Name Solutions UW ID#: ______ Section: ___ Version A

Exam 2 — EE 233

Spring 2010

Mean = 65.4
Median = 69
Std. Dev = 16.4

The test is closed book, with two sheets of notes and calculators allowed. Show all work. Be sure to state all assumptions made and **check** them when possible. The number of points per problem are indicated in parentheses. Total of 100 points in 3 problems on 3 pages (4th page has Laplace transform pairs).

1. The Laplace transform of an output voltage is:

$$V(s) = \frac{5e^{-s}}{(s+1)^2} + \frac{2s+2}{s^2+6s+10}$$

(a) Find $v(t)$ for $t > 0^-$. (20)

$$\mathcal{L}^{-1}\left\{\frac{5}{(s+1)^2}\right\} = 5te^{-t}u(t), \text{ so } \mathcal{L}^{-1}\left\{\frac{5e^{-s}}{(s+1)^2}\right\} = 5(t-1)e^{-(t-1)}u(t-1)$$

$$s^2+6s+10 = (s+3)^2+1 = (s+3-j)(s+3+j), \quad \frac{-6 \pm \sqrt{6^2-4(10)}}{2} = -3 \pm j$$

$$\frac{2s+2}{s^2+6s+10} = \frac{K}{s+3-j} + \frac{K^*}{s+3+j}$$

$$K = \left[\frac{(2s+2)\cancel{(s+3-j)}}{\cancel{(s+3-j)}(s+3+j)}\right]_{s=-3+j} = \left.\frac{2s+2}{s+3+j}\right|_{s=-3+j} = \frac{2(-3+j)+2}{-\cancel{3}+j+\cancel{3}+j}$$

$$= \frac{-4+j2}{j2} = 1 + \frac{-2}{j} = 1+j2 = \sqrt{1+4}\angle\tan^{-1}(2) = \sqrt{5}\angle 63.4^\circ$$

$\alpha = -3$, $\beta = 1$

$$v(t) = 5(t-1)e^{-(t-1)}u(t-1) + \underbrace{2\sqrt{5}}_{|K|}e^{-3t}\cos(t + \underbrace{63.4}_{\theta_K})$$

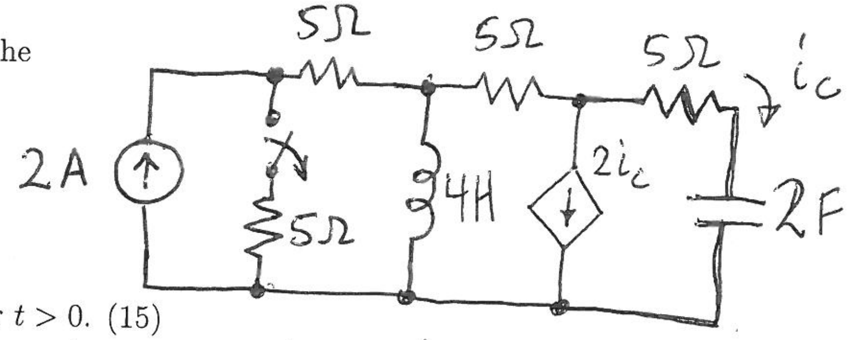
(b) Use the Initial and Final Value Theorems to check your results. (10)

$u(-1) = 0$

$$\lim_{s\to\infty}(sV(s)) = \lim_{s\to\infty}\frac{2s^2+2s}{s^2+6s+10} = 2, \quad \lim_{t\to 0} v(t) = 0 + 2\sqrt{5}\cos(63.4^\circ) = 2.0 \checkmark$$

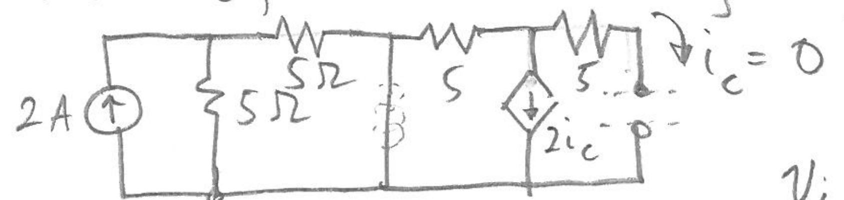
$$\lim_{s\to 0}(sV(s)) = \lim_{s\to 0}\frac{2s^2+2s}{s^2+6s+10} = 0 \quad \lim_{t\to\infty} v(t) = 0 \begin{cases} te^{-(t-1)} \to 0 \\ e^{-3t} \to 0 \end{cases} \checkmark$$

2. For the circuit to the right, note that the switch opens at $t = 0$,



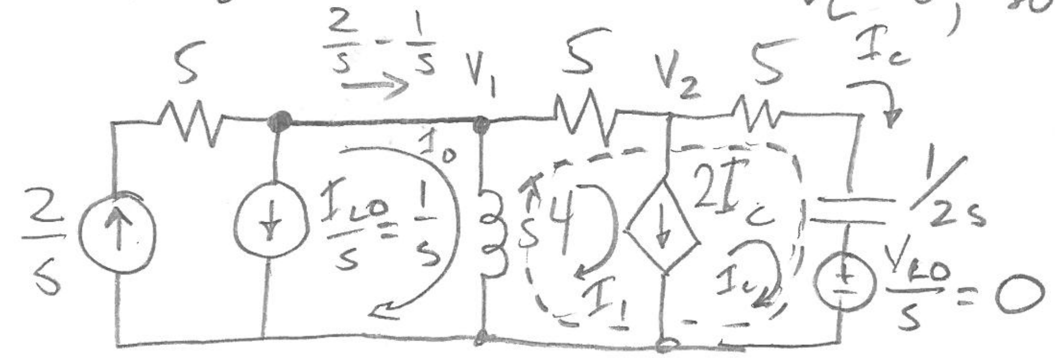
(a) Draw the s-domain circuit valid for $t > 0$. (15)

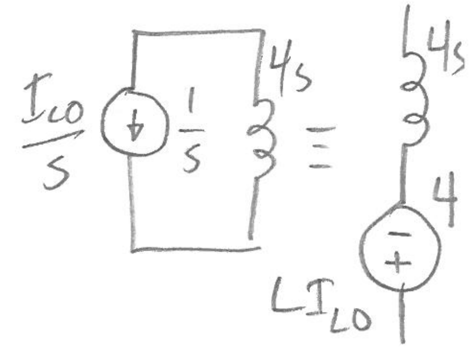
For $t<0$, switch closed for long time, so L is short, C is open



$$i_L = 2A \frac{5\Omega}{5\Omega + 5\Omega} = 1A = I_{L0}$$

$$v_L = 0, \text{ so } v_C = 0 = V_{C0}$$





(b) Find $I_c(s)$ (Laplace transform of capacitor current). (15)

Super Mesh (see above): $4s(I_1 - I_0) + 5I_1 + \left(5 + \frac{1}{2s}\right)I_c = 0$

$I_0 = \frac{2}{s} - \frac{1}{s} = \frac{1}{s}$, $I_1 - I_c = 2I_c$, so $I_1 = 3I_c$

$$4s\left(3I_c - \frac{1}{s}\right) + 5(3I_c) + \left(5 + \frac{1}{2s}\right)I_c = 0$$

$$I_c\left(12s + 15 + 5 + \frac{1}{2s}\right) = 4 \Rightarrow \boxed{I_c = \frac{4}{12s + 20 + \frac{1}{2s}} = \frac{1}{3}\left[\frac{s}{s^2 + \frac{5s}{3} + \frac{1}{24}}\right]}$$

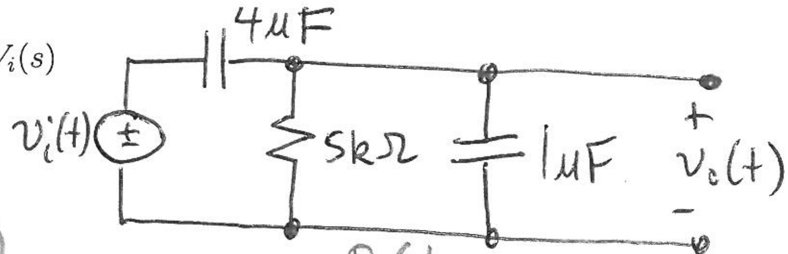
Can also use node voltage:

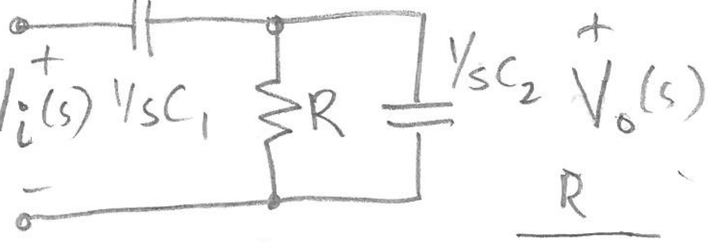
$$V_1: \quad -\frac{2}{s} + \frac{1}{s} + \frac{V_1}{4s} + \frac{V_1 - V_2}{5} = 0$$

$$V_2: \quad \frac{V_2 - V_1}{5} + 2I_c + \frac{V_2}{5 + 1/2s} = 0 \qquad \text{where } I_c = \frac{V_2}{5 + 1/2s}$$

solve 2 x 2 to get same answer

3. (a) Find the transfer function $H(s) = V_o(s)/V_i(s)$ for the circuit to the right. Assume no stored energy in circuit at $t = 0$. (15)





$$Z_{R\|C_2} = \frac{R\left(\frac{1}{sC_2}\right)}{R + 1/sC_2} = \frac{R}{sRC_2+1}$$

$$\frac{V_o(s)}{V_i(s)} = H(s) = \frac{Z_{R\|C_2}}{Z_{C_1} + Z_{R\|C_2}} = \frac{\frac{R}{sRC_2+1}}{\frac{1}{sC_1} + \frac{R}{sRC_2+1}}$$

$$= \frac{sRC_1}{sRC_2 + 1 + sRC_1} = \frac{sRC_1}{sR(C_1+C_2)+1}$$

$$= \frac{0.02s}{0.025s+1} = \underline{\underline{\frac{0.8s}{s+40}}}$$

$$RC_1 = (5\times10^3)(4\times10^{-6}) = 20\times10^{-3} = 0.02$$

$$RC_2 = 5\times10^{-3} = 0.005$$

(b) Calculate the impulse response for this circuit. (10)

$$h(t) = \mathcal{L}^{-1}\{H(s)\}$$

improper

$$H(s) = \frac{0.8s}{s+40} = \frac{0.8(s+40) - 32}{s+40} = 0.8 - \frac{32}{s+40}$$

$$= \mathcal{L}^{-1}\left\{0.8 - \frac{32}{s+40}\right\}$$

$$= 0.8\delta(t) - 32e^{-40t}u(t)$$

(c) If $v_i(t) = u(t) - u(t-50)$, calculate and sketch $v_o(t)$ versus time (seconds). (15)

Option 1: $V_o(s) = V_i(s)H(s) = \left(\frac{1}{\cancel{s}} - \frac{e^{-50s}}{\cancel{s}}\right)\left(\frac{0.8\cancel{s}}{s+40}\right) = \frac{0.8(1-e^{-50s})}{s+40}$

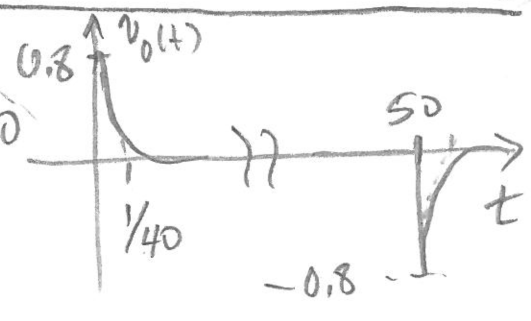
$$v_o(t) = \mathcal{L}^{-1}\left\{\frac{0.8(1-e^{-50s})}{s+40}\right\} = \mathcal{L}^{-1}\left\{\frac{0.8}{s+40} - \frac{0.8e^{-50s}}{s+40}\right\} =$$

$$= \underline{0.8e^{-40t}u(t) - 0.8e^{-40(t-50)}u(t-50)}$$

Option 2: $v_o(t) = v_i(t) * h(t)$

$$= \int_{-\infty}^{\infty} v_i(\lambda - t)\, h(\lambda)\, d\lambda$$

$$= \begin{cases} \int_0^t [0.8\delta(\lambda) - 32e^{-40t}]\, d\lambda = 0.8 + \frac{32}{40}e^{-40t}\Big|_0^t & 0<t<50 \\ \int_{t-50}^t h(\lambda)\, d\lambda = \frac{32}{40}e^{-40}\Big|_{t-50}^t & t>50 \end{cases}$$



End Of Exam