Name Solutions UW ID#: ________ Section: ___ Version A

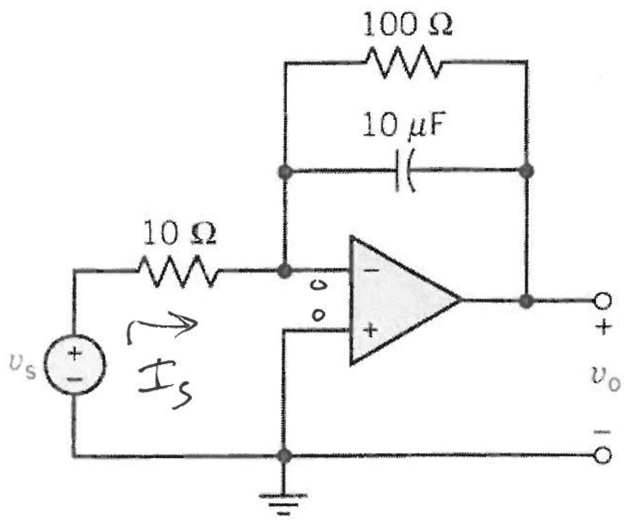
Mean = 31
Median = 33
Std. Dev = 9.5

Exam 1 — EE 233

Spring 2010

The test is closed book, with one sheet of notes and standard calculators (no communications) allowed. Show all work. Be sure to state all assumptions made and **check** them when possible. The number of points per problem are indicated in parentheses. Total of 50 points in 3 problems on 3 pages.

1. If $v_s(t) = 2\cos(1000t - 60^o)$, find $v_o(t)$. (15)



$V_s = 2\angle{-60}\ V$

$\omega = 1000$, so $Z_c = -\frac{j}{\omega C} = -\frac{j}{10^3 10^{-5}} = -j100\ \Omega$

$V^- = V^+ = 0$, so

$I_s = \frac{V_s}{10\Omega} = 0.2\angle{-60}\ A$

$V_o = -I_s Z_{RC}$, where $Z_{RC} = 100\Omega \| Z_c = \frac{(100)(-j100)}{100 - j100}$

$= 100\frac{-j}{1-j} = 100\frac{1\angle{-90}}{\sqrt{2}\angle{-45}}$

$= \frac{100}{\sqrt{2}}\angle{-45} = 50\sqrt{2}\angle{-45}$

$= 50 - j50\ \Omega$

$= -(0.2\angle{-60^\circ})(50\sqrt{2}\angle{-45^\circ})$

$= -10\sqrt{2}\angle{-105^\circ}$

$= 10\sqrt{2}\angle{180-105}$

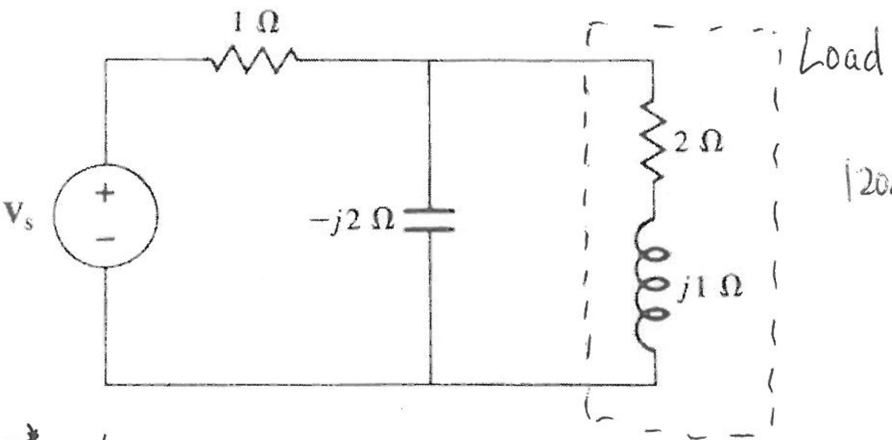
$= 10\sqrt{2}\angle{75^\circ}$

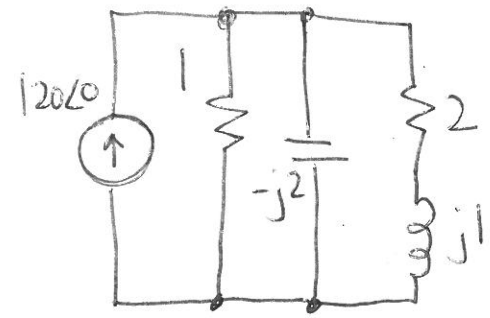
$v_o(t) = 10\sqrt{2}\cos(1000t + 75^\circ)$

$= \underline{14.14\cos(1000t + 75^\circ)\ V}$

Two (of many) approaches shown (left & right)

2. If $V_s = 120\angle 0^o$ V(rms), what is the complex power delivered to RL series load? (15)





$S = I^*_{rms} V_{rms}$

$S_L = I_L^* V_L = \left(\frac{V_L}{Z_L}\right)^* V_L = \frac{|V_L|^2}{Z_L^*}$

$Z_L = 2+j \; ; \; Z_L^* = 2-j$

$V_L = \frac{Z_c \| Z_L}{1 + Z_c \| Z_L} V_s$

$Z_c \| Z_L = \frac{Z_c Z_L}{Z_c + Z_L} = \frac{(-j2)(2+j)}{-j2+2+j}$

$= \frac{-j4+2}{2-j} = \frac{(2-j4)(2+j)}{2^2+1^2}$

$= \frac{4+j2-j8+4}{5} = \frac{8-j6}{5}$

$\frac{V_L}{V_s} = \frac{(8-j6)/5}{1+(8-j6)/5} = \frac{8-j6}{13-j6}$ *

$\frac{|V_L|^2}{|V_s|^2} = \frac{8^2+6^2}{13^2+6^2} = 0.488$

$|V_L|^2 = 0.488(120)^2 = 7024.4$

$S_L = \frac{7024.4}{2-j} = \frac{7024.4}{2^2+1^2}(2+j)$

$= 1404.9(2+j) = \underline{\underline{2810 + j1405 \text{ VA}}}$

$I_L = I_{total} \frac{Y_L}{Y_{total}}$

$Y_L = \frac{1}{Z_L} = \frac{1}{2+j} = \frac{2-j}{2^2+1^2} = \frac{2-j}{5}$

$Y_{total} = 1 + \frac{1}{-j2} + \frac{2-j}{5}$

$= \frac{1}{5}\left[5 + j2.5 + 2 - j\right]$

$= \frac{1}{5}\left[7 + j1.5\right] S$

$I_L = 120\angle 0 \; \frac{2-j}{7+j1.5} = 120 \frac{(2-j)(7-j1.5)}{7^2+1.5^2}$

$= \frac{120}{51.25}\left[14 - j3 - j7 - 1.5\right]$

for rms

$= 2.34\left[12.5 - j10\right]$

$S_L = I_L^* V_L = I_L^*(I_L Z_L) = |I_L|^2 Z_L$

$|I_L|^2 = (2.34)^2(12.5^2 + 10^2) = 1400$

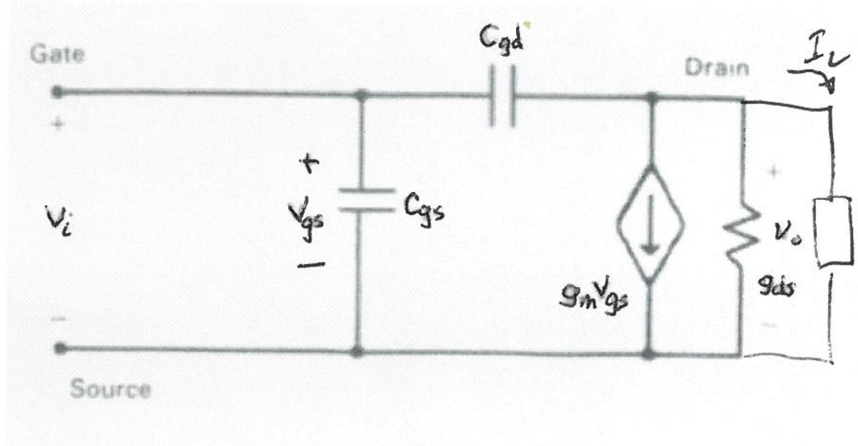
$S_L = 1400(2+j) = 2800 + j1400 \text{VA}$

=

$P = 2800 \text{W}$

$Q = 1400 \text{ VAR}$

3. For the circuit to the right, $C_{gs} = 100\,\text{pF}$, $C_{gd} = 20\,\text{pF}$, $g_m = 0.1\,\text{S}$, and $g_{ds} = 10\,\text{mS}$. If the input voltage is $v_i(t) = 20\cos(10^9 t)$ mV, what load admittance $Y_L = G_L + jB_L$ placed in parallel to g_{ds} (across v_o) would maximize power transfer to the load? (20)



(b)

We can reduce this circuit to its Thevenin or Norton equivalent and power is maximized by making

$$Z_L = Z_{Th}^* \quad \text{or} \quad Y_L = Y_N^* = 1/Z_{Th}^*$$

$$I_{sc} = V_i(j\omega C_{gd}) - g_m V_i = V_i(j\omega C_{gd} - g_m)$$
$$= 20\angle 0\ \text{mV}(j20 - 100\ \text{mS})$$
$$= \underline{\underline{-2 + j0.4\ \text{mA}}}$$

For V_{oc}, use node voltage equation

$$(V_{oc} - V_{gs})(j\omega C_{gd}) + g_m V_{gs} + V_{oc}\, g_{ds} = 0$$

$$V_{oc}(j\omega C_{gd} + g_{ds}) = V_{gs}(j\omega C_{gd} - g_m)$$

$$V_{oc} = V_{gs}\left(\frac{j\omega C_{gd} - g_m}{j\omega C_{gd} + g_{ds}}\right) = V_i\left(\frac{j\omega C_{gd} - g_m}{j\omega C_{gd} + g_{ds}}\right) = 20\angle 0\,\text{mV}\left[\frac{j20 - 100\ \cancel{\text{mS}}}{j20 + 10\ \cancel{\text{mS}}}\right]$$
$$= \underline{\underline{-24 + j88\ \text{mV}}}$$

$$Y_N = \frac{I_{sc}}{V_{oc}} = \frac{\cancel{V_i(j\omega C_{gd} - g_m)}}{\cancel{V_i}\left(\frac{\cancel{j\omega C_{gd} - g_m}}{j\omega C_{gd} + g_{ds}}\right)} = j\omega C_{gd} + g_{ds}$$
$$= 10 + j20\ \text{mS}$$

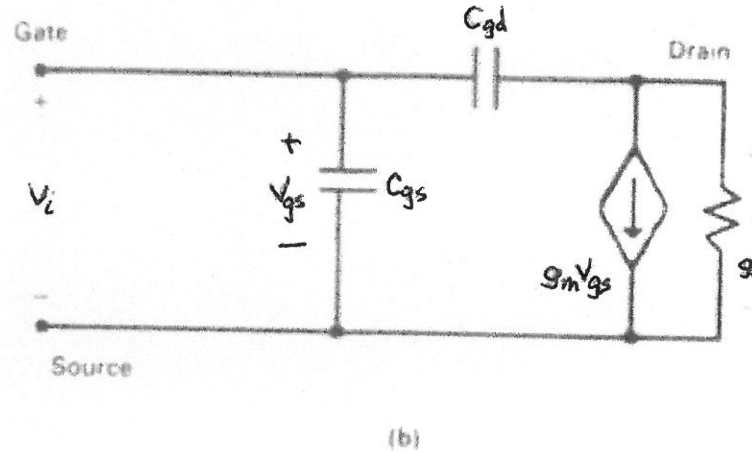
Maximum Power Transfer for $Y_L^{MPT} = Y_N^* = g_{ds} - j\omega C_{gd}$

$$= 10\times 10^{-3} - j(10^9)(20\times 10^{-12})$$
$$= \underline{\underline{10 - j20\ \text{mS}}}$$

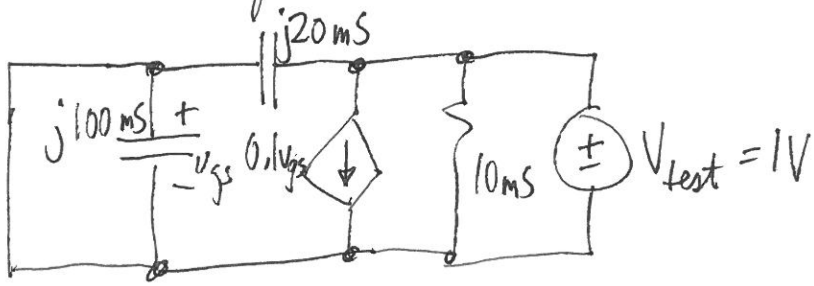
$$Z_L^{MPT} = \frac{1}{Y_L} = \frac{100\cancel{0}\,\Omega}{1\cancel{0} - j2\cancel{0}} = \frac{100(1+j2)\,\Omega}{1^2 + 2^2} = 20 + j40\,\Omega$$

End Of Exam

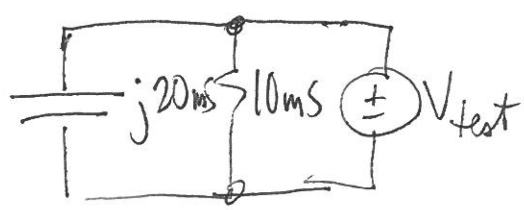
3. For the circuit to the right, $C_{gs} = 100\,\text{pF}$, $C_{gd} = 20\,\text{pF}$, $g_m = 0.1\,\text{S}$, and $g_{ds} = 10\,\text{mS}$. If the input voltage is $v_i(t) = 20\cos(10^9 t)$ mV, what load admittance $Y_L = G_L + jB_L$ placed in parallel to g_{ds} (across v_o) would maximize power transfer to the load? (20)



There is an easier alternate method.
Since only Y_N is needed, just use a test output voltage and zero input



$V_{gs} = 0$ &
C_{gs} is shorted, so can reduce to:



$$Y_N = 10\,\text{mS} + j20\,\text{mS}$$

$$Y_L^{MPT} = Y_N^* = \underline{10 - j20\ \text{mS}}$$

Note: this circuit is a high frequency model for an MOS transistor (note labels of gate, drain and source).

End Of Exam