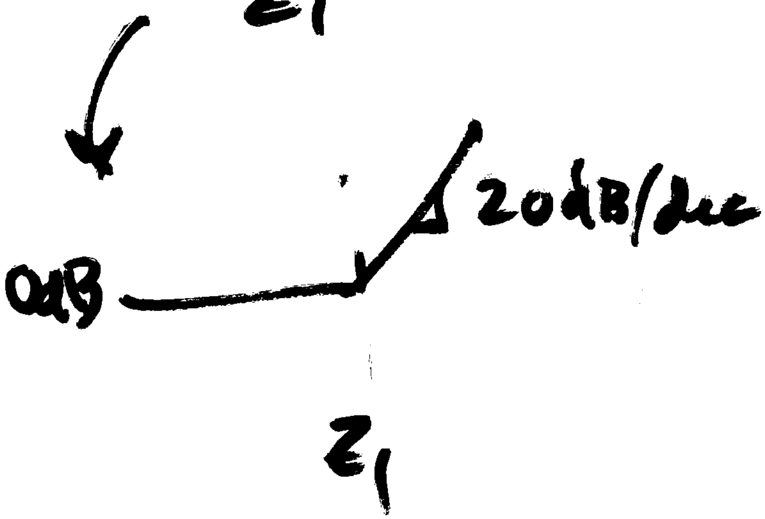
Bode Plot

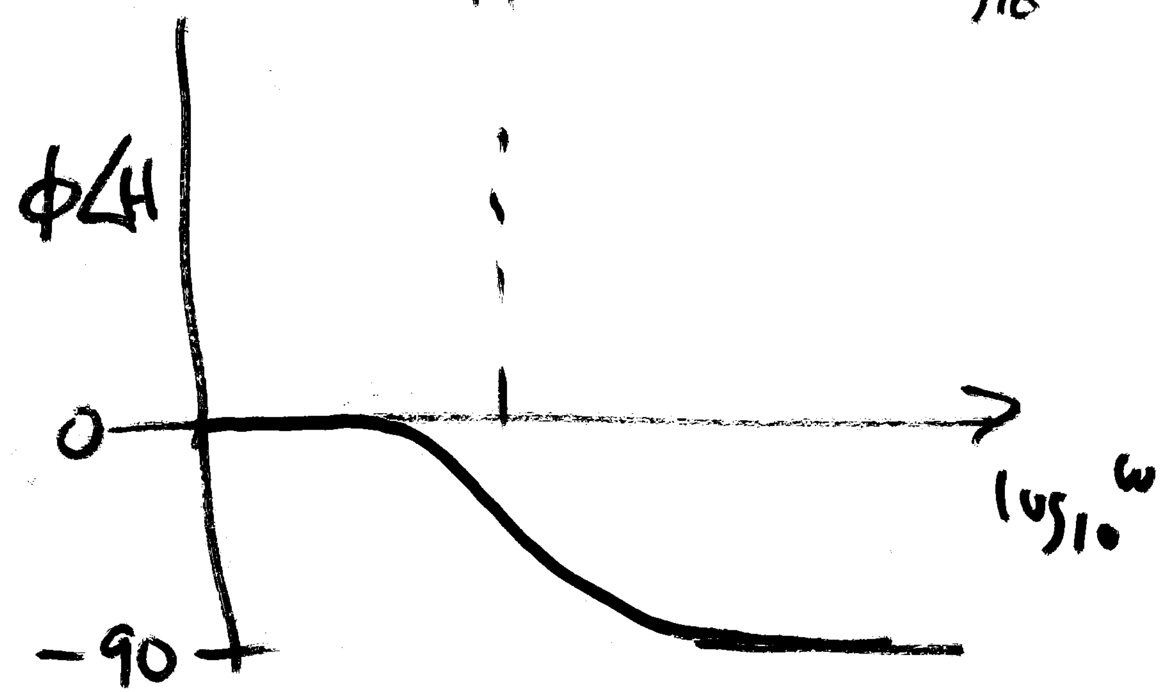
$$\frac{p_1}{s+p_1}$$

$$\frac{p_1}{\cancel{s+}\,j\omega+p_1}$$



$$\frac{s+z_1}{z_1}$$





$$H(s) = \frac{s(s+10^7)\,100}{(s+100)(s+10^4)(s+10^5)}$$

Zeroes @ $0, 10^7$ rad/s

poles @ $100, 10^4, 10^5$ rad/s

$\omega \sim 10^3$

$$\approx \frac{\cancel{(j\omega)}\ \cancel{10^7}\ \cancel{10^2}}{\cancel{(j\omega)}\ \cancel{10^4}\ \cancel{10^5}} \approx 1$$

$\omega \gg 10^7$ rad/s

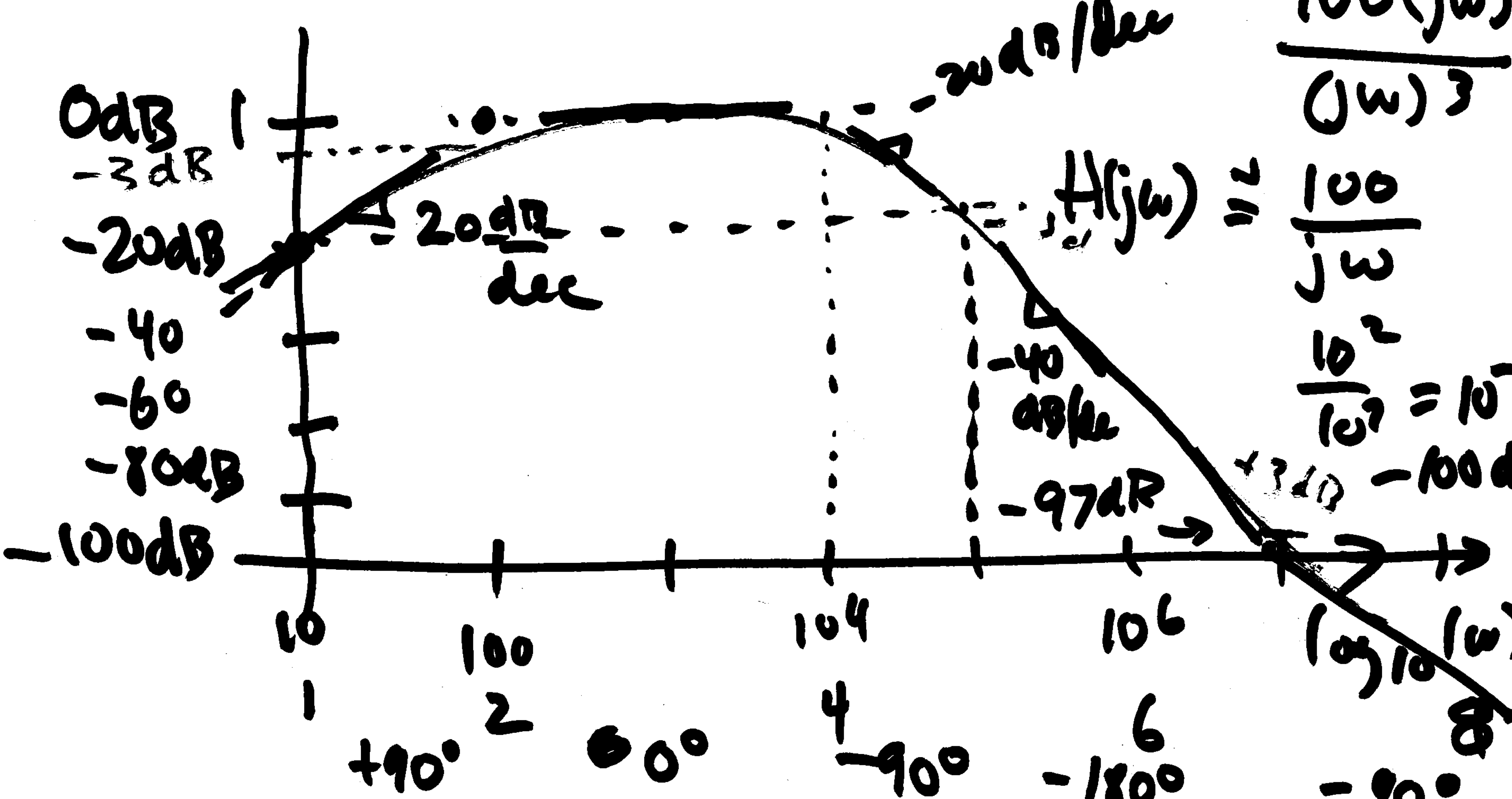
$\omega \to \infty$

$$\frac{100(j\omega)^2}{(j\omega)^3}$$

$$H(j\omega) \approx \frac{100}{j\omega}$$

$$\frac{10^2}{10^7} = 10^{-5}$$

-100 dB



$$s^2 + 2\alpha s + \omega_0^2$$

$$-\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\omega_0^2 = \alpha^2 + \beta^2$$

$$s^2 + b\cancel{\ }s + \omega_0^2$$

bandwidth

$$-\alpha \pm j\beta$$

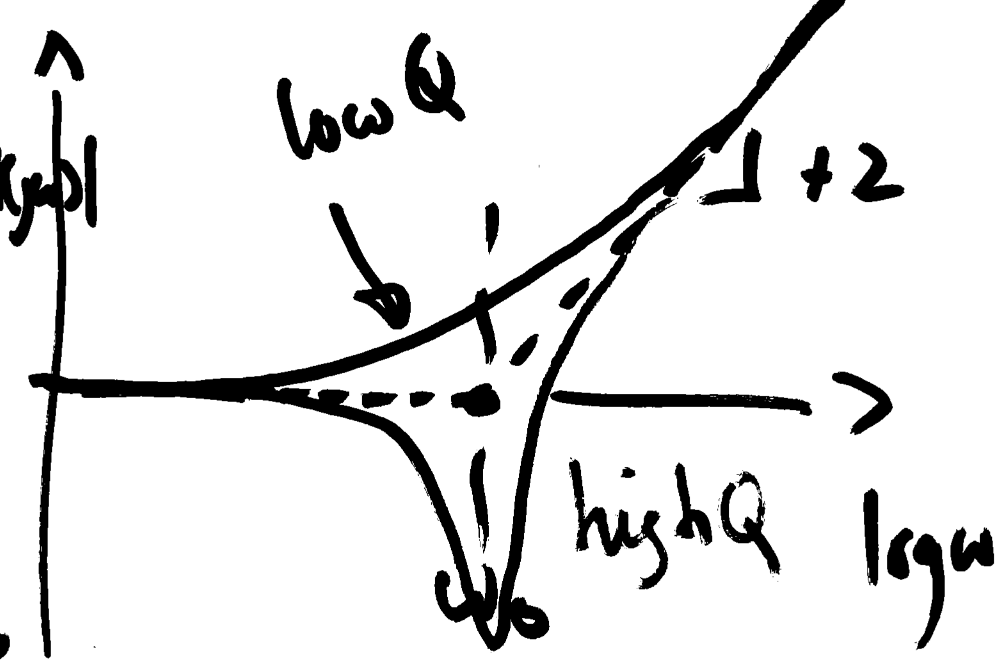
$$2|K|e^{-\alpha t}\cos(\beta t + \theta)$$

$$s^2 + \frac{\omega_0}{Q}s + \omega_0^2$$

$$H(j\omega) = \frac{-\omega^2 + j\frac{\omega\omega_0}{Q} + \omega_0^2}{\omega_0^2}$$

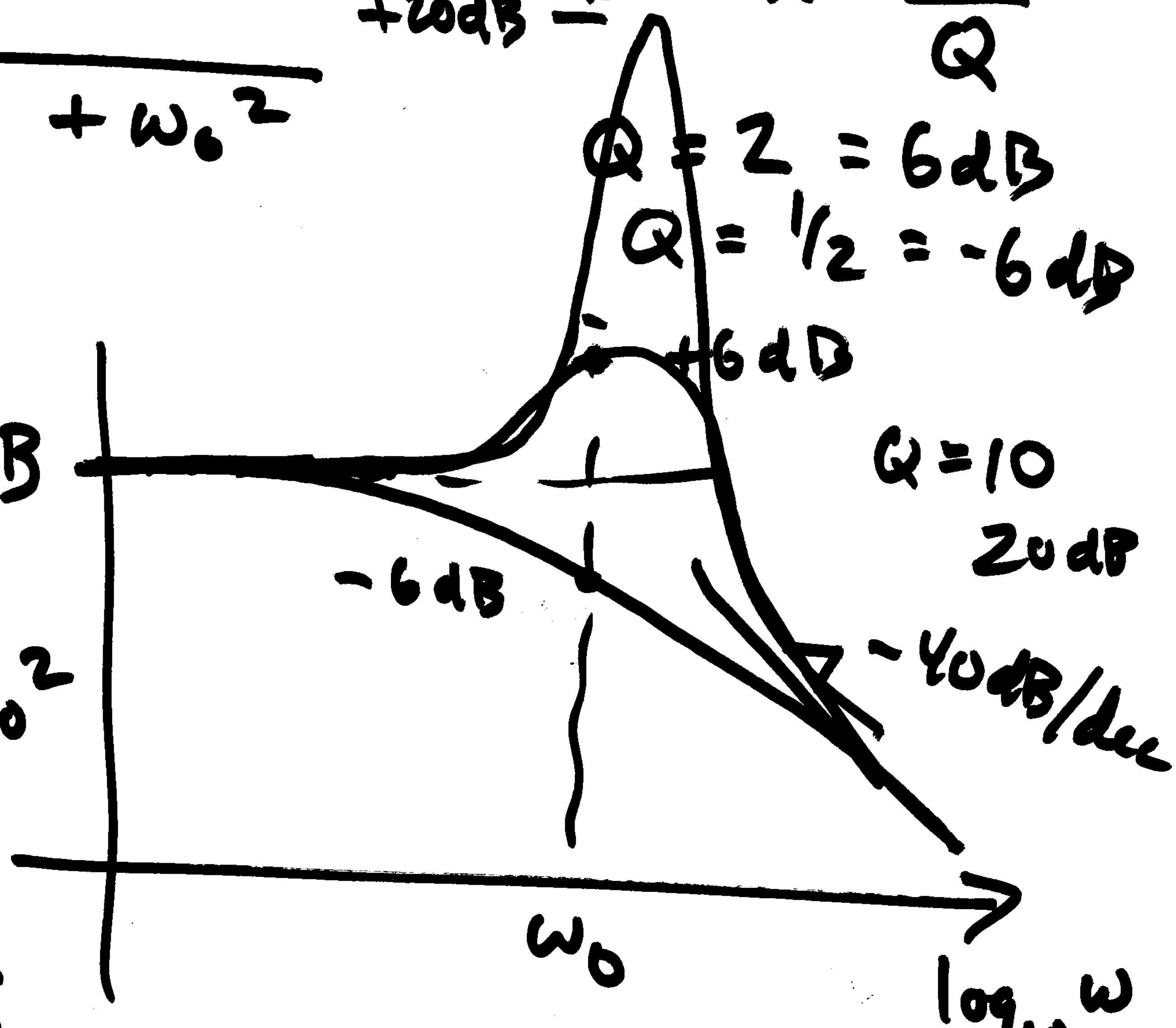
$@\omega = \omega_0$

$$|H(j\omega)| = \frac{j\omega_0\cancel{\omega_0}}{Q\omega_0^{\cancel{2}}} = \frac{1}{Q}\frac{\omega_0}{\omega_0}$$



$$H(s) = \frac{\omega_0^2}{s^2 + b\omega + \omega_0^2}$$

$b = 2\alpha = \frac{\omega_0}{Q}$



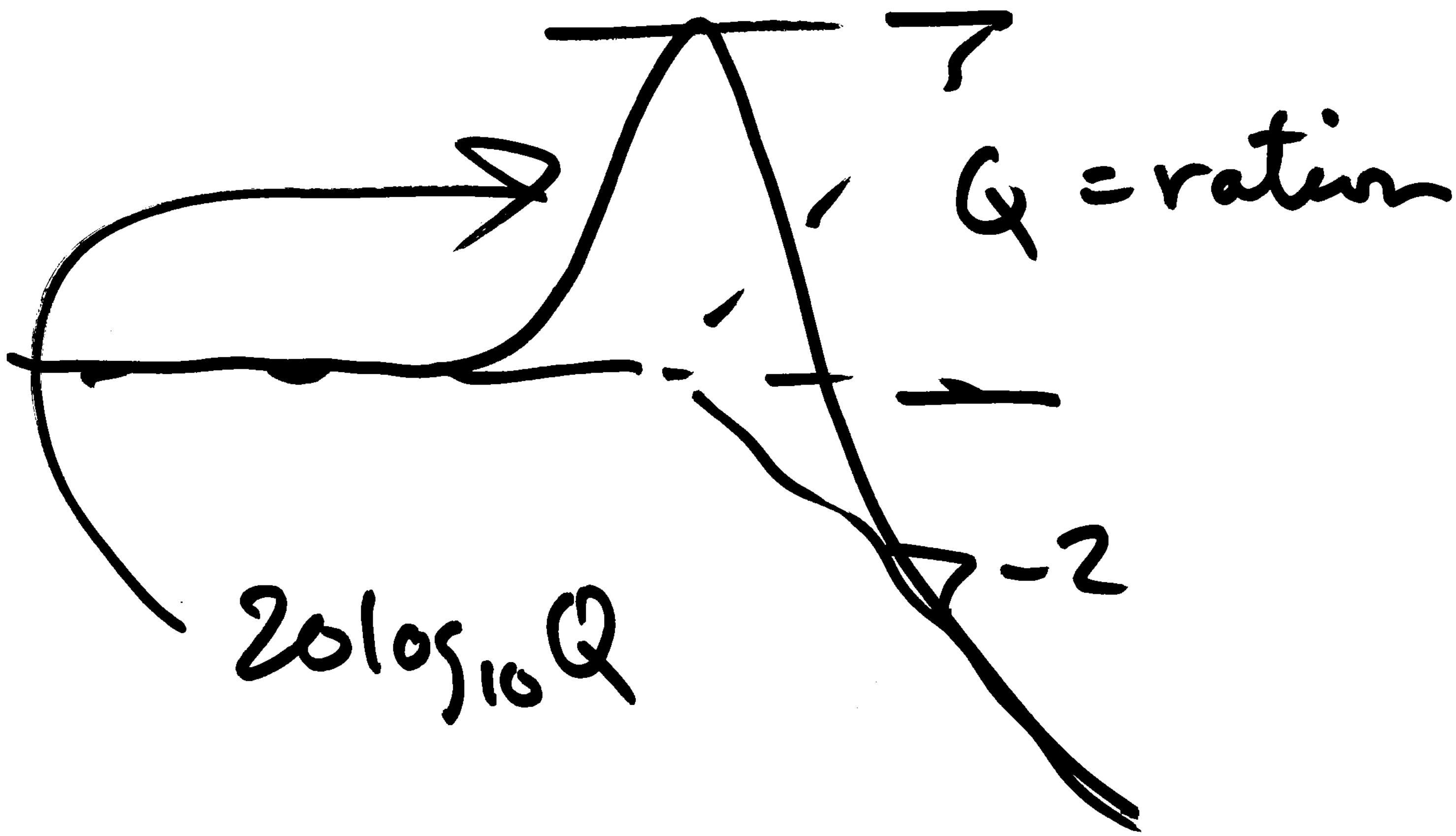
$$H(j\omega_0)$$

$$= \frac{\omega_0^2}{-\omega^2 + \frac{\omega_0\omega}{Q}j + \omega_0^2}$$

$$= \frac{\omega_0^2}{(\omega_0^2 - \omega^2) + \frac{\omega_0\omega}{Q}j}$$

$$H(j\omega_0) = \frac{\omega_0^{\cancel{2}}}{\frac{\cancel{\omega_0}\omega}{Q}j} = \frac{Q\omega_0}{j\omega_0}$$

$0°$ $-90°$ $-180°$


Q = ratio
$20\log_{10} Q$
-2

$$H(s) = \frac{\cancel{s^2+} \frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\frac{\omega_0}{Q} = \frac{R}{L}$$

$|H(j\omega)|$