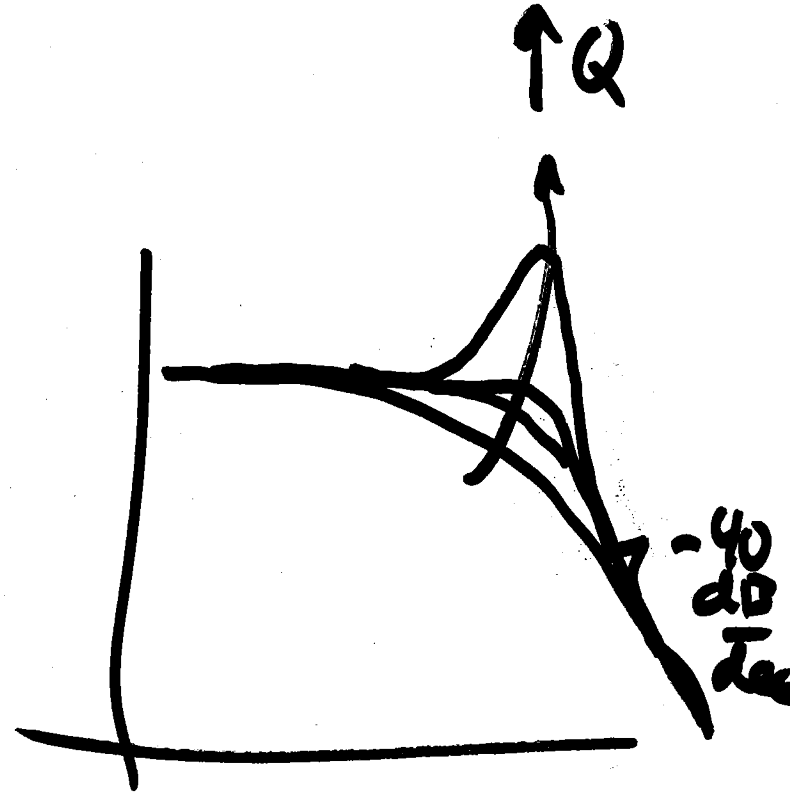
1. Phasors, SSS
2. Laplace, transient
3. Filters, design
4.

$$\frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$



low $Q < 1/2 \Rightarrow$ 2 real roots

$$\frac{\omega_0^2}{(s+\omega_{c1})(s+\omega_{c2})}$$

$\omega_{c1}\omega_{c2} = \omega_0^2$

Transient response

impulse response

$$H(s) = \frac{K_1}{s+\omega_{c1}} + \frac{K_2}{s+\omega_{c2}}$$

$$K_1 e^{-\omega_{c1} t} + K_2 e^{-\omega_{c2} t}$$

$\cos(\omega t)$

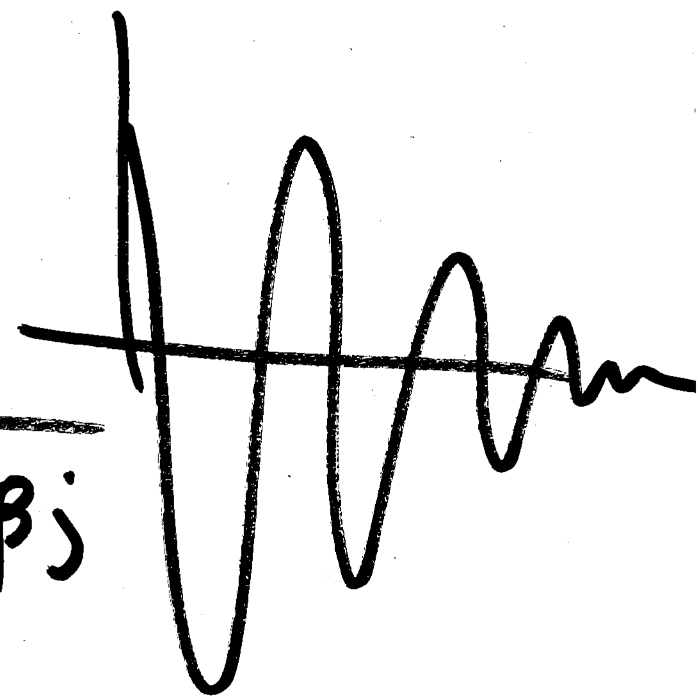
$$H(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0} \quad \frac{s}{(s^2+\omega^2)}$$

$Q_1 > 1 \Rightarrow$ complex conj pair of roots

$$H(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

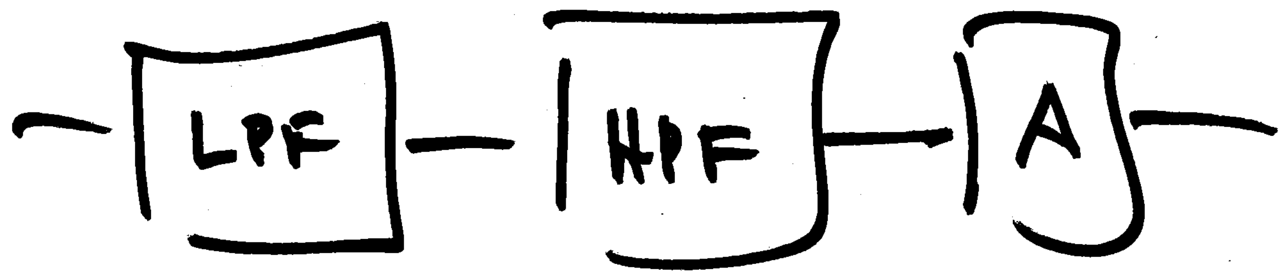
Impulse response

$$H(s) = \frac{K_1}{s - \alpha + \beta j} + \frac{K^*}{s - \alpha - \beta j}$$

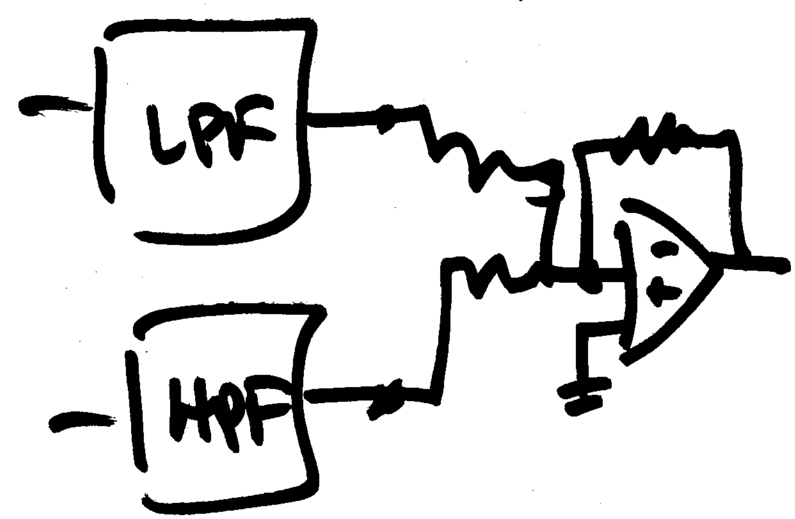


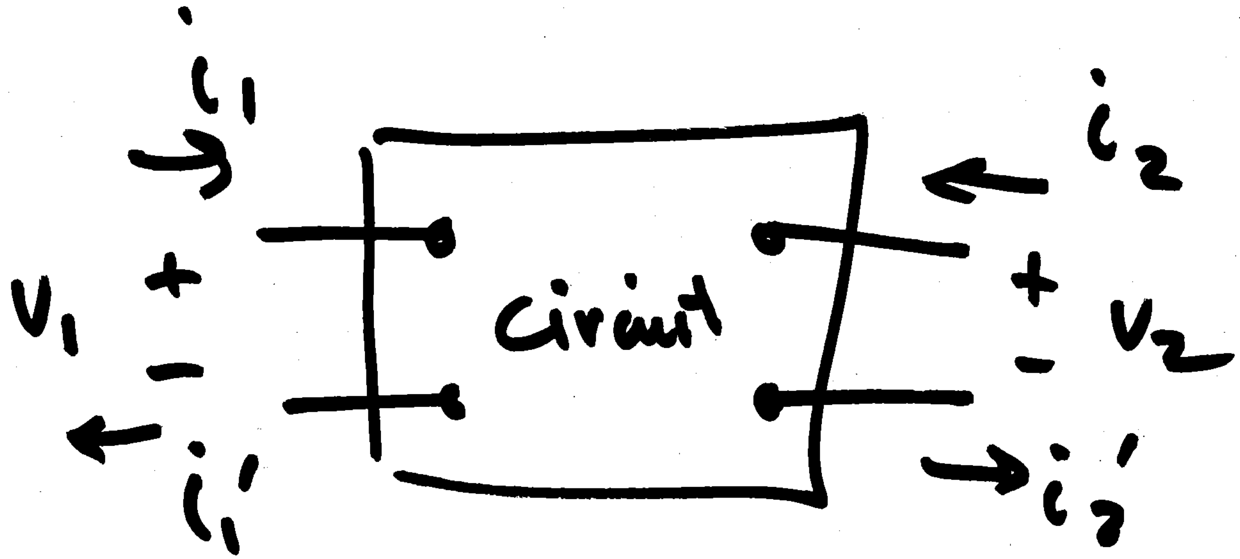
$$h(t) = e^{-\alpha t} \cos(\beta t + \phi)$$

<u>Two ports</u>



Active op-amp circuits had $Z_{out} = 0$





Know any 2 $\Rightarrow$ calculate the other 2

$$V_1 = \underline{z_{11}} I_1 + \underline{z_{12}} I_2$$

$$V_2 = \underline{z_{21}} I_1 + \underline{z_{22}} I_2$$

$$I_2 = 0 \Rightarrow z_{11} = \frac{V_1}{I_1}$$

$$z_{21} = \frac{V_2}{I_1}$$

$$I_1 = 0 \Rightarrow z_{12} = \frac{V_1}{I_2}$$

$$z_{22} = \frac{V_2}{I_2}$$

the upper terminal. This symmetry makes it easier to alysis of a two-port network and is the reason for its uni- iterature.

eral description of the two-port network is carried out in purely resistive networks, the analysis reduces to solving Sinusoidal steady-state problems can be solved either by appropriate s-domain expressions and then replacing s ect analysis in the frequency domain. Here, we write all s domain; resistive networks and sinusoidal steady-state special cases. Figure 18.2 shows the basic building block domain variables I_1, V_1, I_2, and V_2.

r terminal variables, only two are independent. Thus for we specify two of the variables, we can find the two owns. For example, knowing V_1 and V_2 and the circuit e can determine I_1 and I_2. Thus we can describe a two- h just two simultaneous equations. However, there are six which to combine the four variables:

$$\begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2, \\ V_2 &= z_{21}I_1 + z_{22}I_2; \end{aligned} \qquad \bar{V} = \bar{Z}\bar{I} \qquad (18.1)$$

$$\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2, \\ I_2 &= y_{21}V_1 + y_{22}V_2; \end{aligned} \qquad \hat{I} = \bar{Y}\bar{V} \qquad (18.2)$$

$$\left.\begin{aligned} V_1 &= a_{11}V_2 - a_{12}I_2, \\ I_1 &= a_{21}V_2 - a_{22}I_2; \end{aligned}\right\} \text{transmission} \qquad (18.3)$$

$$\left.\begin{aligned} V_2 &= b_{11}V_1 - b_{12}I_1, \\ I_2 &= b_{21}V_1 - b_{22}I_1; \end{aligned}\right\} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix} = \bar{B}\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} \qquad (18.4)$$

$$\left.\begin{aligned} V_1 &= h_{11}I_1 + h_{12}V_2, \\ I_2 &= h_{21}I_1 + h_{22}V_2; \end{aligned}\right\} \text{hybrid} \qquad (18.5)$$

$$\begin{aligned} I_1 &= g_{11}V_1 + g_{12}I_2, \\ V_2 &= g_{21}V_1 + g_{22}I_2. \end{aligned} \qquad (18.6)$$

ts of equations may also be considered as three pairs of relations. The first set, Eqs. 18.1, gives the input and out- unctions of the input and output currents. The second set,

Equations 18.7–18.1(as follows:

- z_{11} is the imped
- z_{12} is a transfer port 2 current w
- z_{21} is a transfer port 1 current w
- z_{22} is the imped

Therefore the i measured by first o V_2/I_1, and then open Example 18.1 illustra tive circuit.

Example 18.1 Fi

Find the z paramet

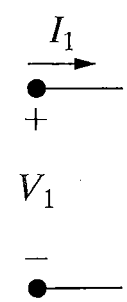
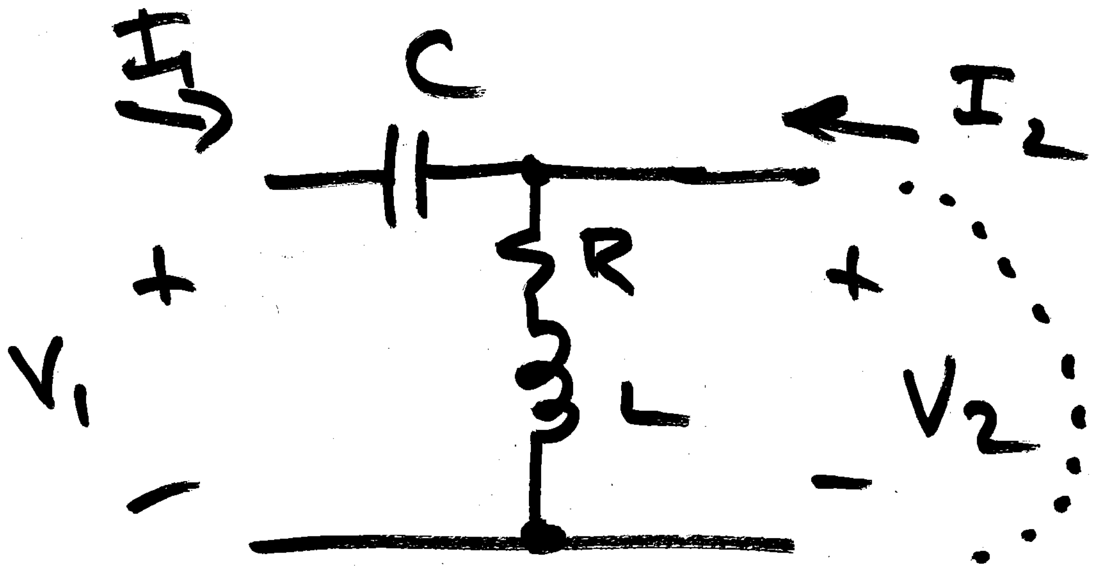


Figure 18.3 ▲ The ci

Solution

The circuit is pur cuit is also purely $I_2 = 0$, the resista 20 Ω resistor in p



Calculate
H-parameters

$h_{11}, h_{21} \Rightarrow V_2 = 0$

$h_{12}, h_{22} \Rightarrow I_1 = 0$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{1}{sC}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -1$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = 1$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{R+sL}$$

$$a_{11} = -\frac{\Delta h}{h_{21}} = \frac{-(h_{11}h_{22} - h_{12}h_{21})}{h_{21}}$$

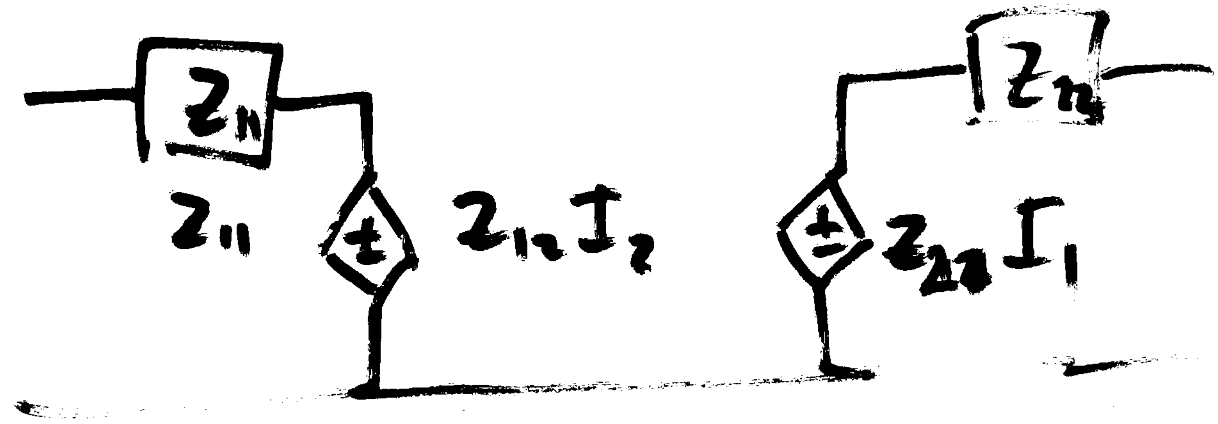
$$= -\left\{ \frac{\frac{1}{sC(R+sL)} + 1}{-1} \right\} = \frac{sC(R+sL)+1}{sC(R+sL)}$$

a_{12}

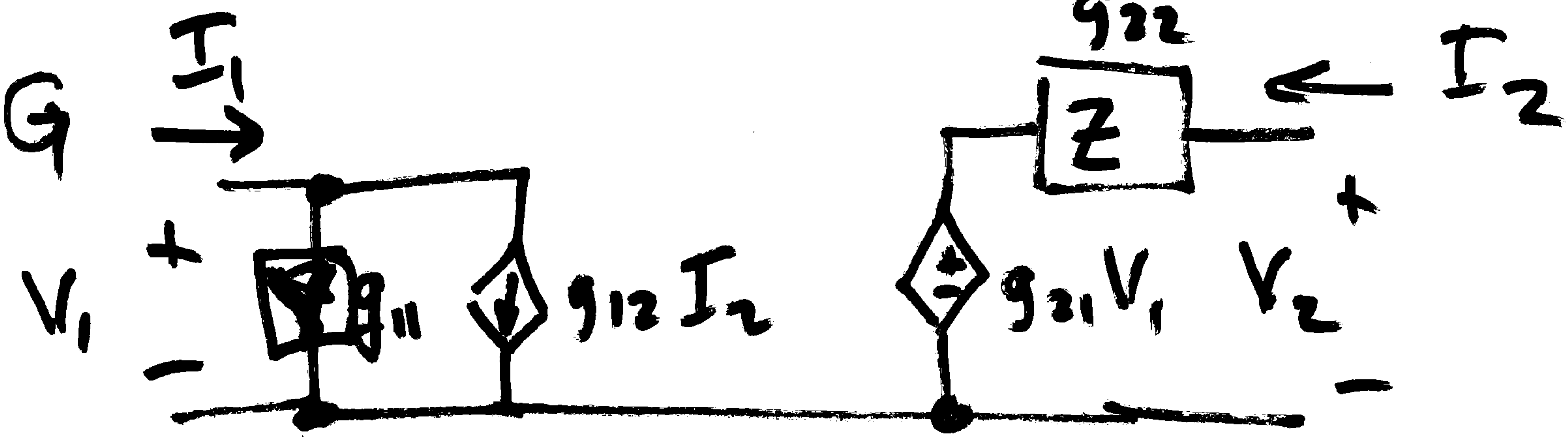
a_{21}

a_{22}

Z :



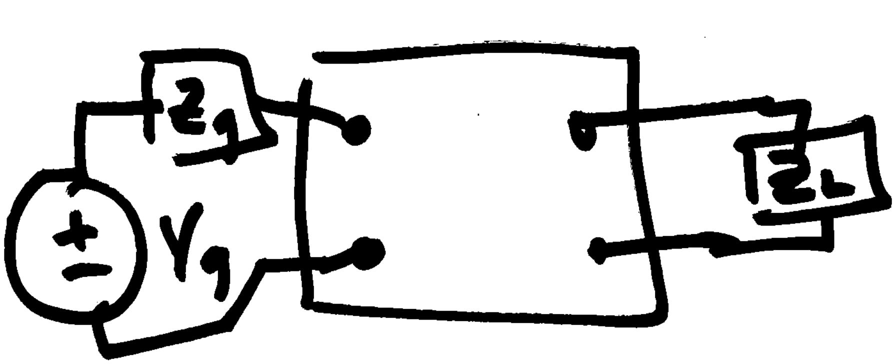
G



$$I_1 = V_1 g_{11} + g_{12} I_2$$

$$V_2 = g_{21} V_1 + g_{22} I_2$$

$$= G \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$



$Z_{in} = \frac{V_1}{I_1}$

output I_1 current I_2

~~output current I_2~~

V_{th}, Z_{th}

I_2 / I_1

V_2 / V_1

V_2 / V_g

TABLE 18.2 Terminated Two-Port Equations

***z* Parameters**

$$Z_{in} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$$

$$I_2 = \frac{-z_{21}V_g}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$$

$$V_{Th} = \frac{z_{21}}{z_{11} + Z_g}V_g$$

$$Z_{Th} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g}$$

$$\frac{I_2}{I_1} = \frac{-z_{21}}{z_{22} + Z_L}$$

$$\frac{V_2}{V_1} = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta z}$$

$$\frac{V_2}{V_g} = \frac{z_{21}Z_L}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$$

***y* Parameters**

$$Y_{in} = y_{11} - \frac{y_{12}y_{21}Z_L}{1 + y_{22}Z_L}$$

$$I_2 = \frac{y_{21}V_g}{1 + y_{22}Z_L + y_{11}Z_g + \Delta y Z_g Z_L}$$

$$V_{Th} = \frac{-y_{21}V_g}{y_{22} + \Delta y Z_g}$$

$$Z_{Th} = \frac{1 + y_{11}Z_g}{y_{22} + \Delta y Z_g}$$

$$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$$

$$\frac{V_2}{V_1} = \frac{-y_{21}Z_L}{1 + y_{22}Z_L}$$

$$\frac{V_2}{V_g} = \frac{y_{21}Z_L}{y_{12}y_{21}Z_g Z_L - (1 + y_{11}Z_g)(1 + y_{22}Z_L)}$$

***a* Parameters**

$$Z_{in} = \frac{a_{11}Z_L + a_{12}}{a_{21}Z_L + a_{22}}$$

$$I_2 = \frac{-V_g}{a_{11}Z_L + a_{12} + a_{21}Z_g Z_L + a_{22}Z_g}$$

$$V_{Th} = \frac{V_g}{a_{11} + a_{21}Z_g}$$

$$Z_{Th} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g}$$

$$\frac{I_2}{I_1} = \frac{-1}{a_{21}Z_L + a_{22}}$$

$$\frac{V_2}{V_1} = \frac{Z_L}{a_{11}Z_L + a_{12}}$$

$$\frac{V_2}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

***b* Parameters**

$$Z_{in} = \frac{b_{22}Z_L + b_{12}}{b_{21}Z_L + b_{11}}$$

$$I_2 = \frac{-V_g\Delta b}{b_{11}Z_g + b_{21}Z_g Z_L + b_{22}Z_L + b_{12}}$$

$$V_{Th} = \frac{V_g\Delta b}{b_{22} + b_{21}Z_g}$$

$$Z_{Th} = \frac{b_{11}Z_g + b_{12}}{b_{21}Z_g + b_{22}}$$

$$\frac{I_2}{I_1} = \frac{-\Delta b}{b_{11} + b_{21}Z_L}$$

$$\frac{V_2}{V_1} = \frac{\Delta b Z_L}{b_{12} + b_{22}Z_L}$$

$$\frac{V_2}{V_g} = \frac{\Delta b Z_L}{b_{12} + b_{11}Z_g + b_{22}Z_L + b_{21}Z_g Z_L}$$

***h* Parameters**

$$Z_{in} = h_{11} - \frac{h_{12}h_{21}Z_L}{1 + h_{22}Z_L}$$

$$I_2 = \frac{h_{21}V_g}{(1 + h_{22}Z_L)(h_{11} + Z_g) - h_{12}h_{21}Z_L}$$

$$V_{Th} = \frac{-h_{21}V_g}{h_{22}Z_g + \Delta h}$$

$$Z_{Th} = \frac{Z_g + h_{11}}{h_{22}Z_g + \Delta h}$$

$$\frac{I_2}{I_1} = \frac{h_{21}}{1 + h_{22}Z_L}$$

$$\frac{V_2}{V_1} = \frac{-h_{21}Z_L}{\Delta h Z_L + h_{11}}$$

$$\frac{V_2}{V_g} = \frac{-h_{21}Z_L}{(h_{11} + Z_g)(1 + h_{22}Z_L) - h_{12}h_{21}Z_L}$$

***g* Parameters**

$$Y_{in} = g_{11} - \frac{g_{12}g_{21}}{g_{22} + Z_L}$$

$$I_2 = \frac{-g_{21}V_g}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$$

$$V_{Th} = \frac{g_{21}V_g}{1 + g_{11}Z_g}$$

$$Z_{Th} = g_{22} - \frac{g_{12}g_{21}Z_g}{1 + g_{11}Z_g}$$

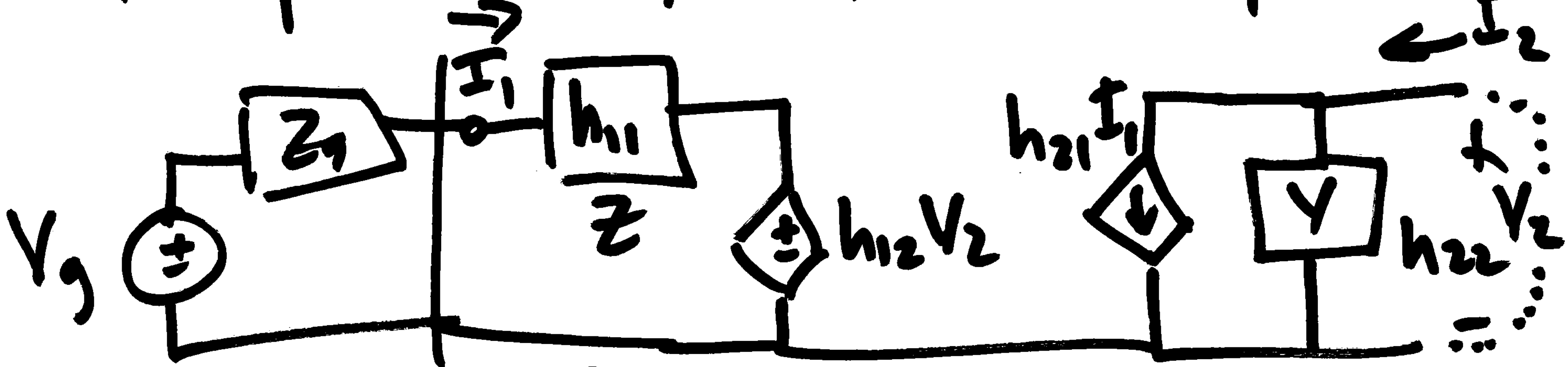
$$\frac{I_2}{I_1} = \frac{-g_{21}}{g_{11}Z_L + \Delta g}$$

$$\frac{V_2}{V_1} = \frac{g_{21}Z_L}{g_{22} + Z_L}$$

$$\frac{V_2}{V_g} = \frac{g_{21}Z_L}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$$

4 eq = 4 unknowns

Example: V_{th}, Z_{th} from h-parameters



$\underline{\underline{I_{sc}}}$, V_{oc} $\stackrel{SC}{=}$ $I_1 = \dfrac{V_g}{Z_g + h_{11}} \Rightarrow I_2 = +h_{21}I_1$

$I_{sc} = -I_2 = \dfrac{-h_{21}V_g}{Z_g + h_{11}}$

oc: $V_2 = -\dfrac{h_{21} I_1}{h_{22}}$

$$I_1 = \frac{V_g - h_{12} V_2}{Z_g + h_{11}}$$

$$V_2 = \frac{-h_{21}/h_{22}\,(V_g - h_{12} V_2)}{Z_g + h_{11}}$$

$$V_{oc} = V_2 = V_{th} \Rightarrow V_2\left(Z_g + h_{11} - \frac{h_{12} h_{21}}{h_{22}}\right) = -\frac{h_{21}}{h_{22}} V_g$$

$$V_{oc} = V_2 = V_{th} = \frac{-h_{21} V_g}{h_{22} Z_g + h_{11} h_{22} - h_{12} h_{21}}$$

$$= -h_{21} V_g / (h_{22} Z_g + \Delta h)$$

Cascade $\quad A$ or B

$A = A_1 A_2 \quad , \quad B = B_2 B_1$

Series $\quad Z = Z_1 + Z_2$

Parallel $\quad Y = Y_1 + Y_2$

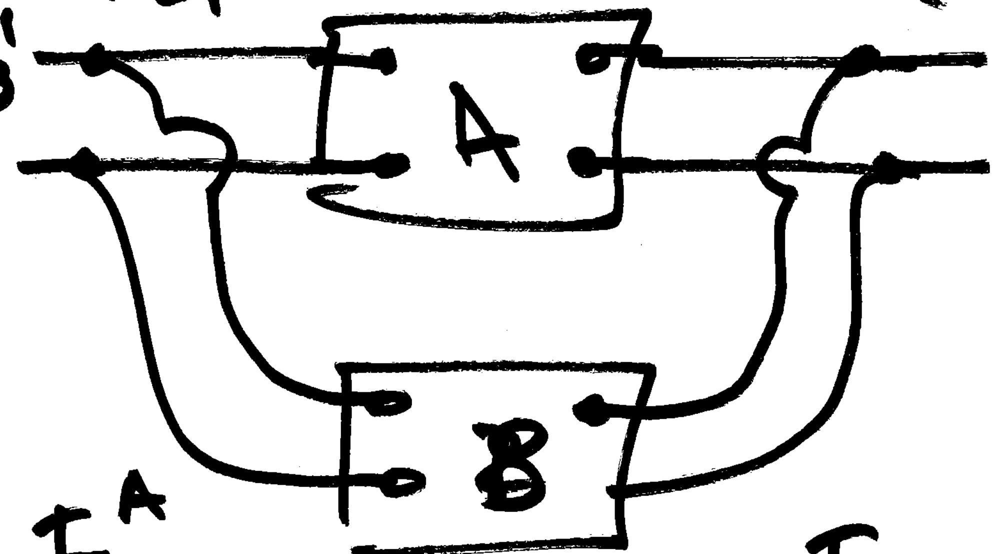
Hybrid: more complicated

$I_1 = I_1^A + I_1^B$

$V_1 \overset{+}{\underset{-}{=}} V_1^A = V_1^B$

$\longleftarrow I_2 = I_2^A + I_2^B$

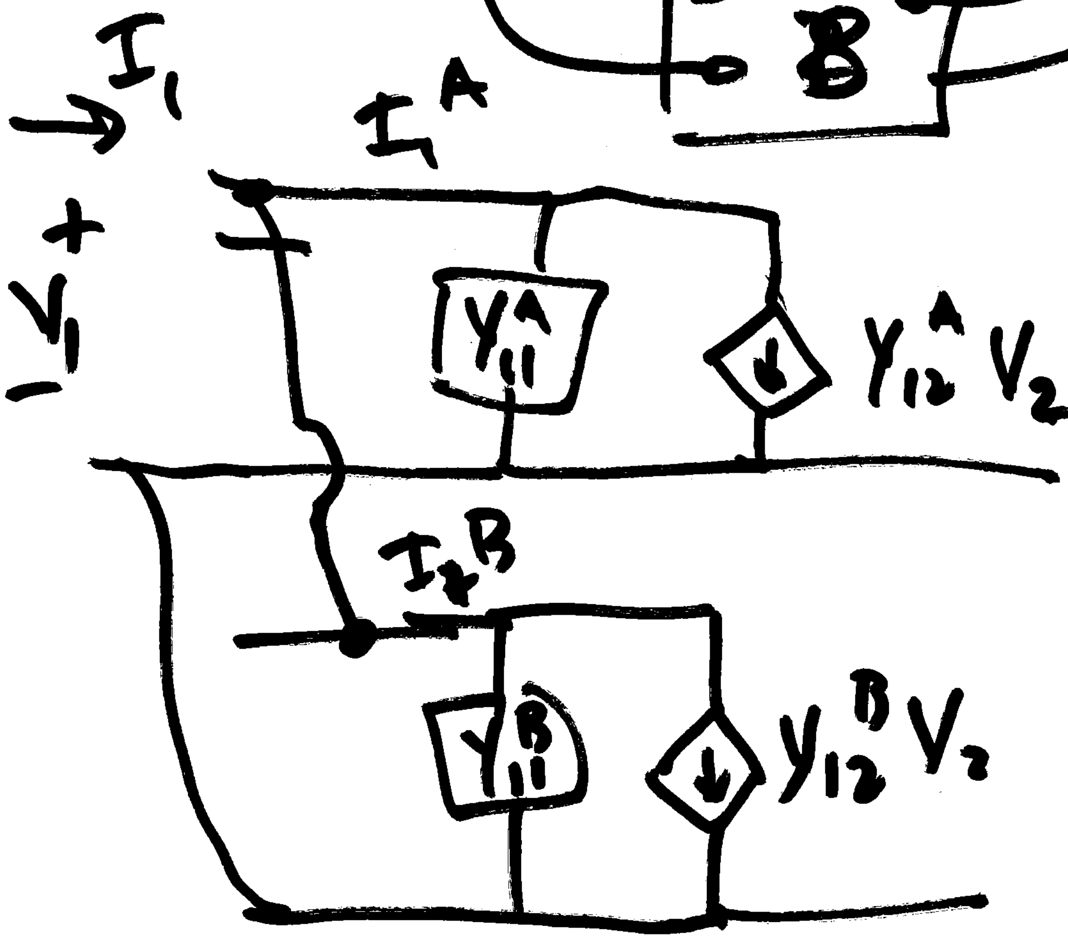
$V_2 = V_2^A = V_2^B$



$$Y = Y^A + Y^B$$

$$\overbrace{}$$

$$I_1 = V_1\left(Y_{11}^A + Y_{11}^B\right)$$
$$+ V_2\left(Y_{12}^A + Y_{12}^B\right)$$



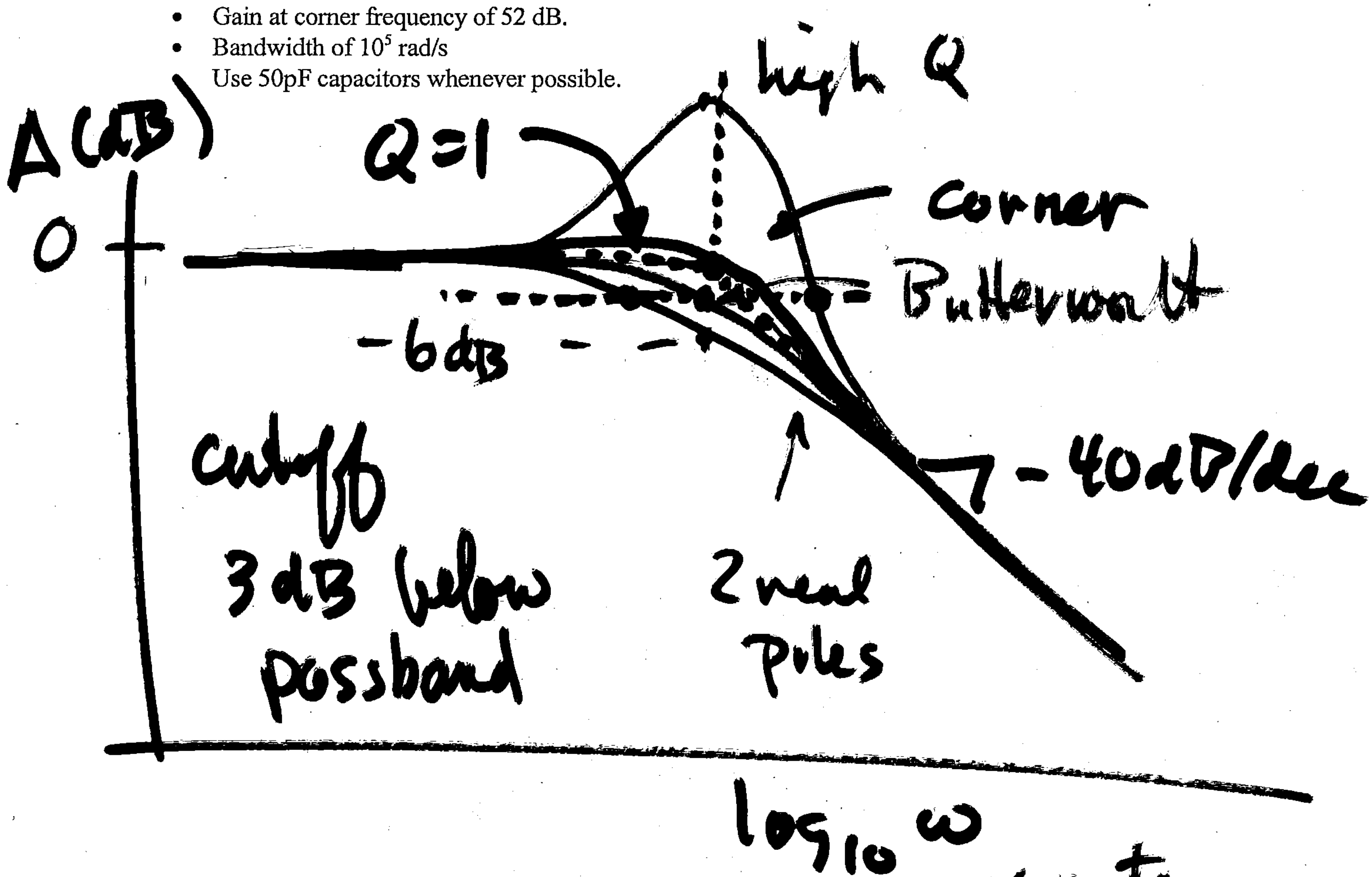
$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \left(\overline{\overline{Y_A + Y_B}}\right)\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

3. A filter has the transfer function

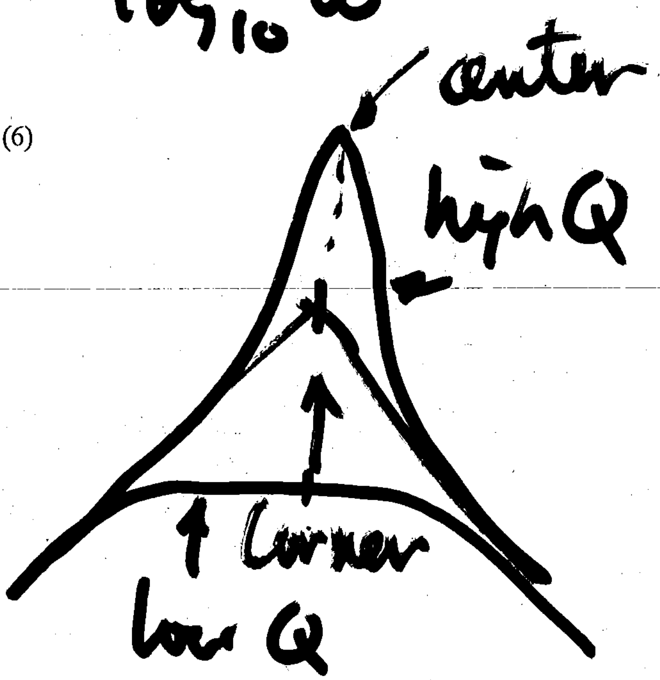
$$H(s) = \frac{-\left(\frac{1}{R_2 C_1}\right) s}{s^2 + \frac{s}{R_1 C_1} + \frac{1}{R_1 R_2 C_1 C_2}}.$$

(a) Choose values for R_1, R_2, C_1 and C_2 to satisfy the following conditions: (12)

- Center frequency of $2\text{x}10^6$ rad/s.
- Gain at corner frequency of 52 dB.
- Bandwidth of 10^5 rad/s
- Use 50pF capacitors whenever possible.



(b) Calculate the cutoff frequency or frequencies? (6)



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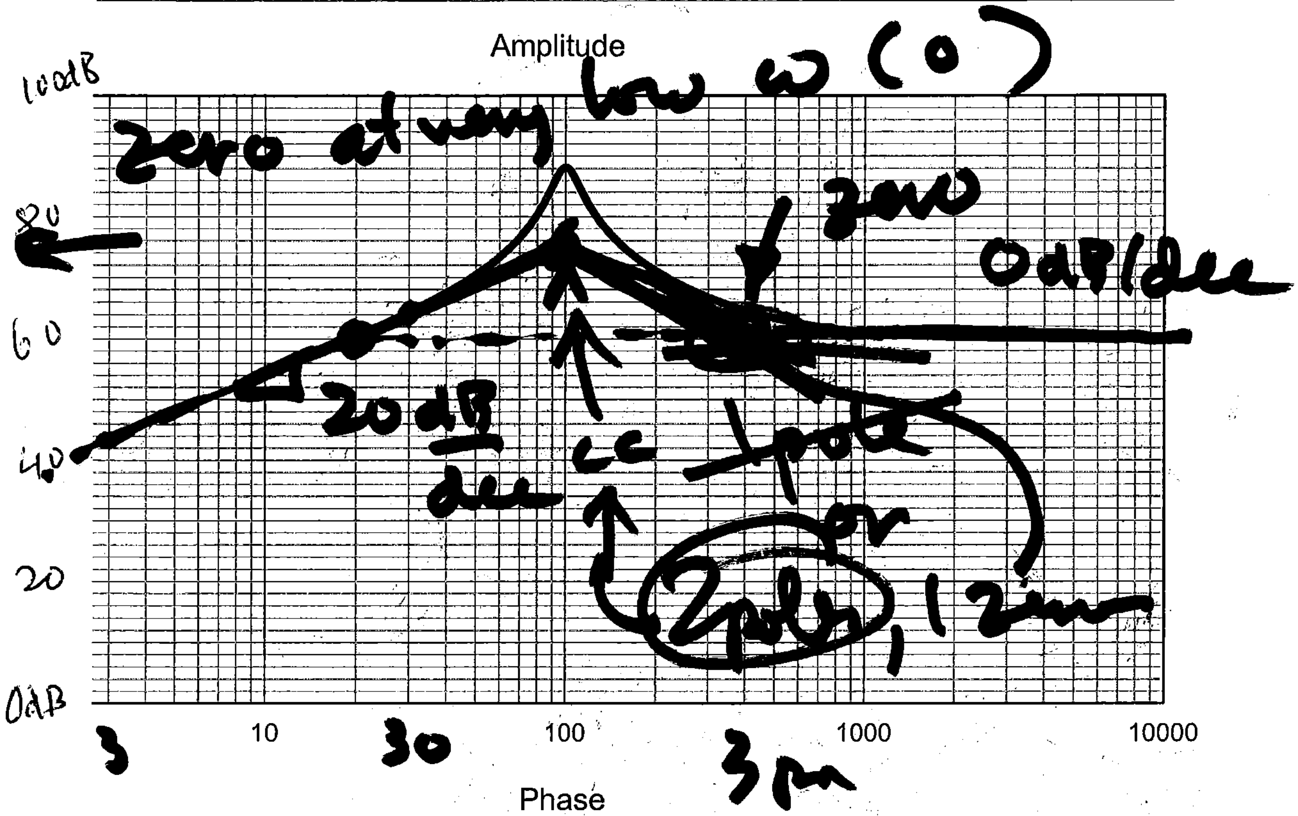
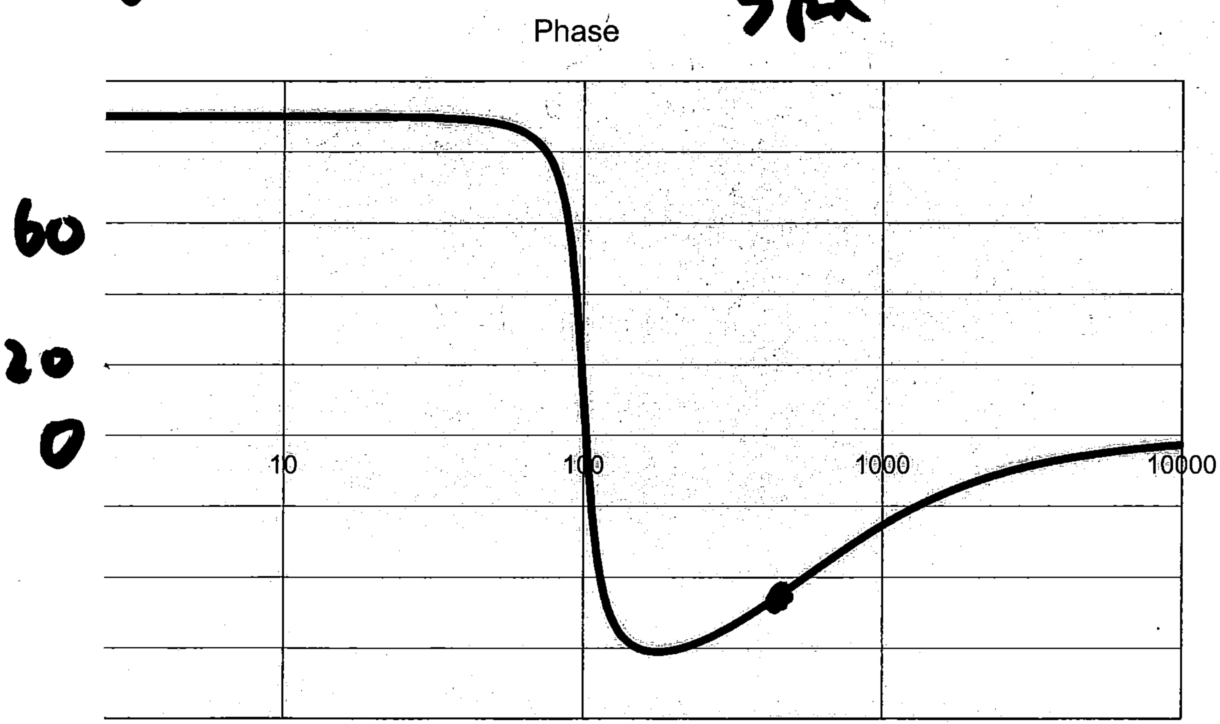

Amplitude
100dB
80
60
40
20
0dB
3
10
30
100
1000
10000
3pm
zero at very low ω (0)
zero
0dB/dec
20 dB/dec
cc
1 pole
or
2 poles, 1 zero


Phase
60
20
0
10
100
1000
10000

1) Phasor transform and SSS
- Working with complex impedances & ~~sources~~ currents/voltages

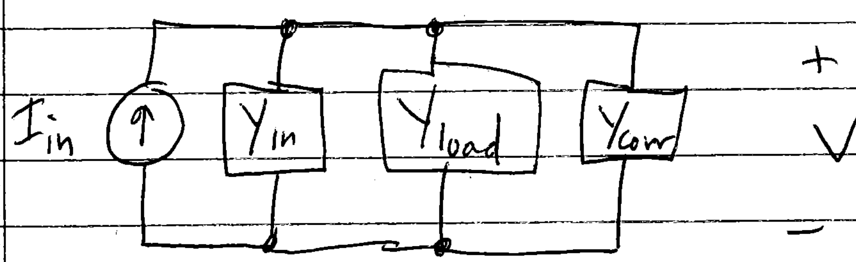
2) Complex power:
- Calculating & optimizing power transfer

3) Laplace transform
- General method for DEqs => AEqs
- Functional & Operational Transforms
- Use for circuits (s-domain models of R, L, C, etc.)
- Inverse Laplace transform (partial fractions)
- Impulse response and convolution transfer function

4) Filters and H(jω) from H(s)
- Loading of passive filters
- Designing passive and active filters
- Multistage filters
- Bodeplots mag plot <=> H(s)

5) Two ports

$$S = \underline{I^*_{rms} V_{rms}} = \frac{I_m^* V_m}{2}$$



$$I_{load} = Y_{load} V, \quad V = I_{in} / (Y_{in} + Y_{load} + Y_{corr})$$

$$I^*_{Load} V_{Load} = Y^*_{load} V^* V = Y^* |V|^2$$

$$|V|^2 = \frac{|I_{in}|^2}{|Y_{in} + Y_{load} + Y_{corr}|^2}$$

$$= \frac{|I_{in}|^2}{\left[(G_{in} + G_{Load} + G_{corr})^2 + (B_{in} + B_{Load} + B_{corr})^2 \right.}$$

Maximum when $B_{in} + B_{Load} + B_{corr} = 0$

$$B_{corr} = -(B_{in} + B_{Load})$$

$$G_{corr} = 0$$

$$S = Y^* |V|^2$$

$$= \frac{G_{Load} |I_{in}|^2}{(G_{in} + G_{Load})^2} \Rightarrow S =$$

$G_{Load} = G_{in}$