Analyze & Design Circuits with Sinusoidal Signals

Learning goals

- Understand physical meanings of sine / cosine signals
- Analyze simple circuits (R,L,C,opamp) with sinusoidal inputs
  - Apply 3 alternatives to solve problems
  - Enhance problem-solving skills
- Design simple circuits with sinusoidal inputs

Physical meanings

- Real-life use of sine / cosine signals
  - Power transmission to homes
    - 110 V, 60 Hz in North America
  - Music / stereo systems
- $V(t) = V_m \cos(\omega t + \phi)$
  - Amplitude V_m : soft / loud sound
  - Frequency ω: musical notes A, B, C, …
    - Frequency f, period T
  - Phase ϕ: surround-sound stereo system

Analysis vs. Design

Task	Input signal	Circuit diagram	Component values	Output signal
Analysis	Given	Given	Given	Compute
Design (easy)	Given	Given	Compute	Given
Design	Given	Create	Compute	Given

Analysis goal

- Sinusoidal input form
  - $V_{in}(t) = V_m \cos(\omega t + \phi)$ (or current $I_{in}(t)$)
- Output form (voltage or current)
  - Linear circuits: R, L, C, ideal opamp
  - $V_{out}(t) = V_x \cos(\omega t + \phi_x)$
    - Frequency ω does NOT change.
    - Analyze = find output amplitude V_x and phase ϕ_x
- Key tasks: find V_x and ϕ_x

Analysis procedures

- Two methods using differential equations
- One method using complex math and phasors to avoid differential equations

DE-based methods

- Use KVL and KCL to write a set of equations (nodes, mesh, mixed nodes-mesh)
- Manipulate equations to simplify
  - end up with one or a set of differential equations involving V(t) and / or I(t)
- Solve differential equations
  - two methods

Solving differential equations (1)

- Assume sinusoidal signals
- Method 1: look up Math tables, or run Maple (or symbolic math software)
- Method 2 (a):
  - Assume output form: $V_{out}(t) = V_x \cos(\omega t + \phi_x)$
  - Substitute this form into differential equations to evaluate V_x and ϕ_x
  - END: solution found. Check answer.

Solving differential equations (2)

- Method 2(b):
  - Assume output form: $V_{out}(t) = A \cos(\omega t) + B \sin(\omega t)$
  - Substitute this form into differential equations to evaluate A and B
  - END: solution found. Check answer.

- Tedious boring step:
  - Convert $V_{out}(t) = A \cos(\omega t) + B \sin(\omega t)$ to $V_{out}(t) = V_x \cos(\omega t + \phi_x)$ or vice versa

Method 3: Complex math

- Key idea: $\cos(\omega t+\phi) = \mathrm{Re}\,[e^{\,j(\omega t+\phi)}]$
- Method 3:
  - Re-write source signal(s) as Re of $e^{j(\omega t+\phi)}$
  - Use KVL and KCL to write a set of equations (nodes, mesh, mixed nodes-mesh)
  - Use complex form of source signals and simplify equations
  - Solve for output functions in complex form (algebraic manipulations only, no DE)
  - Take Re part to get final answer. END.

Variation of Method 3: Phasor

- Phasor = short-hand notation to emphasize only magnitude and phase
  - $V_m e^{\,j(\omega t + \phi)} = V_m \,/\underline{\phi}$
  - Signal $V(t) = V_m \cos(\omega t + \phi) = V_m \,/\underline{\phi}$
  - Use full complex form in differentiation, integration, addition, subtraction then convert back to phasor form
    - polar form <--> rectangular form conversion

Method 3(b): Phasor technique

- Write source signal(s) as phasor(s)
- Write KVL and KCL as a set of phasor equations
  - phasor equations for R, L, C
- Manipulate equations to simplify
- Solve equations to get output phasor
  - Algebraic solution, no DE
- Final answer = Re part of phasor. END.

Working with large circuits

- Simplify circuit topology by combining elements
  - Impedance & admittance concept
  - Parallel and series combinations
- Simplify circuit topology by combining sources and elements
  - Thevenin equivalent
  - Norton equivalent
- Superposition: one source at a time

Impedance & Admittance

- Component behavior in frequency
  - R --> R
  - L --> $j\omega L$
  - C --> $1/j\omega C$
- Impedance Z
  - General representation in V = Z I
- Admittance Y
  - General representation in I = Y V

Circuit simplifications with Z & Y

- Represent each component with either Z or Y
- Z behaves similarly to resistance R
  - Series combination
  - Parallel combination
- Y behaves similarly to conductance G

Simplifications: sources / elements

- Thevenin equivalent V_T, Z_T of a black box
  - Write sources in phasor form and elements as impedance Z and / or admittance Y
  - Use same procedure (as in time-domain) to find V_T, Z_T
- Norton equivalent I_N, Z_N of a black box
  - Write sources in phasor form and elements as impedance Z and / or admittance Y
  - Use same procedure (as in time-domain) to find I_N, Z_N

Simplifications: superposition

- Apply one source at a time
  - Replace the other independent sources by shorts or opens
- Find output corresponding to each source
  - Use any method you know
- Sum all outputs to get final answer
  - Might need some simplification to get answer into a conventional form