

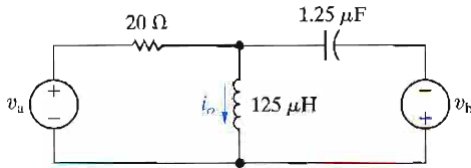
9.70 For the circuit in Fig. P9.57, suppose

$$v_a = 5 \cos 80,000t \text{ V}$$

$$v_b = -2.5 \cos 320,000t \text{ V.}$$

- What circuit analysis technique must be used to find the steady-state expression for $i_o(t)$?
- Find the steady-state expression for $i_o(t)$?

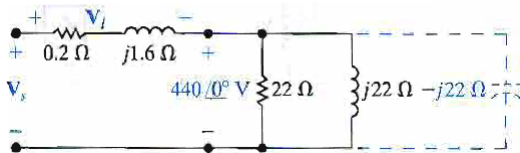
Figure P9.57



- For the circuit shown in Fig. P9.83, compute V_s and V_L .
- Construct a phasor diagram showing the relationship between V_s , V_L , and the load voltage of $440 \angle 0^\circ \text{ V}$.
- Repeat parts (a) and (b), given that the load voltage remains constant at $440 \angle 0^\circ \text{ V}$, when a capacitive reactance of -22Ω is connected across the load terminals.

Also determine the power factor of the source with and without the $-j22 \Omega$

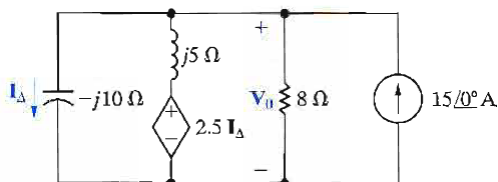
Figure P9.83



- Show that the maximum value of the instantaneous power given by Eq. 10.9 is $P + \sqrt{P^2 + Q^2}$ and that the minimum value is $P - \sqrt{P^2 + Q^2}$.

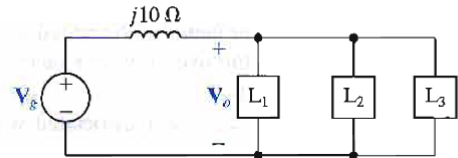
- Calculate the real and reactive power associated with each circuit element in the circuit in Fig. P9.56.
- Verify that the average power generated equals the average power absorbed.
- Verify that the magnetizing vars generated equal the magnetizing vars absorbed.

Figure P9.56



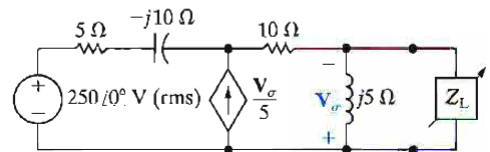
- The three parallel loads in the circuit shown in Fig. P10.24 can be described as follows: Load 1 is absorbing an average power of 24 kW and 18 kVAR of magnetizing vars; load 2 is absorbing an average power of 48 kW and generating 30 kVAR of magnetizing reactive power; load 3 consists of a 60Ω resistor in parallel with an inductive reactance of 480Ω . Find the rms magnitude and the phase angle of V_s if $V_o = 2400 \angle 0^\circ \text{ V(rms)}$.

Figure P10.24



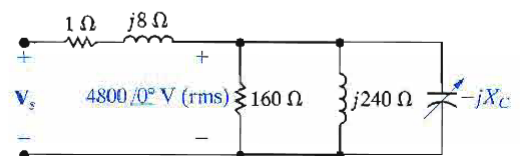
- The load impedance Z_L for the circuit shown in Fig. P10.43 is adjusted until maximum average power is delivered to Z_L .
 - Find the maximum average power delivered to Z_L .
 - What percentage of the total power developed in the circuit is delivered to Z_L ?

Figure P10.43



- The sending-end voltage in the circuit seen in Fig. P10.51 is adjusted so that the rms value of the load voltage is always 4800 V. The variable capacitor is adjusted until the average power dissipated in the line resistance is minimum.
 - If the frequency of the sinusoidal source is 60 Hz, what is the value of the capacitance in microfarads?
 - If the capacitor is removed from the circuit, what percentage increase in the magnitude of V_s is necessary to maintain 4800 V at the load?
 - If the capacitor is removed from the circuit, what is the percentage increase in line loss?

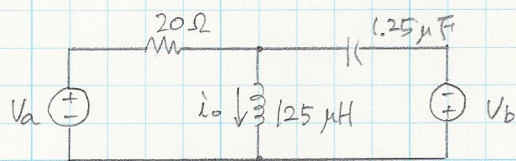
Figure P10.51



9.70

- (a) Superposition must be used because the frequencies of the two sources are different.

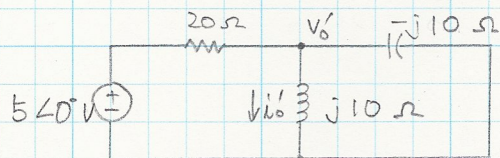
(b)



$$V_a = 5 \cos 80,000t \text{ V}$$

$$V_b = -2.5 \cos 320,000t \text{ V}$$

- i) For V_a ($\omega = 80,000 \text{ rad/s}$)



$$Z_C = 1/j\omega C = -j/\omega C = -j10 \Omega$$

$$Z_L = j\omega L = j10 \Omega$$

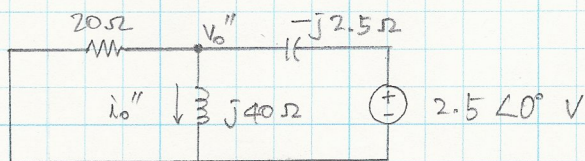
$$\frac{V_o' - 5}{20} + \frac{V_o'}{j10} - \frac{V_o'}{j10} = 0$$

$$V_o' \left(\frac{1}{20} + \frac{1}{j10} - \frac{1}{j10} \right) = \frac{5}{20}$$

$$\therefore V_o' = 5 \angle 0^\circ \text{ V}$$

$$\rightarrow i_o' = V_o' / j10 = \frac{5 \angle 0^\circ}{10 \angle 90^\circ} = \underline{0.5 \angle -90^\circ \text{ A}} \quad \left(\begin{aligned} &= 0.5 \cos(80,000t - 90^\circ) \\ &= 0.5 \sin 80,000t \end{aligned} \right)$$

- ii) For V_b ($\omega = 320,000 \text{ rad/s}$)



$$Z_C = -j/\omega C = -j2.5 \Omega$$

$$Z_L = j\omega L = j40 \Omega$$

$$\frac{V_o''}{20} + \frac{V_o''}{j40} - \frac{V_o'' - 2.5}{j2.5} = 0$$

$$V_o'' \left(\frac{1}{20} + \frac{1}{j40} - \frac{1}{j2.5} \right) = \frac{-1}{j} = j$$

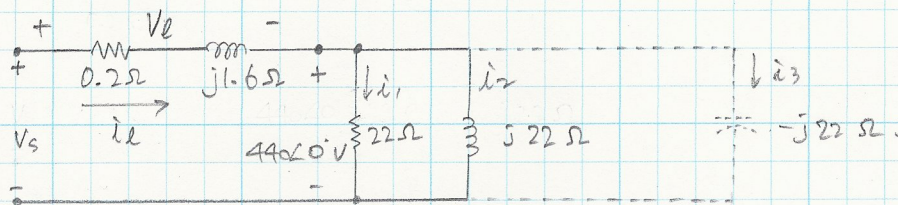
$$V_o'' (j2 + 1 - 16) = -40$$

$$\therefore V_o'' = \frac{-40}{-15 + j2} = \frac{40 \angle 180^\circ}{\sqrt{15^2 + 2^2} \angle 17.4^\circ} = 2.643 \angle 7.6^\circ \text{ V}$$

$$\rightarrow i_o'' = V_o'' / j40 = \frac{2.643 \angle 7.6^\circ}{40 \angle 90^\circ} = \underline{0.0661 \angle -82.4^\circ \text{ A}}$$

$$= \underline{66.1 \cos(320,000t - 82.4^\circ)}$$

Thus, $i_o(t) = i_o' + i_o'' = 500 \sin 80,000t + 66.1 \cos(320,000t - 82.4^\circ) \text{ mA}$



(a) Compute V_s and V_L

$$i_l = i_2 + i_3 = \frac{440 \angle 0^\circ}{22} + \frac{440 \angle 0^\circ}{j22} = 20 - j20 \text{ A.}$$

$$\therefore V_L = i_l (0.2 + j1.6)$$

$$= (20 - j20)(0.2 + j1.6)$$

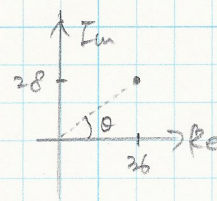
$$= 4 - j4 + j32 + 32$$

$$= 36 + j28 = \sqrt{36^2 + 28^2} \angle \theta$$

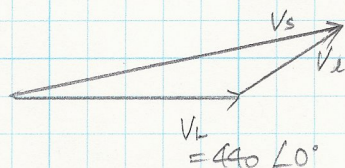
$$= \underline{45.61 \angle 37.87^\circ \text{ V}}$$

$$V_s = V_L + 440 \angle 0^\circ$$

$$= 36 + j28 + 440 = 476 + j28 = \underline{476.82 \angle 3.37^\circ}$$



(b) Phasor diagram
($V_s, V_L, 440 \angle 0^\circ$)



(c) with $-j22 \Omega$,

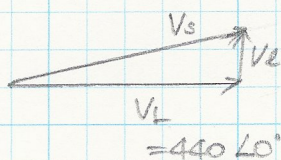
$$i_l = i_1 + i_2 + i_3 = \frac{440 \angle 0^\circ}{22} + \frac{440 \angle 0^\circ}{j22} - \frac{440 \angle 0^\circ}{j22} = 20 \text{ A}$$

$$\therefore V_L = i_l (0.2 + j1.6)$$

$$= 20(0.2 + j1.6) = 4 + j32 = \underline{32.25 \angle 82.87^\circ}$$

$$V_s = V_L + 440 \angle 0^\circ$$

$$= 4 + j32 + 440 = 444 + j32 = \underline{445.15 \angle 4.12^\circ}$$



* the power factor of the source

(i) with and (ii) without the $-j22\ \Omega$.

(i) with $-j22\ \Omega$.

$$\text{from (c)} \quad \left(\begin{array}{l} V_s = 445.15 \angle 4.12^\circ \text{ V} \\ i_e = 20 \angle 0^\circ \text{ A} \end{array} \right)$$

$$\begin{aligned} \therefore \text{pf} &= \cos(\theta_v - \theta_i) \\ &= \cos 4.12^\circ = \underline{\underline{0.997}} \end{aligned}$$

(ii) without $-j22\ \Omega$.

$$\text{from (a)} \quad \left(\begin{array}{l} V_s = 476.82 \angle 3.37^\circ \\ i_e = 20 - j20 = 28.28 \angle 45^\circ \end{array} \right)$$

$$\begin{aligned} \therefore \text{pf} &= \cos(\theta_v - \theta_i) \\ &= \cos(3.37^\circ - 45^\circ) = \underline{\underline{0.747}} \end{aligned}$$

10.1 (Eg. 10.9) $p = P + P \cos 2\omega t - Q \sin 2\omega t$

→ maximum: $P + \sqrt{P^2 + Q^2}$

minimum: $P - \sqrt{P^2 + Q^2}$

soln. by differentiating.

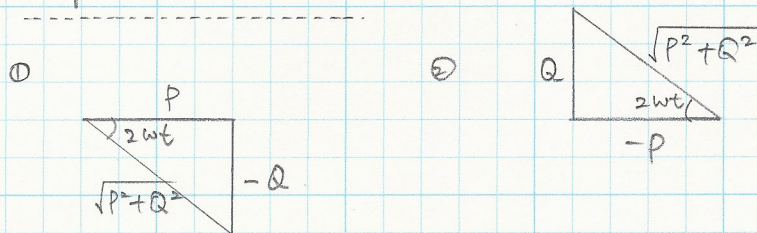
$$\frac{dp}{dt} = -2\omega P \sin 2\omega t - 2\omega Q \cos 2\omega t$$

$$\frac{dp}{dt} = 0. \quad (\text{for maximum or minimum})$$

$$\therefore -2\omega P \sin 2\omega t = 2\omega Q \cos 2\omega t$$

$$-P \sin 2\omega t = Q \cos 2\omega t$$

$$\frac{\sin 2\omega t}{\cos 2\omega t} = -\frac{Q}{P} = \tan 2\omega t$$



for ①,

$$\text{Eg. 10.9} \rightarrow p = P + P \cdot \frac{P}{\sqrt{P^2 + Q^2}} - Q \cdot \frac{-Q}{\sqrt{P^2 + Q^2}}$$

$$= P + \frac{P^2 + Q^2}{\sqrt{P^2 + Q^2}} = \underline{P + \sqrt{P^2 + Q^2}} \quad ; \text{ maximum}$$

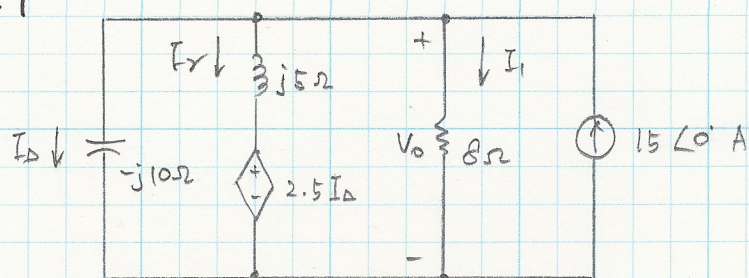
for ②,

$$\text{Eg. 10.9} \rightarrow p = P + P \cdot \frac{-P}{\sqrt{P^2 + Q^2}} - Q \cdot \frac{Q}{\sqrt{P^2 + Q^2}}$$

$$= P + \frac{-(P^2 + Q^2)}{\sqrt{P^2 + Q^2}} = \underline{P - \sqrt{P^2 + Q^2}} \quad ; \text{ minimum}$$

10.9

5



(a) We need to find V_o .

using KCL,

$$\left(-15 + \frac{V_o}{8} + \frac{V_o - 2.5 I_D}{j5} + \frac{V_o}{-j10} = 0 \right)$$

$$I_D = \frac{V_o}{-j10}$$

$$-15 + \frac{V_o}{8} + \frac{V_o}{j5} - \frac{2.5 \cdot V_o}{j5 \cdot -j10} + \frac{V_o}{-j10} = 0$$

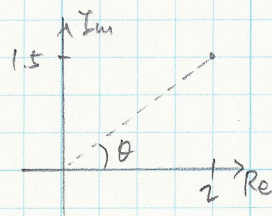
$$-15 + \frac{V_o}{8} + \frac{V_o}{j5} + \frac{V_o}{(j2)(j10)} - \frac{V_o}{j10} = 0$$

by multiplying $j20$

$$-j300 + V_o(j2.5 + 4 - j - 2) = 0$$

$$\therefore V_o = \frac{j300}{2 + j1.5} = \frac{300 \angle 90^\circ}{\sqrt{2^2 + 1.5^2} \angle \theta}$$

$$= \frac{300 \angle 90^\circ}{2.5 \angle 36.87^\circ} = 120 \angle 53.13^\circ$$



i) $I_1 = \frac{V_o}{8} = 15 \angle 53.13^\circ \text{ A} \rightarrow P_{8\Omega} = \frac{1}{2} I_1^2 R = \frac{1}{2} \cdot (15^2) \cdot 8 = \underline{900 \text{ W}}$

∴ The 8Ω resistor is absorbing 900 W .

ii) $I_D = \frac{V_o}{-j10} = \frac{120 \angle 53.13^\circ}{10 \angle -90^\circ} = 12 \angle 143.13^\circ$

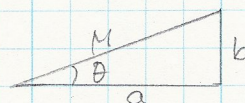
$$\rightarrow Q_{\text{cap}} = \frac{1}{2} (12)^2 (-10) = \underline{-720 \text{ VAR}}$$

∴ The $-j10\Omega$ capacitor is delivering 720 magnetizing vars.

iii) $I_2 = \frac{V_o - 2.5 I_D}{j5} = \frac{120 \angle 53.13^\circ - 2.5 \times 12 \angle 143.13^\circ}{j5}$

$$= \frac{(72 + j96) + (24 - j18)}{j5}$$

$$= 15.6 - j19.2 = 24.738 \angle -50.91^\circ \text{ A}$$



$$M \angle \theta = M \cos \theta + j M \sin \theta$$

$$\rightarrow Q_{IND} = \frac{1}{2} (24.738)^2 5 = \underline{1530 \text{ VAR}}$$

i The $j5\Omega$ inductor is absorbing 1530 magnetizing vars.

(b) For the independent current source.

$$S_g = -\frac{1}{2} V_o I_g^* = -\frac{1}{2} (120 \angle 53.13^\circ) (15 \angle 0^\circ) \\ = -900 \angle 53.13^\circ = \underline{-540 - j720 \text{ VA}}$$

i The independent current source is delivering 540 W and 720 magnetizing vars.

For the dependent voltage source,

$$S_{2.5I_A} = \frac{1}{2} (2.5 I_A) I_2^* = \frac{1}{2} (2.5 \times 12 \angle 143.13^\circ) (24.72 \angle +50.91^\circ) \\ = 370.8 \angle 194.04^\circ \\ = \underline{-360 - j90 \text{ VA}}$$

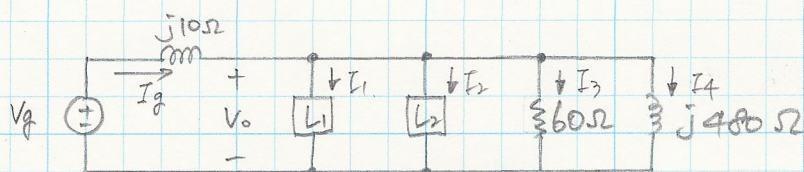
i The dependent voltage source is delivering 360 W and 90 magnetizing vars.

$$\text{Thus, } \Sigma P_{gen} = 360 + 540 = 900 \text{ W} = \Sigma P_{abs}$$

$$(c) \quad \Sigma Q_{gen} = \underset{(cap.)}{720} + \underset{(2.5I_A)}{90} + \underset{(I_g)}{720} = 1530 \text{ VAR} = \Sigma Q_{abs} \underset{(IND)}{}$$

10.24

7



$$V_o = 2400 \angle 0^\circ \text{ V (rms)}$$

L_1 : absorbing an average power of 24 kW and 0 kVAR.

L_2 : absorbing 48 kW and generating 30 kVAR

L_2 : $60 \Omega \parallel j480 \Omega$

→ Find the rms magnitude and the phase angle of V_g .

For I_1 : $2400 I_1^* = 24000 + j18000$

$$I_1^* = \frac{24000 + j18000}{2400} = 10 + j7.5 \text{ A (rms)}$$

$$\therefore \underline{I_1 = 10 - j7.5 \text{ A (rms)}}$$

For I_2 : $2400 I_2^* = 48000 - j30000$

$$I_2^* = \frac{48000 - j30000}{2400} = 20 - j12.5 \text{ A (rms)}$$

$$\therefore \underline{I_2 = 20 + j12.5 \text{ A (rms)}}$$

For I_3 : $I_3 = \frac{2400 \angle 0^\circ}{60} = 40 \text{ A}$

For I_4 : $I_4 = \frac{2400 \angle 0^\circ}{j480} = -j5 \text{ A}$

$$\therefore I_g = I_1 + I_2 + I_3 + I_4$$

$$= (10 - j7.5) + (20 + j12.5) + 40 - j5$$

$$= \underline{70 \text{ A}}$$

$$\therefore V_g = (j10) I_g + V_o$$

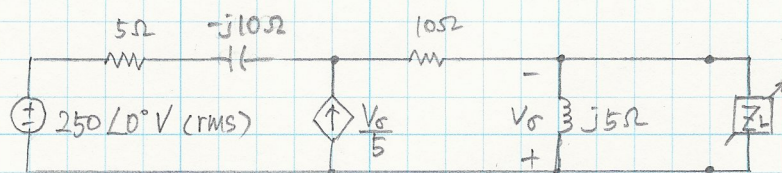
$$= j10(70) + 2400 = 2400 + j700$$

$$= \sqrt{2400^2 + 700^2} \angle \tan^{-1}(700/2400)$$

$$= \underline{2500 \angle 16.26^\circ \text{ V (rms)}}$$

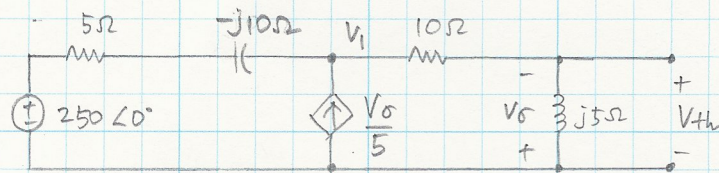
10.43

8



Z_L is adjusted until maximum power is delivered to Z_L .

(a) Using Thevenin equivalent circuit



at V_1 node,

$$\frac{V_1 - 250}{5 - j10} - \frac{V_o}{5} + \frac{V_1}{10 + j5} = 0 \quad \text{--- (1)}$$

$$V_o = \frac{-j5 V_1}{10 + j5} \quad \text{--- (2)}$$

$$\text{from (2)} \quad -\frac{V_o}{5} = \frac{jV_1}{10 + j5} = \frac{-V_1}{-5 + j10} = \frac{V_1}{5 - j10}$$

$$\therefore \text{ (1)} : V_1 \left(\frac{1}{5 - j10} + \frac{1}{5 - j10} + \frac{1}{10 + j5} \right) = \frac{250}{5 - j10}$$

$$V_1 \left(\frac{2}{5 - j10} + \frac{-j}{5 - 10j} \right) = \frac{250}{5 - j10}$$

$$\therefore V_1 = 250 \times \frac{1}{2 - j} = 250 \cdot \frac{(2 + j)}{(2 - j)(2 + j)}$$

$$= 250 \cdot \frac{2 + j}{5} = \underline{100 + j50}$$

$$\left(= \sqrt{100^2 + 50^2} \angle 26.565^\circ = 111.8 \angle 26.565^\circ \right)$$

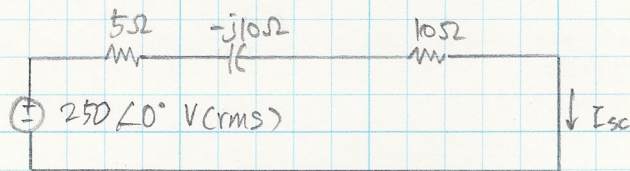
$$\therefore V_{th} = \frac{j5}{10 + j5} V_1$$

$$= \frac{j5}{10 + j5} (100 + j50) = j50 = \underline{50 \angle 90^\circ \text{ V (rms)}}$$

W

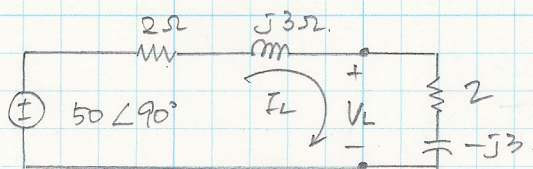
For short circuit current ,

$$V_o = 0.$$



$$I_{sc} = \frac{250}{5 - j10 + 10} = \frac{250}{15 - j10}$$

$$\therefore Z_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{j50 \left(\frac{15 - j10}{250} \right)}{1} = \frac{j15 + 10}{5} = \underline{2 + j3 \Omega}.$$



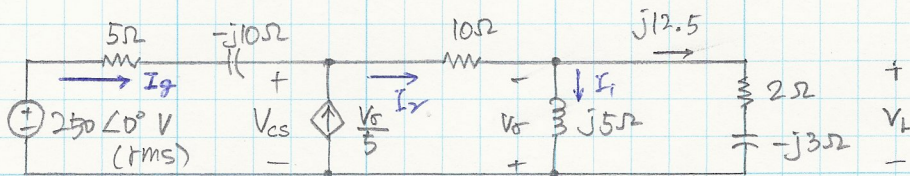
Thus, for the maximum power,

$$\underline{Z_L = 2 - j3}.$$

$$I_L = \frac{50 \angle 90^\circ}{(2 + j3) + (2 - j3)} = 12.5 \angle 90^\circ$$

$$\therefore \underline{P = (12.5)^2 (2) = 312.5 \text{ W.}}$$

(b)



For current source, $S_{as} = -V_{os} \cdot I_{as}^*$

$$V_{os} = 10 I_2 + V_L$$

$$\text{i) } I_2 = I_1 + j12.5 = \frac{V_L}{j5} + j12.5$$

$$= \frac{j12.5(2 - j3)}{j5} + j12.5$$

$$= (5 - j7.5) + j12.5 = 5 + j5 \text{ A (rms)}$$

$$\text{ii) } V_L = j12.5(2 - j3) = 37.5 + j25 \text{ V (rms)}$$

From i), ii),

$$\begin{aligned} V_{cs} &= 10 I_2 + V_L \\ &= 10(5 + j5) + (37.5 + j25) \\ &= 87.5 + j75 \text{ V (rms)} \end{aligned}$$

$$\begin{aligned} \therefore S_{cs} &= -V_{cs} \cdot I_{sc}^* \\ &= -(87.5 + j75) \left(\frac{V_s}{5} \right)^* \\ &= -(87.5 + j75) \left(\frac{-V_L}{5} \right)^* \\ &= -(87.5 + j75) \left(\frac{-37.5 - j25}{5} \right)^* \\ &= -(87.5 + j75)(-7.5 + j5) = 1031.25 + j125 \text{ VA} \end{aligned}$$

i) The current source is absorbing 1031.25 W and 125 magnetizing VARs.

→ Only the independent voltage source is developing power.

For the independent voltage source,

$$\begin{aligned} I_g &= I_2 - V_s/5 \\ &= (5 + j5) - (-V_L/5) \\ &= (5 + j5) + \frac{37.5 + j25}{5} \\ &= (5 + j5) + (7.5 + j5) = 12.5 + j10 \end{aligned}$$

$$\begin{aligned} S_{vs} &= -V_g I_g^* \\ &= (-250 \angle 0^\circ)(12.5 + j10)^* \\ &= -250(12.5 - j10) \\ &= -3125 + j2500 \text{ VA} \end{aligned}$$

∴ Developed Power

$$P_{dev} = 3125 \text{ W.}$$

10% of the developed power is delivered to the load.

$$\therefore \% \text{ delivered} = \frac{312.5}{3125} \times 100\% = \underline{\underline{10\%}}$$

check.

$$P_{10\Omega} = (\sqrt{5^2 + 5^2})^2 \cdot 10 = 500 \text{ W.}$$

$$P_{2\Omega} = 312.5 \text{ W.}$$

$$P_{5\Omega} = (\sqrt{12.5^2 + 10^2})^2 \cdot 5 = 1281.25 \text{ W.}$$

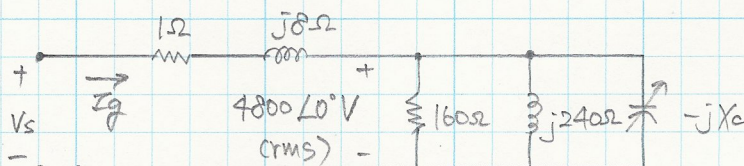
$$\text{For current source } P_{CS} = 1031.25 \text{ W}$$

$$\Sigma P_{abs} = 500 + 312.5 + 1281.25 + 1031.25$$

$$= 3125$$

$$= \Sigma P_{dev}$$

10.51



The load voltage is always $4800 \angle 0^\circ \text{ V (rms)}$.

The variable cap. is adjusted until the average power dissipated in the line resistance is minimum.

$$(a) \quad f = 60 \text{ Hz.} \rightarrow \omega = 2\pi f = 120\pi.$$

minimum power dissipation

\equiv maximum power transfer

with fixed load voltage.

$$\therefore -jX_0 = j240.$$

$$X_0 = \frac{1}{\omega C} = 240. \rightarrow C = \frac{1}{(120\pi \times 240)}$$

$$= \underline{\underline{11.05 \mu\text{F}}}.$$

(b) without cap.

$$I_g = \frac{4800}{160} + \frac{4800}{j240} = 30 - j20 \text{ A (rms)}$$

$$V_s = I_g(1 + j8) + 4800 = (30 - j20)(1 + j8) + 4800$$

$$= 30 + j220 + 160 + 4800$$

$$= 4990 + j220$$

$$= \underline{\underline{4994.85 \angle 2.52^\circ \text{ V (rms)}}}$$

magnitude.

C9

With cap.

$$I_g = \frac{4800}{160} + \frac{4800}{j240} - \frac{4800}{j240} = 30 \text{ A (rms)}$$

$$\begin{aligned} V_s &= I_g (1 + j8) + 4800 \\ &= 30 + j240 + 4800 \\ &= 4830 + j240 \\ &= \underline{4835.96} \angle 2.84^\circ \text{ V (rms)} \\ &\quad \text{magnitude} \end{aligned}$$

$$\% \text{ increase} = \frac{4994.85}{4835.96} \times 100 = \underline{3.29 \%}$$

(c) line loss without cap.

$$\begin{aligned} P_e &= |I_g|^2 R = |30 - j20|^2 \cdot 1 \\ &= |\sqrt{30^2 + 20^2}|^2 \cdot 1 = \underline{1300 \text{ W.}} \end{aligned}$$

line loss with cap.

$$P_e = |I_g|^2 R = |30|^2 \cdot 1 = \underline{900 \text{ W.}}$$

$$\% \text{ Increase} = \frac{1300}{900} \times 100 = \underline{44.44 \%}$$