Step Response of RC Circuits

1. Objectives

- Measure the internal resistance of a signal source (e.g. a function generator).

- Measure the output waveform of simple RC circuits excited by step functions.

- Calculate and measure various timing parameters of switching waveforms (time constant, delay time, rise time, and fall time) common in computer systems.

- Compare theoretical calculations and experimental data, and explain any discrepancies.

2. Reference

The step response of RC circuits is covered in the textbook. Review the appropriate sections, look at signal waveforms, and review the definition and formula for the time constant.

Review the usage of laboratory instruments.

3. Circuits

Figure 1 shows a simple circuit of a function generator driving a resistive load. This circuit is used to illustrate and measure the internal resistance of a function generator.

Figure 2 shows the first-order RC circuit whose step response will be studied in this lab.

Figure 3 shows two sections of the first-order RC circuit connected in series to illustrate a simple technique to model computer bus systems (PCI bus, SCSI bus, etc.).
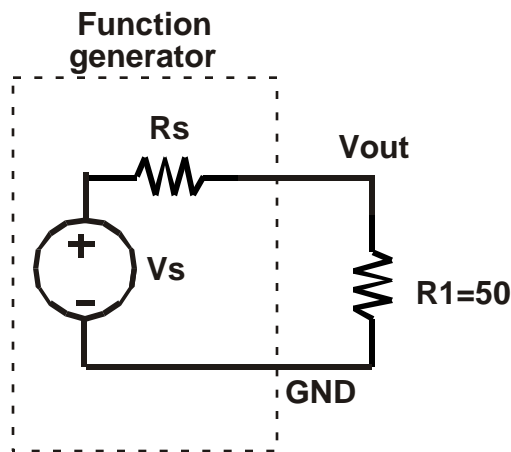
Figure 1. Internal resistance of a function generator.
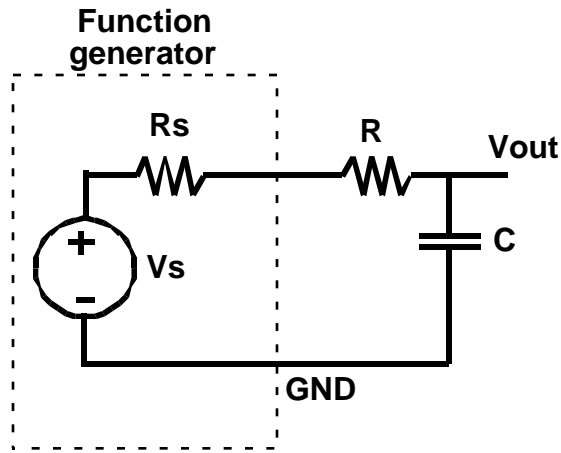
Figure 2. Simple RC circuit.
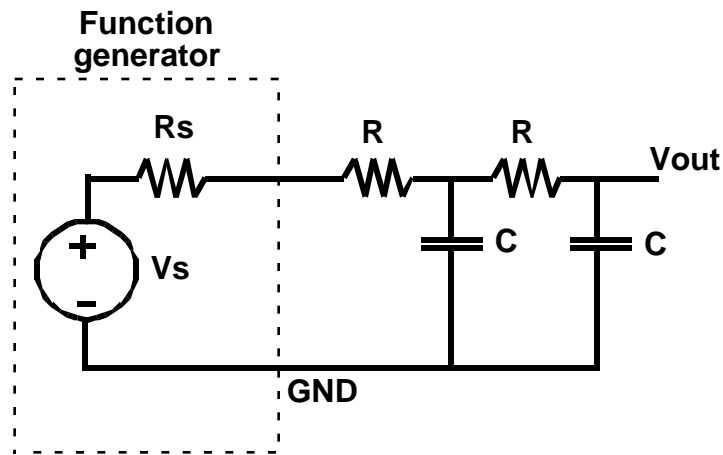
Figure 3. Two cascaded RC sections.

4. Components and specifications

Quantity	*Description*	*Comments*
1	50 Ω resistor	
2	10 KΩ resistor	
1	27 KΩ resistor	
2	0.01 µF capacitor	

5. Discussion

5.1 Source resistance

The function generator is a voltage source V_s with a finite non-zero source resistance R_s. The generator assumes a loading of 50 Ω connected to its output (see Figure 1). The display of signal

amplitude on the front panel reflects this assumption. The output voltage V_{out} might be very different to V_s depending on the resistive load R_1.

5.2 Step response and timing parameters

The step response of a simple RC circuit, illustrated in Figure 4, is an exponential signal with time constant = RC. Besides this timing parameter, four other timing parameters are important in describing how fast or how slow an RC circuit responds to a step input. These timing parameters are marked in Figure 4, at three voltage levels:

a. The 10%-point is the point at which the output voltage is 10% of the maximum output voltage.

b. The 50%-point is the point at which the output voltage is 50% of the maximum output voltage.

c. The 90%-point is the point at which the output voltage is 90% of the maximum output voltage.
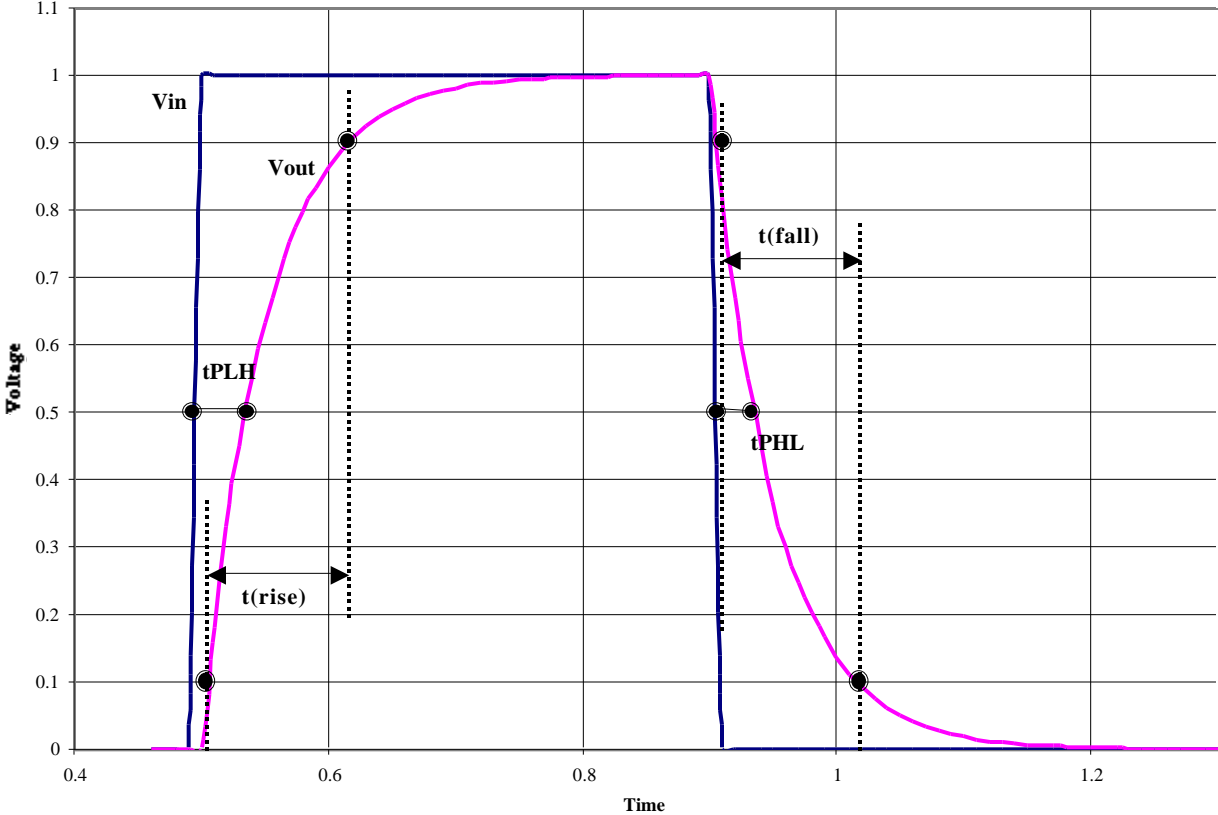
Figure 4. Timing parameters of signal waveforms.

The three timing parameters are defined as follows:

a. Rise time: the time interval between the 10%-point and the 90%-point of the waveform when the signal makes the transition from low voltage (L) to high voltage (H). Notation: t_r.

b. Fall time: the time interval between the 90%-point and the 10%-point of the waveform when the signal makes the transition from high voltage (H) to low voltage (L). Notation: t_f.

c. Delay time (or propagation delay time): the time interval between the 50%-point of the input signal and the 50%-point of the output signal when both signals make a transition. There are two delay times depending on whether the output signal is going from L to H (delay notation t_{PLH}) or from H to L (delay notation t_{PHL}). The subscript P stands for "propagation."

Note that the rise time and the fall time are defined using a single waveform (the output waveform) while the delay time is defined between two waveforms: the input waveform and the corresponding output waveform.

5.3 Delay models of gates and interconnects using RC circuits

RC circuits are frequently used to model the timing characteristics of computer systems. When one logic gate drives another gate, the input circuit of the second gate can be modeled as an RC load. The propagation delay through the first gate can then be calculated assuming ideal square wave input and the RC load. The longer the delay time, the slower the circuit can be switched and the slower the computer is. Conversely, the shorter the delay time, the faster the computer is. This delay time is called "gate delay" since it relates to driving characteristics of a logic gate.

Another use of RC circuits is to model wiring characteristics of bus lines on integrated circuits (IC) or on printer-circuit boards (PCB). A wire can be modeled as many cascaded sections of simple RC circuits as shown in Figure 3 using 2 sections. When a square wave is applied to one end of the bus, it takes time for the signal to propagate to the other end. This delay time due to the wire can be calculated based on the values of R and C in each section and the number of sections used to model the wire. The longer the wire, the more sections are needed for accurate model. A wire is also referred to as "interconnect" and the delay due to a wire is also called "interconnect delay." In high-frequency systems, the interconnect delay tends to dominate the gate delay and is a fundamental constraint on how fast a computer can operate.

6. Prelab

6.1 Equation for extracting source resistance

1. Derive an equation for V_{out} in Figure 1 in terms of V_s, R_s, and R_1.

2. For the circuit in Figure 1, given that $V_s = 1$ V, compute V_{out} for these two cases: (a) $R_1 = R_s$, and (b) $R_1 \gg R_s$.

6.2 Equations for timing parameters of the step response

The input signal to the circuit in Figure 2 is a perfect square wave with amplitude A (from 0 V to A), and period T where $T \gg RC$. You may also assume that $R \gg R_s$ (the internal resistance of the function generator). Using only symbolic parameters (e.g. R, C, A; not numerical values), derive the equations for the following quantities:

a. V_{out} (t). What is the maximum value of $V_{out}(t)$? What is the minimum value of $V_{out}(t)$?

b. The time values when the output reaches 10%, 50%, and 90% of its final value.

c. Rise time t_r of $V_{out}(t)$.

d. Fall time t_f of $V_{out}(t)$.

e. Delay times t_{PHL} and t_{PLH}.

6.3 Parameter extraction via linear least-square-fit technique

The important parameters of $V_{out}(t)$ are the maximum amplitude and the time constant . The maximum amplitude is easily measured using the oscilloscope. To measure the time constant directly and accurately is more difficult since the waveform is an exponential function of time. A linear least-square-fit procedure can be used in the lab to extract the time constant from measured voltage and time values as follows.

From the equation for $V_{out}(t)$ during the time interval when $V_{out}(t)$ falls with time, write the equation for log{ $V_{out}(t)$} as function of t. This equation should be linear in terms of t. What is the slope of this line in terms of the time constant ?

Alternatively, from the equation for $V_{out}(t)$ during the time interval when $V_{out}(t)$ rises with time, manipulate this equation so that the final form looks like:

$$ 1 - \frac{V_{out}(t)}{A} = e^{-t/} $$

where A is the amplitude of the step. Now you can write the equation for log{1- $V_{out}(t)/A$} as function of t. This equation should be linear in terms of t. What is the slope of this line in terms of the time constant ? Either technique can be used to extract the time constant.

In the lab, you will measure a set of data points (t,V). These values, after the appropriate manipulation as above, can be used to plot a straight line, whose slope is a function of . You can use any procedure or a calculator to plot and extract the slope. The slope value will then be used to calculate the time constant .

Make sure you understand this procedure and be ready to use it in the lab. Note that the more points you measure, the more accurate the extracted value for .

6.4 Values of resistors

Use the digital multimeter (DMM) to measure the correct values of the resistors used in this lab. Record these measurements. Also record the internal resistance R_s of the function generator when it is measured in the lab and used it whenever necessary.

7. Experimental procedures

7.1 Instruments needed for this experiment

The instruments needed for this experiment are: a function generator, a multimeter, and an oscilloscope.

7.2 Effect of internal resistance of function generator

1. Build the circuit in Figure 1 using a 50   resistor as load. Set the function generator to provide a square wave with amplitude 400 mV and frequency 1 KHz.

2. Use the scope to display the signal V_{out} on channel 1, using DC coupling. Set the horizontal timebase to display 3 or 4 complete cycles of the signal.

3. Use the scope to measure the amplitude of V_{out}. Record this value in your report. Is it the same as the amplitude displayed by the function generator? Explain any difference.

4. Vary the square wave amplitudes from 400 mV to 1 V, using 100 mV step size (e.g. the amplitudes are 400 mV, 500 mV,… up to 1 V). Repeat step 3 to measure the amplitude of V_{out} on the scope for each setting. Get a hardcopy for the case of 500 mV amplitude only.

5. Remove the 50 resistor and replace it with a 27 K resistor. Repeat the steps 1 through 4 above. Observe and explain any difference in signal amplitudes when the loading on the function generator is changed from 50 to 27 K .

7.3 Step response of first-order RC circuits

1. Build the circuit in Figure 2 using R = 10 K and C = 0.01 µF. Set the function generator to provide a square wave input as follows:

> a. Period T 3 ms (to ensure that T >> RC). This value of T guarantees that the output signal has sufficient time to reach a final value before the next input transition. Record the value of T.

> b. Amplitude from 0 V to 5 V. Note that you need to set the offset to achieve this waveform. Use the oscilloscope to display this waveform on Channel 1 to make sure the amplitude is correct. We use this amplitude since it is common in computer systems.

2. Use Channel 2 of the oscilloscope to display the output signal waveform. Adjust the timebase to display 2 complete cycles of the signals. Record the maximum and the minimum values of the output signal.

3. Use the measurement capability of the scope to measure the period T of the input signal, the time value of the 10%-point of V_{out}, the time value of the 90%-point of V_{out}, and the time value of the 50%-point of V_{out}.

4. Get a hardcopy output from the scope display with both waveforms and the measured values. Turn this hardcopy in as part of your lab report.

5. Clear all the measurements. Use the measurement capability of the scope to measure the rise time of V_{out}, the fall time of V_{out}, and the two delay times t_{PHL} and t_{PLH} between the input and output signals.

6. Get a hardcopy output from the scope display with both waveforms and the measured values. Turn this hardcopy in as part of your lab report.

7. Clear all the measurements. Use the paired measurement capability of the scope to measure the voltage and time values at 10 points on the V_{out} waveform during one interval when V_{out} rises or falls with time (pick one interval only). Note that the time values should be referred to time t = 0 at the point where the input signal rises from 0 V to 5 V or falls from 5 V to 0 V. Record these 10 measurements.

7.4 Step response of cascaded RC sections

1. Build the circuit in Figure 3, using 2 identical resistors R = 10 K   and 2 identical capacitors C = 0.01 µF. Use the same square input as in item 1, section 7.3 above and display it on Channel 1.

2. Display V_{out} on Channel 2 and adjust the timebase to display 2 complete cycles of the signals.

3. Use the scope measurement capability to measure the two delay times t_{PHL} and t_{PLH} between the input and output signals.

4. Get a hardcopy output from the scope display with both waveforms and the measured values. Turn this hardcopy in as part of your lab report.

7.5 Manufacturing test time and test cost considerations

1. The more points you measure on a waveform, the more accurate the measured results but this also takes more time and increases the test cost. This is an important tradeoff in measurement accuracy and test cost. Given the circuit in Figure 2, ten data points per waveform were collected in section 7.3 item 7. Should you collect more or fewer than 10 points to extract a "good" estimate of the rise or fall time of the circuit (as defined above)? "Good" estimate means the estimated value is within 10% of the correct value (from computation or simulation).

2. What is the minimum number of data points do you need to collect to get a good estimate? Compare your minimum number of points with other teams' answers. Do they collect more or fewer points? Are their results "better" than yours? Include this discussion in your report.

8. Data analysis

8.1 Extracting internal resistance of function generator

1. From the equation for V_{out} and using the amplitude of V_s as 500 mV, compute the amplitude of V_{out} for both cases $R_1 = 50$ and $R_1 = 27$ K   . Do these values agree with the recorded data in the lab? Include the hardcopies from the scope display to justify your answers.

2. From the data recorded in section 7.2, can you compute the internal resistance R_s of the function generator? What is the estimated value of R_s?

3. Are the values for V_s (as displayed by the function generator panel) and the measured values on the scope the same? Explain any difference.

8.2 Step response of first-order RC circuits

Use the component values in section 7.3 for the following data analysis. Remember to use the measured values of the external resistors in section 6.4 if they are significantly different than the marked values (more than 5%).

1. From the Pre-lab equation for $V_{out}(t)$, compute the maximum and minimum values of the output. Compare with the measured values in section 7.3 item 2. Explain the sources of errors leading to the differences.

2. Using the numerical values for R and C in the Prelab equations in section 6.2 item b, compare the time values when the output voltage reaches the 10%-point, the 50%-point, and the 90%-point

with the measured values in section 7.3 item 3. Explain the sources of errors leading to the differences. How much error is committed by neglecting the internal resistance of the function generator?

3. Using the numerical values for R and C in the Prelab equations in section 6.2 items c, d, and e, compare the computed values of the 4 timing parameters (the rise time, the fall time , and the two delay times t_{PHL} and t_{PLH}) with the measured values in section 7.3 item 5. Explain the sources of errors leading to the differences. How much error is committed by neglecting the internal resistance of the function generator?

4. From the 10 data points (V,t) measured in section 7.3 item 7, select 5 data points and use the best-fit line technique to extract the time constant   of the output signal. Compare this value with the expected value RC and compute the difference in percent. Plot the best-fit line and the measured data points on the same plot.

5. Use all 10 data points in section 7.3 item 7, and repeat the best-fit line technique to extract the time constant. Is this new value different than the value computed in item 4 above by more than 5%? Plot the best-fit line and the measured data points on the same plot. Assuming that this new value for   is the more accurate, use it to compute the correct value of the capacitor C. Compare the computed value of the capacitor with the marked value.

8.3 Step response of cascaded RC sections

1. From the measurements in section 7.4 item 3, are the delay times for the cascaded circuit in Figure 3 (of 2 identical RC sections) twice as large as the delay times for the simple RC circuit? Do the delay times scale with the number of sections? Explain.

2. Estimate the number of cascaded RC sections so that the propagation delay time is about T/2 (T is the period of the input square wave). Explain how you arrive at this estimate. How would you verify that this is a good estimate (i.e. within 10% of the correct number of sections)? This estimate basically is related to the longest bus line that can be driven by the signal before there is a timing error in the system.

9. Further research

You do not have to turn this section in.

1. Check papers in industry trade journals (Electronic Design, EDN, etc.) and research journals (e.g. IEEE Transactions on Circuits and Systems, IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems) in the Engineering Library to find formulas for estimating the propagation delay as function of the number of cascaded RC sections. Use one formula and set the number of sections to 6. Calculate the delay using this formula and compare to the measured value in section 7.4 above.

2. Below the frequency 500 MHz or so, the RC model for wire delay is very good. For very high-frequency systems (f > 1 GHz), the RC model is not as accurate. Another circuit incorporating the inductive effect (using the inductor L in series with the resistor R in Figure 2) is used to model interconnect delay. Check the papers in the journals mentioned above to see how the delay is calculated using this model.

10. Self-test

1. Use a different input signal (e.g. a ramp waveform from the function generator) in Figure 2. Repeat the measurements of the timing parameters and see what the waveforms look like on the scope.

2. How do you check your own measurements to reduce errors? Can you come up with two different ways to measure the same parameter so that the results can be compared?