Name Solutions UW ID#: ____________ Section: ___ Version A

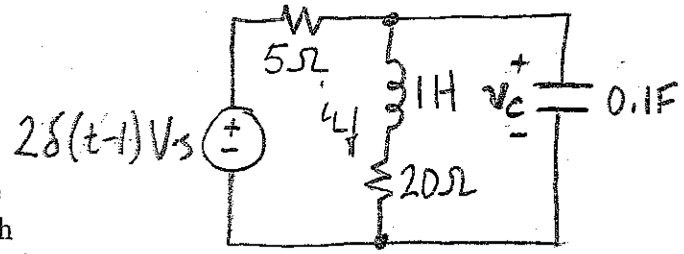
Final Exam — EE 233

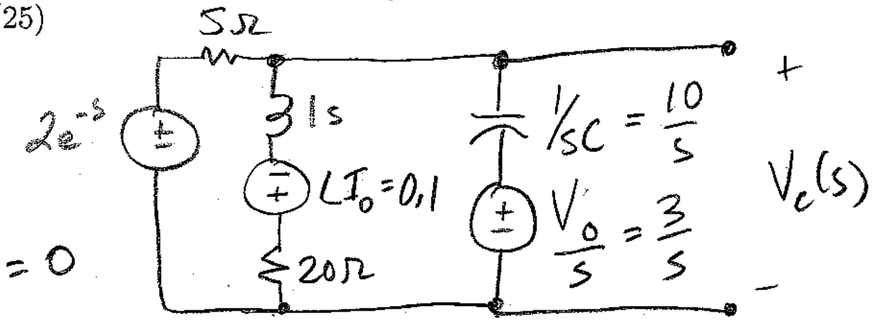
Fall 2006

The test is closed book, with two sheets of 8.5 by 11 inch notes and standard (non-graphing) calculators allowed. Show all work. Be sure to state all assumptions made and **check** them when possible. The number of points per problem are indicated in parentheses. Total of 150 points in 6 problems on 6 pages. A table of Laplace transform pairs are attached as page 7.

1. In the circuit to the right, $v_C = 3\,\text{V}$ and $i_L = 0.1\,\text{A}$ at $t = 0^-$.

   Draw the s-domain circuit valid for $t > 0$ and determine $V_C(s)$. Check your result with the Initial Value Theorem. (25)





$$\mathcal{L}\{2\delta(t-1)\} = 2e^{-s}$$

$$\frac{V_c - 2e^{-s}}{5} + \frac{V_c + 0.1}{s+20} + \frac{V_c - 3/s}{10/s} = 0$$

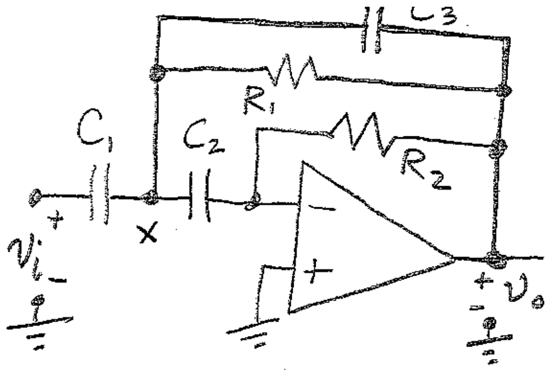
$$\frac{(V_c - 2e^{-s})(s+20)(2) + 10(V_c + 0.1) + (sV_c - 3)(s+20)}{10(s+20)} = 0$$

$$V_c\left[2s + 40 + 10 + s^2 + 20s\right] = (4s+80)e^{-s} - 1 + 3s + 60$$

$$\underline{V_c = \frac{(4s+80)e^{-s} + 3s + 59}{s^2 + 22s + 50}}$$

$$\lim_{s\to\infty} sV_c(s) = \lim_{s\to\infty}\left[\frac{(4s^2+80s)e^{-s} + 3s^2 + 59s}{s^2 + 22s + 50}\right] = \frac{3s^2}{s^2} = 3 \quad \underline{\text{checks}}$$

2. A filter is constructed using the circuit at the right. Find the transfer function $H(s)$ in terms of R_1, R_2, C_1, C_2, and C_3. What kind of filter is it? (25)



Node x:

$$(V_i - V_x)sC_1 = V_x(sC_2) + (V_x - V_o)\left(sC_3 + \frac{1}{R_1}\right)$$

Node V^- $\quad V_x(sC_2) = -V_o/R_2 \implies V_x = -\frac{V_o}{sR_2C_2}$

substitute:

$$V_i(sC_1) = V_x\left(sC_1 + sC_2 + sC_3 + \frac{1}{R_1}\right) - V_o\left(sC_3 + \frac{1}{R_1}\right)$$

$$= -\frac{V_o}{sR_2C_2}\left[s(C_1 + C_2 + C_3) + \frac{1}{R_1}\right] - V_o\left(sC_3 + \frac{1}{R_1}\right)$$

$$\frac{V_o}{V_i} = \frac{-sC_1(sR_2C_2)}{s(C_1+C_2+C_3) + \frac{1}{R_1} + sR_2C_2\left(sC_3 + \frac{1}{R_1}\right)}$$

$$= -\frac{s^2R_2C_1C_2}{s^2R_2C_2C_3 + s\left(C_1 + C_2 + C_3 + \frac{R_2}{R_1}C_2\right) + \frac{1}{R_1}}$$

$$= -\frac{C_1}{C_3}\left[\frac{s^2}{s^2 + \left[\frac{C_1+C_2+C_3}{R_2C_2C_3} + \frac{1}{R_1C_3}\right]s + \frac{1}{R_1R_2C_2C_3}}\right]$$

2nd order High Pass Filter

3. Design the s-domain transfer function for a low-pass filter with a corner frequency of 10^4 rad/s, a pass-band gain of 50, a gain at the corner frequency of 37 dB, and a gain less than 0.5 at 8×10^4 rad/s. (25)

LPF $\omega_c = 10^4 \frac{rad}{s}$

$20\log_{10}(0.5) = -6dB$, $20\log_{10}50 = 34dB$

$$H(s) = \frac{A}{(s+10^4)(s^2+\frac{10^4}{Q}s+10^8)}$$

Need drop of $34-(-6) = 40$ dB over $\log_{10} 8 = 0.9$ decades

$\frac{40dB}{0.9dec} = 44.4$ dB/dec $> 40dB$

need 3rd order LPF

At corner real pole gives drop of 3dB, but need increase of 3dB = 37dB − 34dB from pass band, so

$Q = 2$ ($20\log_{10} 2 = 6dB$)

In passband ($\omega << 10^4$) $H(s) \cong \frac{A}{10^{12}}$, so $A = 5\times10^{13}$

$$H(s) = \frac{5\times10^{13}}{(s+10^4)(s^2+5000s+10^8)}$$

4. Find the time-domain behavior of the following s-domain functions:

(a) $V_1(s) = \frac{2s+1}{s+3}$ (10)

$$\frac{2s+1}{s+3} = 2 - \frac{5}{s+3}$$

$$\mathcal{L}^{-1}\left(2 - \frac{5}{s+3}\right) = 2\delta(t) - 5e^{-3t}u(t) = v_1(t)$$

(b) $V_2(s) = \frac{10s+5}{s^2+9s+13}$. (15)

roots of $s^2+9s+13$ are $\frac{-9 \pm \sqrt{9^2-4(13)}}{2}$

$$= -\frac{9}{2} \pm \frac{\sqrt{29}}{2} = -1.81, -7.19$$

$$V_2(s) = \frac{K_1}{s+1.81} + \frac{K_2}{s+7.19}$$

$$K_1 = \frac{10s+5}{s^2+9s+13}(s+1.81)\Big|_{s=-1.81} = \frac{10s+5}{s+7.19}\Big|_{s=-1.81}$$

1.73

$$= \frac{-18.1+5}{-1.81+7.19} = -2.43$$

Check: $\underline{s=0}$

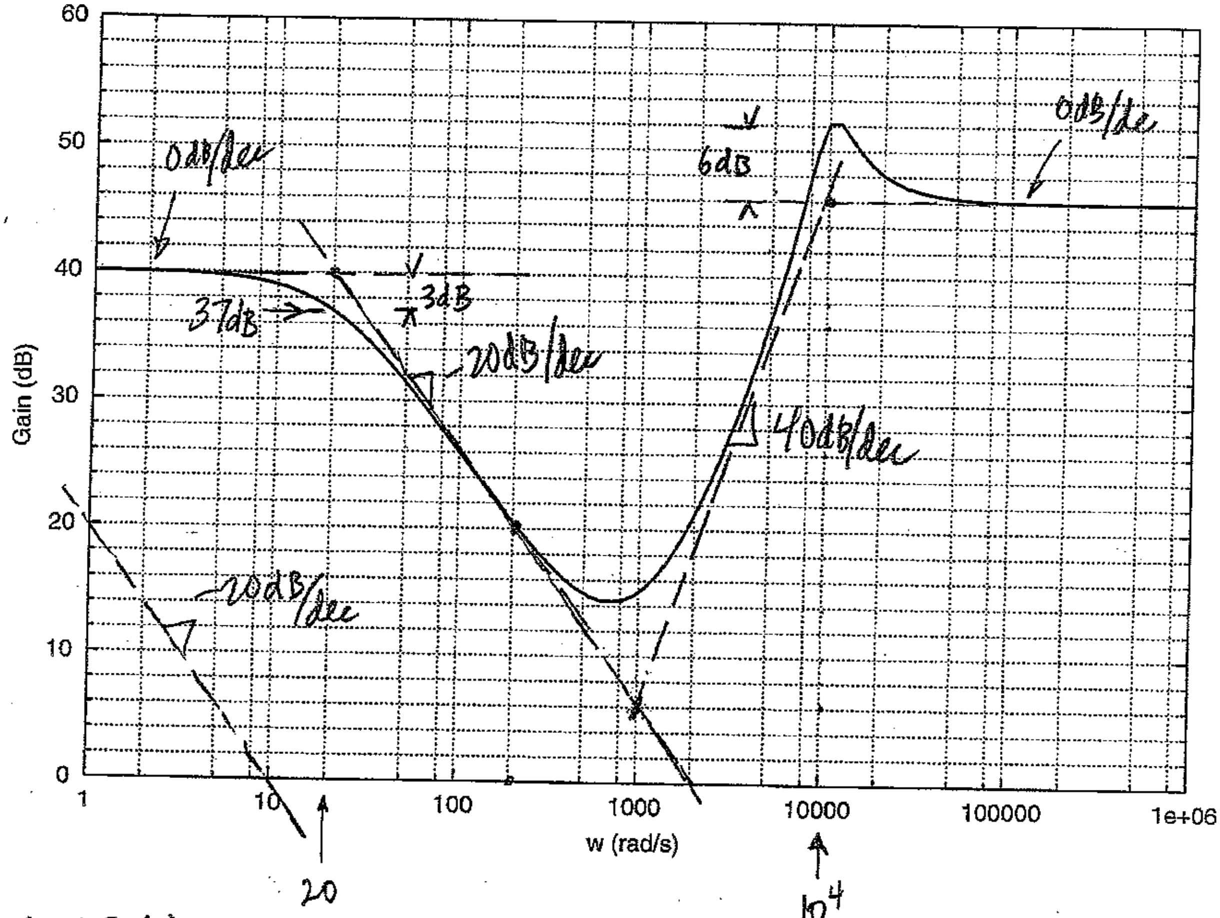
$$\frac{-2.43}{1.81} + \frac{12.43}{7.19} = 0.38 = \frac{5}{13}$$

$$K_2 = \frac{10s+5}{s+1.81}\Big|_{s=-7.19} = \frac{-71.9+5}{-7.19+1.81} = 12.43$$

$$v_2(t) = -2.43e^{-1.81t} + 12.43e^{-7.19t}$$

Check w/ IVT: $\lim_{s\to\infty} sV_2(s) = 10 = v_2(0)$ ✓

5. Determine the s-domain transfer function associated with Bode magnitude plot below. Show your work on the plot and in workspace. (25)



pole (single) at ~20 rad/s
triple zero at ~1000 rad/s
double pole at ~10^4 rad/s

$$H(s) = \frac{A(s+1000)\left(s^2 + \frac{1000}{Q_Z} + 10^6\right)}{(s+20)\left(s^2 + \frac{10^4}{Q_P} + 10^8\right)}$$

$Q_P = 2$ $\quad 20\log_{10} Q_P = 6\text{dB}$ at upper corner frequency

Gain at 1000 3dB above corner due to 1st order term
Additional 6dB due to 2nd order term $\Rightarrow Q_Z = \frac{1}{2}$ ← pair of real zeroes $(s+10^4)^2$

For low frequency $|H(j\omega)| = \dfrac{A(10^9)}{2\times 10^9} = 100 \Rightarrow |A| = 200$

$$H(s) = \frac{200(s+1000)^3}{(s+20)(s^2+5000s+10^8)}$$

Check: at very high frequency
$|H(j\omega)| = \dfrac{200s^3}{s^3} = 200 \Rightarrow 46\text{dB}$