Bode Plots

Real poles/roots

$$H(s) = \frac{K(s+z_1)\cdots}{(s+p_1)(s+z_2)\cdots}$$

zeroes at $-z_1 \ldots$

poles at $-p_1, -p_2$

$$H(j\omega) = \frac{K(j\omega+z_1)}{(j\omega+p_1)(j\omega+p_2)}$$

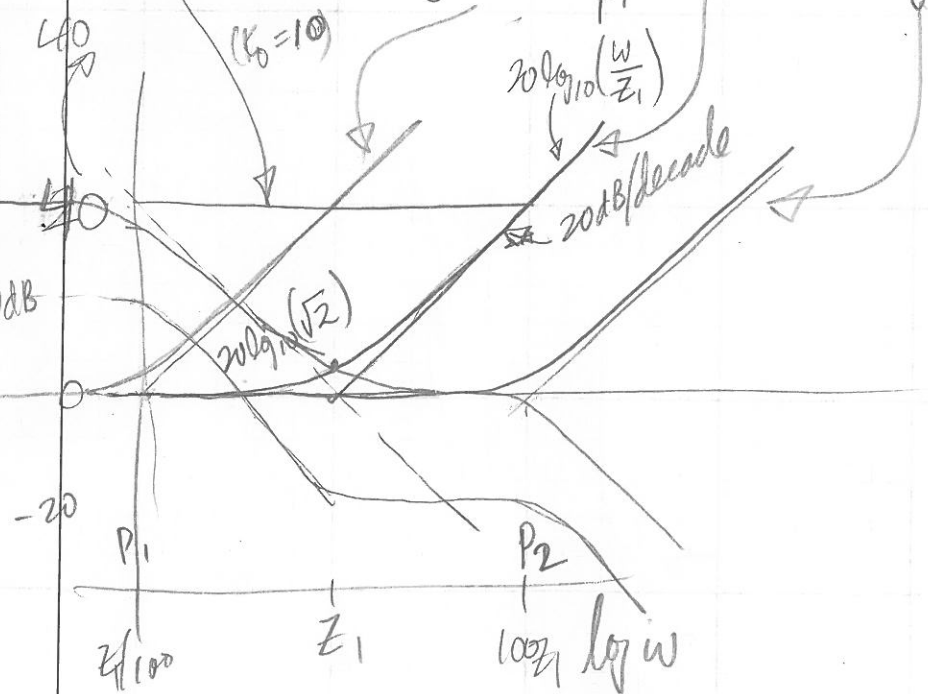
$$= \underbrace{\frac{K z_1}{p_1 p_2}}_{K_0}\left[\frac{1+j\omega/z_1}{(1+j\omega/p_1)(1+j\omega/p_2)}\right]$$

$$A_{dB} = 20\log_{10}|H(j\omega)|$$

$$= 20\log_{10}\left[K_0\,\frac{(1+j\omega/z_1)}{(1+j\omega/p_1)(1+j\omega/p_2)}\right]$$

$$= 20\log_{10}K_0 + 20\log_{10}\left|1+\frac{j\omega}{z_1}\right|$$

$$- 20\log_{10}\left|1+\frac{j\omega}{p_1}\right| - 20\log_{10}\left|1+\frac{j\omega}{p_2}\right|$$



$$H(j\omega) = \frac{\cancel{\angle}(1 + j\omega/z_1)}{\cancel{\angle}(1 + j\omega/p_1)} = A_{z_1} \angle \theta_{z_1}(\omega)$$

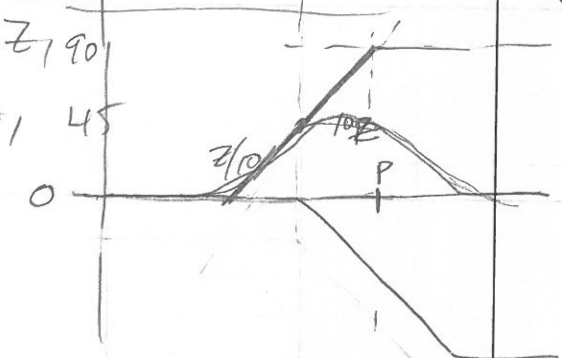
$$= \frac{A_1 \angle \phi_1 \; A_2 \angle \phi_2}{(B_1 \angle \phi_1)(B_2 \angle \phi_2)} = \frac{A_1 A_2}{B_1 B_2} \angle \phi_1 + \phi_2 - \phi_1 - \phi_1$$

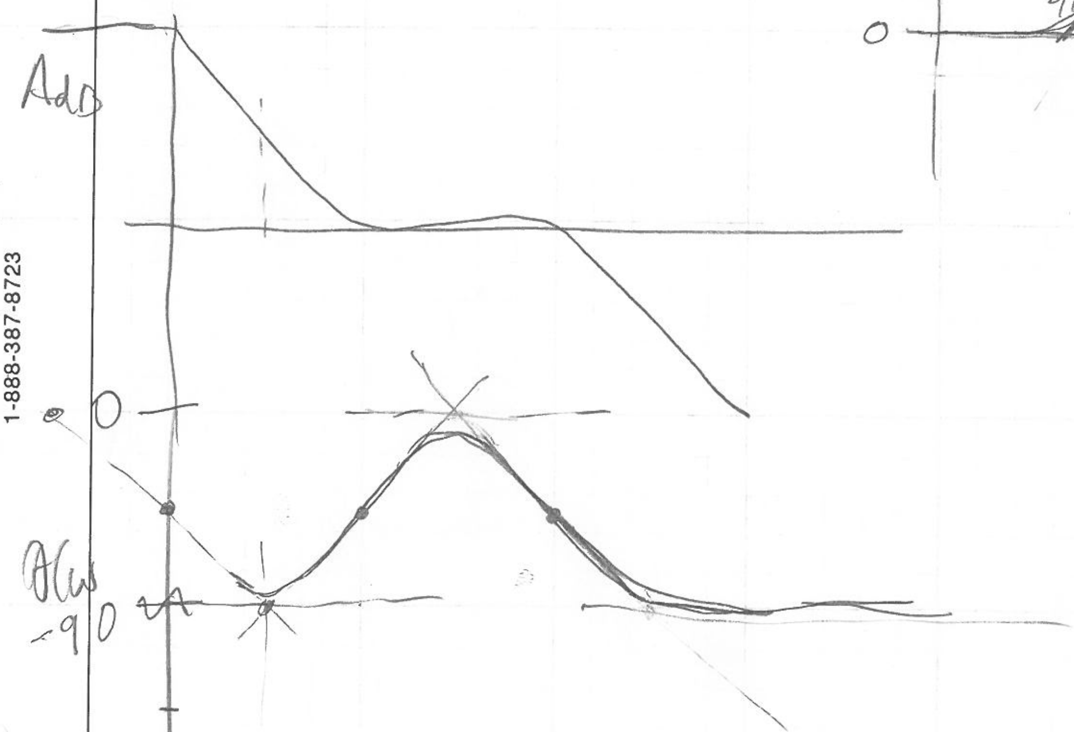
$\rightarrow$ angles from each term also add

$$H(j\omega) = |H(j\omega)| \angle \theta(\omega) \qquad H(s) \left(\frac{s+z}{s+p}\right)$$

$$H(j\omega) = \cancel{(1+z j\omega)} \left(1 + \frac{j\omega}{z_1}\right)$$

$$\theta(\omega) = \begin{cases} \cong 0 & \omega \ll z_1 \\ \cong +90 & \omega \gg z_1 \quad 90 \\ = +45 & \omega = z_1 \quad 45 \end{cases}$$





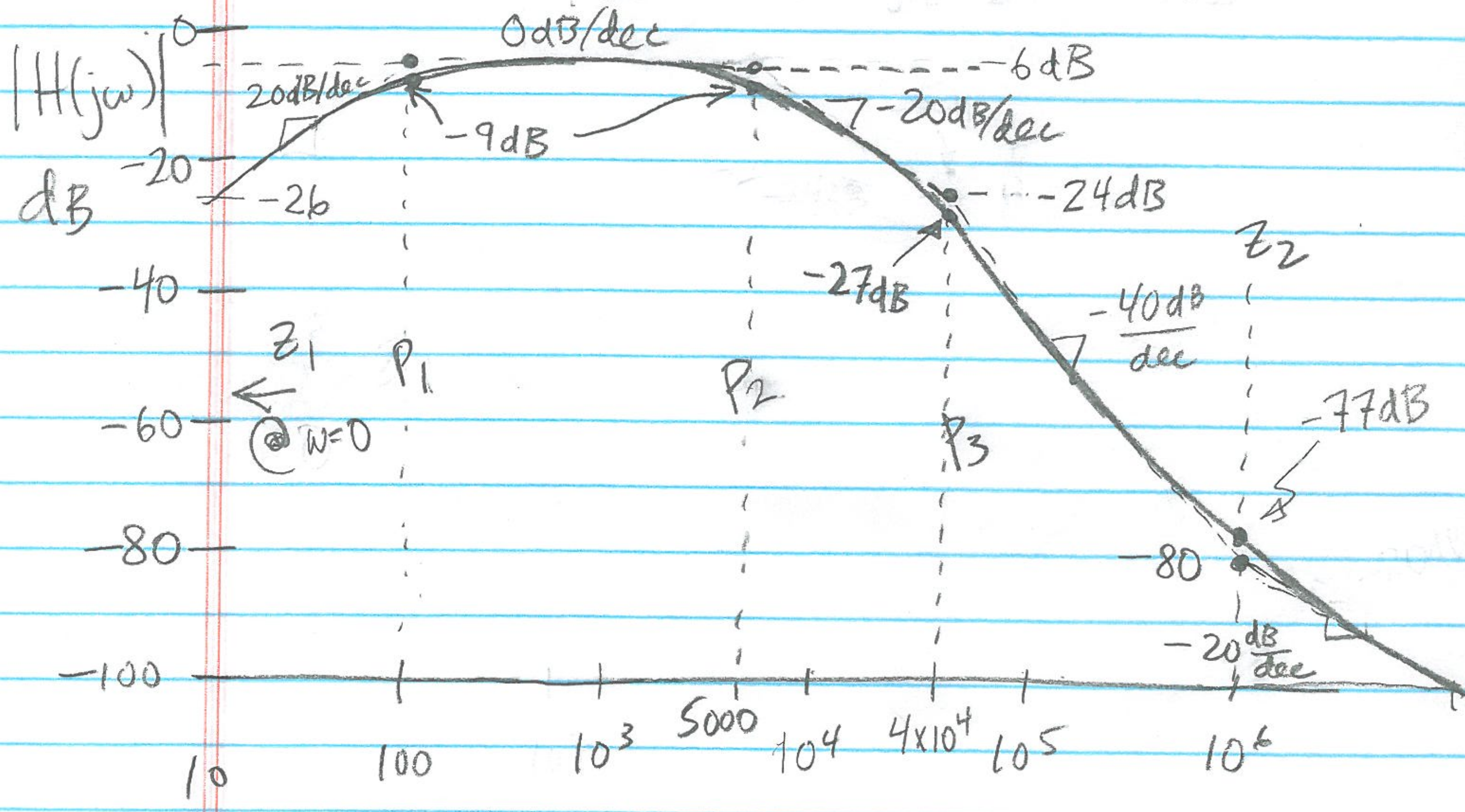
Bode Plot Example

Zeroes @ $\omega = 0$ & 10^6
poles @ $\omega = 100, 5000$ & 4×10^4

$$H(s) = \frac{100\, s\,(s+10^6)}{(s+100)(s+5000)(s+4\times10^4)}$$

$$H(j\omega) = \frac{100(j\omega)(j\omega+10^6)}{(j\omega+100)(j\omega+5000)(j\omega+4\times10^4)}$$

$$= \underbrace{\frac{\cancel{(100)}(10^6)}{\cancel{(100)}(5000)(4\times10^4)}}_{1/200 \;\Rightarrow\; -26\text{dB} \;@\;\omega=1} \left[\frac{j\omega\left(1+\frac{j\omega}{10^6}\right)}{\left(1+\frac{j\omega}{100}\right)\left(1+\frac{j\omega}{5000}\right)\left(1+\frac{j\omega}{4\times10^4}\right)}\right]$$



$\log_{10}\frac{40000}{5000} = 0.9$ – change of -18 dB for slope of -20dB/decade

$\log_{10}\left(\frac{10^6}{4\times10^4}\right) = 1.4$, change of -56dB for slope of $-40\frac{\text{dB}}{\text{dec}}$

For $\omega << 100$ $H(j\omega) \cong \frac{1}{200}\left(\frac{j\omega(1)}{(1)(1)(1)}\right) = \frac{j\omega}{200}$

$100 << \omega << 5000$ $H(j\omega) \cong \frac{1}{200}\frac{(j\omega)(1)}{(j\omega/100)(1)(1)}$

$\cong 1/2$

$5000 << \omega << 40{,}000$ $H(j\omega) \cong \frac{1}{200}\frac{(j\omega)(1)}{\left(\frac{j\omega}{100}\right)\left(\frac{j\omega}{5000}\right)(1)}$

$\cong \frac{2{,}500}{j\omega}$

$40{,}000 << \omega << 10^6$

$H(j\omega) \cong \frac{1}{200}\frac{(j\omega)(1)}{\left(\frac{j\omega}{100}\right)\left(\frac{j\omega}{5000}\right)\left(\frac{j\omega}{40{,}000}\right)}$

$\cong \frac{10^8}{-\omega^2}$

$10^6 << \omega$

$H(j\omega) \cong \frac{1}{200}\frac{(j\omega)(j\omega/10^6)}{\left(\frac{j\omega}{100}\right)\left(\frac{j\omega}{5000}\right)\left(\frac{j\omega}{40{,}000}\right)}$

$\cong \frac{100}{j\omega}$

$$H'(s) = \frac{K}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$\zeta \geq 1$ $(Q < \frac{1}{2})$ $\Rightarrow$ factor into real poles

$$= \underbrace{\frac{K}{\omega_n^2}}\left[\frac{1}{1+(s/\omega_n)^2+2\zeta(s/\omega_n)}\right]$$

$$H'(j\omega) = K_0\left[\frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2+j\left(2\zeta\frac{\omega}{\omega_n}\right)}\right]$$

$$H(j\omega) = \frac{H'(j\omega)}{K_0}$$

for

$\omega \ll \omega_n$ $\quad H(j\omega) \cong \angle 0$

$\omega \gg \omega_n$ $\quad H(j\omega) \cong \frac{}{-(\omega/\omega_n)^2} = \left(\frac{\omega_n}{\omega}\right)^2 \angle -180^\circ$

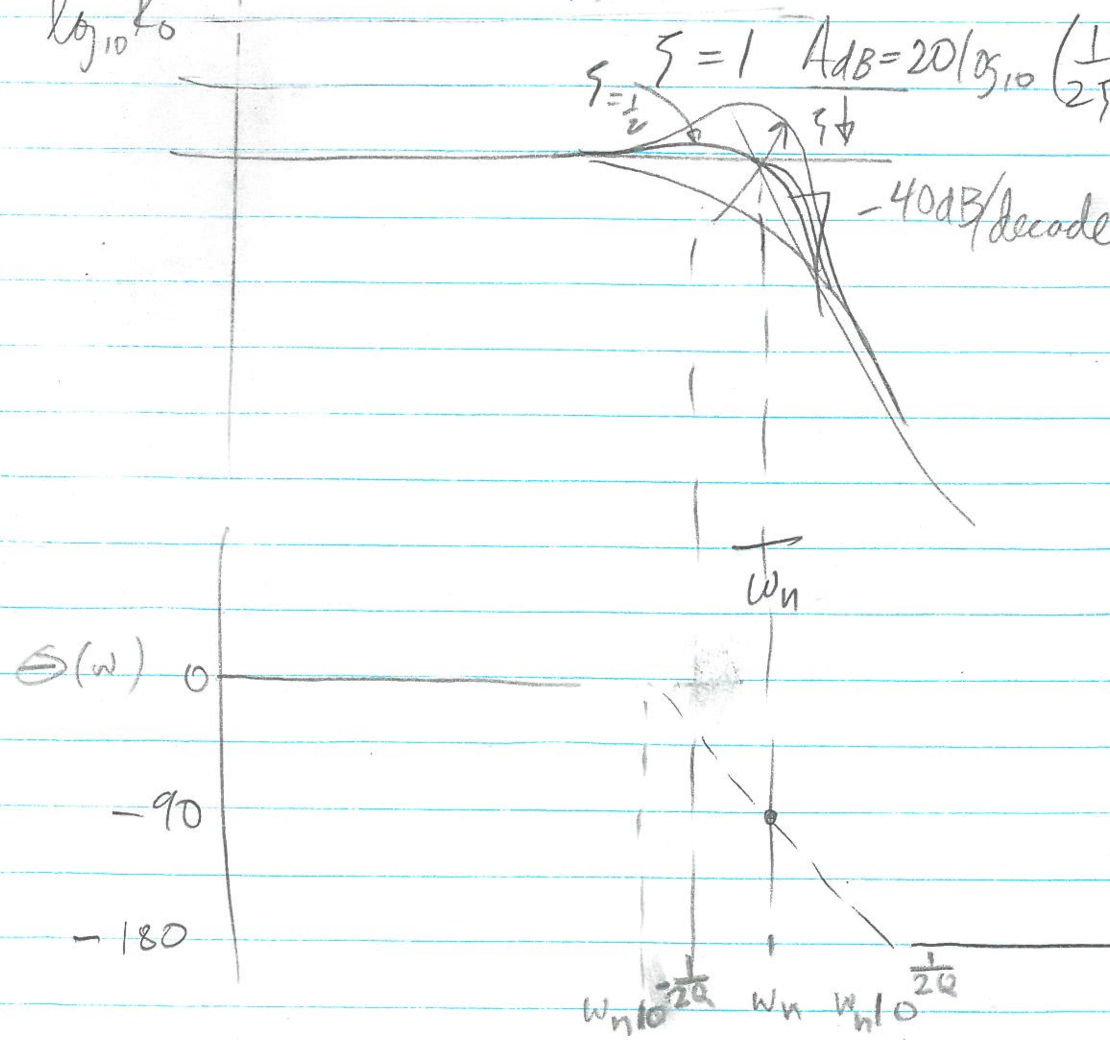
$\omega = \omega_n$ $\quad H(j\omega_n) = \frac{\cancel{K_0}1}{j(2\zeta)} = \frac{\cancel{K_0}1}{2\zeta}\angle -90$

$\omega \times 10 \Rightarrow |H(j\omega)| \times 10^{-2}$

$20\log_{10}(10^{-2}) = -40\,dB$

$\Rightarrow -40\,dB/dec$

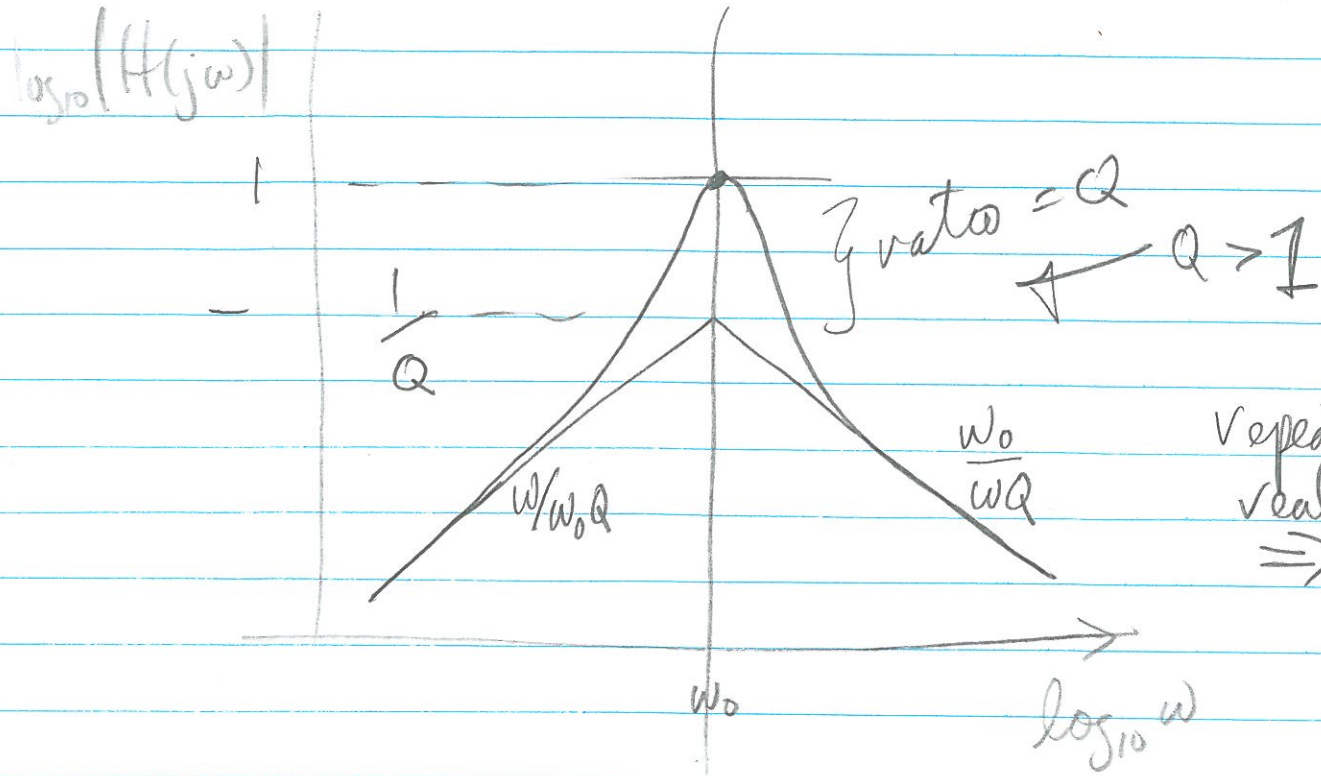
$A_{dB} = 20\log_{10}\left(\frac{1}{2\zeta}\right) = -6\,dB$



Q	ζ	dB
1	$\zeta = \frac{1}{2}$	0dB
2	$\zeta = \frac{1}{4}$	6dB
10	$\zeta = \frac{1}{20}$	20dB

$Q = \frac{1}{2\zeta} =$

$1/2$	$-6\,dB$



repeated real roots

$\Rightarrow Q = 1/2$

$$\frac{j\omega\omega_0/Q}{-\omega^2 + j\frac{\omega\omega_0}{Q} + \omega_0^2}$$

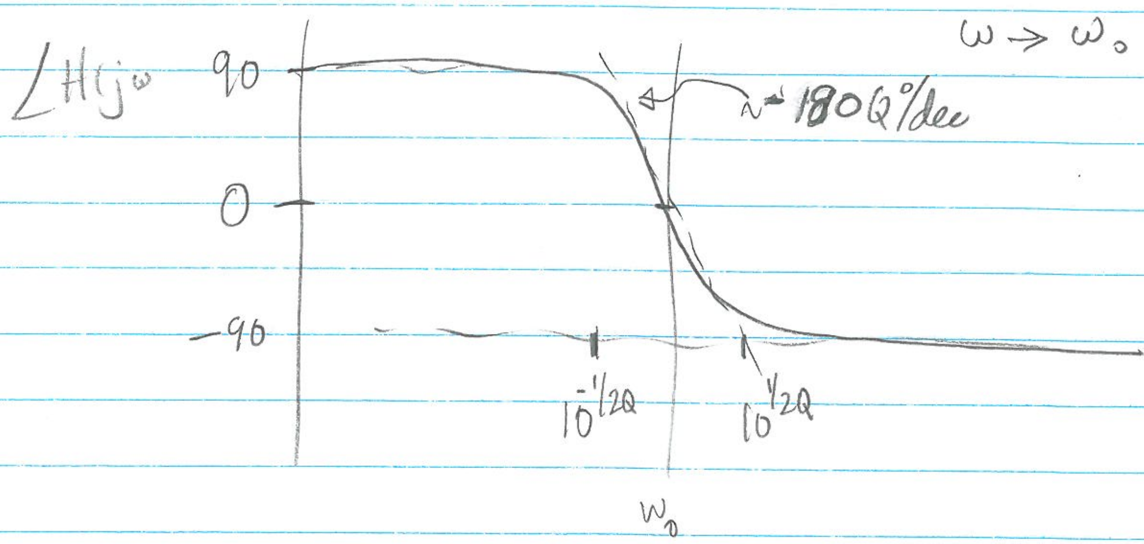
$\omega \to 0 \quad H(j\omega) \to \frac{j\omega}{\omega_0 Q}$

$\frac{\omega}{\omega_0 Q} \angle 90$

$\omega \to \infty \quad H(j\omega) \to -j\frac{\omega_0}{\omega Q}$

$\frac{\omega_0}{\omega Q} \angle -90$

$\omega \to \omega_0 \quad H(j\omega) = 1$



$$H(\omega) = \frac{200 + j\omega}{(1 + 0.02j\omega)\left(1 + 0.5j(\omega/1000) - (\omega/1000)^2\right)}$$

canonical form

$$= 200\left[\frac{1 + j(\omega/200)}{(1 + j\omega/50)\left[1 + 2(0.25)j(\omega/1000) - (\omega/1000)^2\right]}\right]$$

$\zeta < 1$. (if $\zeta > 1$, factor)

Zeroes: $\omega = 200$
poles: $\omega = 50$, 1000 (double)

~~Can check to factor $-0.5j \pm \sqrt{(0.5)^2 -}$~~

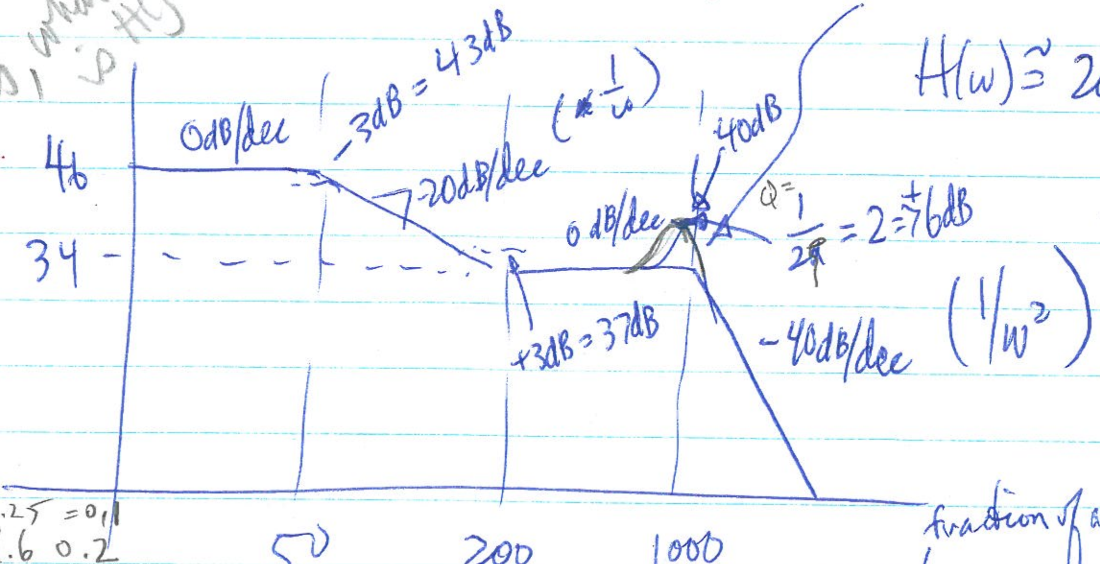
$\omega \Rightarrow 0 \quad H(\omega) \cong 200 \Rightarrow G(\omega) = 20\log_{10} 200 = 46$
$\phi(\omega) = 0°$

$\omega = 50$ corner frequency

Given this, what is $H(j\omega)$

$200 < \omega < 1000$

$$H(\omega) \cong 200\left(\frac{j\omega/200}{(j\omega/50)(1)}\right) = 50 \text{ (34dB)}$$



fraction of a decade

$\log_{10}$
$(1.25) = 0.1$
$(1.6) = 0.2$
$(2) = 0.3$
$(2.5) = 0.4$
$(4) = 0.6$
$(5) = 0.7$
$(6.25) = 0.8$
$(8) = 0.9$

$\underbrace{50 \quad 200}_{\text{factor of 4}} \quad \log_{10}(4) = 0.6$ dec

$G(\omega) = 46\text{dB} + (0.6\text{ dec})(-20\text{dB/dec}) = 34\text{dB}$