12.10 a) Show that

$$\int_{-\infty}^{\infty} f(t)\delta'(t-a)\,dt = -f'(a).$$

(*Hint:* Integrate by parts.)

b) Use the formula in (a) to show that

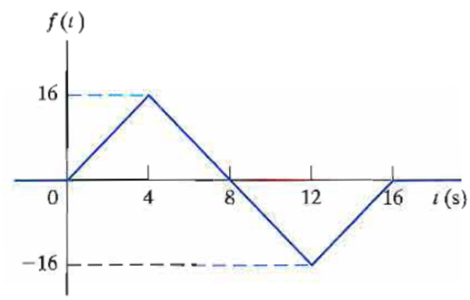
$$\mathcal{L}\{\delta'(t)\} = s.$$

12.19 a) Find the Laplace transform of the function illustrated in Fig. P12.19.

b) Find the Laplace transform of the first derivative of the function illustrated in Fig. P12.19.

c) Find the Laplace transform of the second derivative of the function illustrated in Fig. P12.19.

Figure P12.19



12.23 Find the Laplace transform for (a) and (b).

a) $f(t) = \dfrac{d}{dt}(e^{-at}\sin \omega t)$.

b) $f(t) = \displaystyle\int_{0^-}^{t} e^{-ax}\cos \omega x\,dx$.

c) Verify the results obtained in (a) and (b) by first carrying out the indicated mathematical operation and then finding the Laplace transform.

12.27 The switch in the circuit in Fig. P12.27 has been open for a long time. At $t = 0$, the switch closes.

a) Derive the integrodifferential equation that governs the behavior of the voltage v_o for $t \geq 0$.

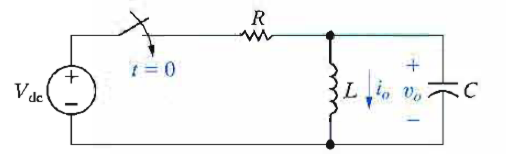
b) Show that

$$V_o(s) = \frac{V_{dc}/RC}{s^2 + (1/RC)s + (1/LC)}.$$

c) Show that

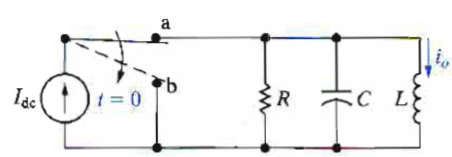
$$I_o(s) = \frac{V_{dc}(1/RLC)}{s[s^2 + (1/RC)s + (1/LC)]}.$$

Figure P12.27



12.37 PSPICE The circuit parameters in the circuit in Fig. P12.30 are $R = 50\,\Omega$, $L = 31.25$ mH, and $C = 2\,\mu$F. If $I_{dc} = 100$ mA, find $i_o(t)$ for $t \geq 0$.

Figure P12.30



/

12.10

(a) Show that $\int_{-\infty}^{\infty} f(t)\,\delta'(t-a)\,dt = -f'(a)$

sol). let $\delta'(t-a)\,dt = dV$ $\rightarrow$ $\delta(t-a) = V$

let $u = f(t)$ $\rightarrow$ $du = f'(t)\,dt$

from. $(u \cdot v)' = u'v + uv'$

$\therefore\ uv' = (uv)' - u'v$

$\rightarrow \int uv'\,dt = uv - \int u'v\,dt$

Therefore,

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-a)\,dt = f(t)\,\delta(t-a)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f'(t)\,\delta(t-a)\,dt$$

$$= \left(\underbrace{f(\infty)\,\delta(\infty-a)}_{=0} - \underbrace{f(-\infty)\,\delta(-\infty-\alpha)}_{=0} \right) - f'(a)$$

$$= 0 - f'(a)$$

(b) Show that $\mathcal{L}\{\delta'(t)\} = s$

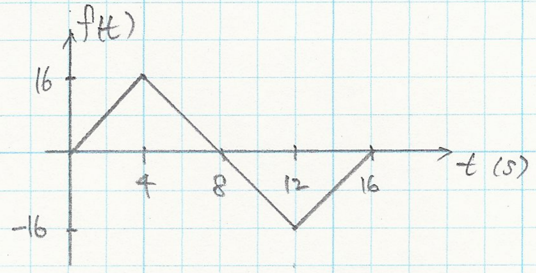
sol). $\mathcal{L}\{\delta'(t)\} = \int_{0^-}^{\infty} \delta'(t)\,e^{-st}\,dt$

$$= -\left[\frac{d[e^{-st}]}{dt}\right]_{t=0}$$

$$= -[-s\,e^{-st}]_{t=0}$$

$$= +s.$$

12.19



(a). $f(t) = 4t\,[u(t) - u(t-4)]$

$+ (32-4t)\,[u(t-4) - u(t-12)]$

$+ (-64+4t)\,[u(t-12) - u(t-16)]$

$= 4t\cdot u(t) + (-4t + 32 - 4t)\,u(t-4)$

$+ (-32+4t-64+4t)\,u(t-12) + (64-4t)\,u(t-16)$

$= 4t\cdot u(t) - 8(t-4)\,u(t-4) + 8(t-12)\,u(t-12) - 4(t-16)\,u(t-16)$

* Laplace transforms of the Unit step function. *

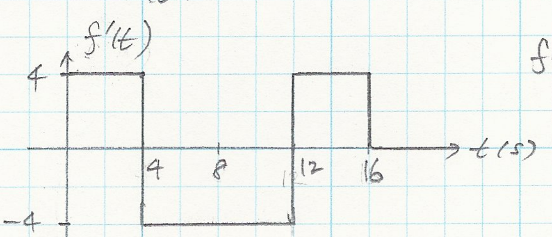
$$\mathcal{L}[u(t)] = 1/s$$

$$\mathcal{L}[u(t-a)] = \frac{e^{-as}}{s}$$

$$\mathcal{L}[u(t-a)\,g(t-a)] = e^{-as}\cdot G(s). \qquad G(s) = \mathcal{L}[g(t)]$$

$$\therefore\ F(s) = \frac{4\left(1 - 2\cdot e^{-4s} + 2\cdot e^{-12s} - e^{-16s}\right)}{s^2}$$

(b) First derivative



$f'(t) = 4\,[u(t) - u(t-4)]$

$-4\,[u(t-4) - u(t-12)]$

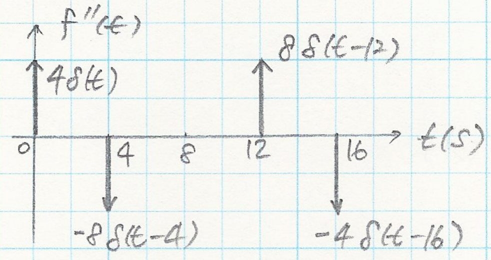
$+4\,[u(t-12) - u(t-16)]$

$= 4u(t) - 8u(t-4) + 8u(t-12)$

$- 4u(t-16)$

$$\therefore\ \mathcal{L}[f'(t)] = \frac{4\left(1 - 2e^{-4s} + 2e^{-12s} - e^{-16s}\right)}{s}$$

(c) Second derivative.



$$f''(t) = 4\delta(t) - 8\delta(t-4) + 8\delta(t-12) - 4\delta(t-16)$$

* Laplace transform of the unit impulse function *.

$$\mathcal{L}[\delta(t)] = 1$$

$$\mathcal{L}[\delta(t-a)] = e^{-as}$$

$$\therefore \mathcal{L}[f''(t)] = \underline{4(1-2e^{-4s}+2e^{-12s}-e^{-16s})}$$

12.23

(a) $f(t) = \dfrac{d(e^{-at}\sin wt)}{dt}$

* Laplace transform of Differentiation *.

$$\mathcal{L}[f'(t)] = sF(s) - f(0)$$

$$\mathcal{L}[f''(t)] = s^2F(s) - sf(0) - f'(0)$$

let $f_1(t) = e^{-at}\sin wt$

$$\mathcal{L}[f_1'(t)] = sF_1(s) - f_1(0)$$

$$F_1(s) = \frac{w}{(s+a)^2+w^2} \quad , \quad f_1(0) = 0.$$

$$\therefore \mathcal{L}[f(t)] = \frac{sw}{(s+a)^2+w^2}$$

(b) $f(t) = \int_{0^-}^{t} e^{-ax}\cos wx\, dx$

* Laplace transform of Integration *.

$$\mathcal{L}\left[\int_0^t f(\tau)d\tau\right] = \frac{1}{s}F(s)$$

let $f_1(t) = e^{-at}\cos wt$

$$\mathcal{L}\left[\int_0^t f_1(\tau)d\tau\right] = \frac{1}{s}F_1(s) = \frac{1}{s}\cdot\frac{s+a}{(s+a)^2+w^2}$$

(c) for (a),

$$f(t) = -ae^{-at}\sin wt + we^{-at}\cos wt$$

$$\therefore F(s) = \frac{-aw}{(s+a)^2+w^2} + \frac{w(s+a)}{(s+a)^2+w^2} = \frac{sw}{(s+a)^2+w^2}$$

for (b),

$$f(t) = \int_{0^-}^{t} e^{-ax}\cos wx\, dx$$

Ce→

using $\cos\omega x = \frac{1}{2}(e^{j\omega x} + e^{-j\omega x})$ & $\sin\omega x = \frac{1}{j2}(e^{j\omega x} - e^{-j\omega x})$

$$\therefore f(t) = \int_{0^-}^{t} e^{-ax}\frac{1}{2}(e^{j\omega x} + e^{-j\omega x})\,dx$$

$$= \frac{1}{2}\int_{0^-}^{t} e^{(-a+j\omega)x} + e^{(-a-j\omega)x}\,dx.$$

$$= \frac{1}{2}\left[\frac{e^{(-a+j\omega)x}}{-a+j\omega} - \frac{e^{(-a-j\omega)x}}{a+j\omega}\right]_0^t$$

$$= \frac{1}{2}\left[\frac{ae^{(-a+j\omega x)} + j\omega e^{(-a+j\omega x)} + ae^{(-a-j\omega)x} - j\omega e^{(-a-j\omega x)}}{-a^2-\omega^2}\right]_0^t$$

$$= \frac{1}{-2(a^2+\omega^2)}\left[a\cdot e^{-ax}(e^{j\omega x} + e^{-j\omega x}) + j\omega e^{-ax}(e^{j\omega x} - e^{-j\omega x})\right]_0^t$$

$$= \frac{1}{-2(a^2+\omega^2)}\left[ae^{-ax}2\cos\omega x + j\omega e^{-ax}(j2)\sin\omega x\right]_0^t$$

$$= \left[\frac{-ae^{-ax}\cos\omega x + \omega e^{-ax}\sin\omega x}{a^2+\omega^2}\right]_0^t$$

$$= \frac{-ae^{-at}\cos\omega t + \omega e^{-at}\sin\omega t}{a^2+\omega^2} - \frac{-a}{a^2+\omega^2}$$

$$= \frac{-ae^{-at}\cos\omega t + \omega e^{-at}\sin\omega t + a}{\omega^2+a^2}$$

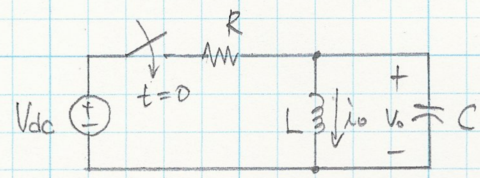
$$\therefore F(s) = \frac{1}{\omega^2+a^2}\left[\frac{\omega^2}{(s+a)^2+\omega^2} - \frac{a(s+a)}{(s+a)^2+\omega^2} + \frac{a}{s}\right]$$

$$= \frac{1}{\omega^2+a^2}\;\frac{\omega^2 s - as^2 - a^2 s + a(s^2+2s+a^2) + \omega^2 a}{s[(s+a)^2+\omega^2]}$$

$$= \frac{1}{\cancel{\omega^2+a^2}}\;\frac{s(\cancel{\omega^2+a^2}) + a(\cancel{\omega^2+a^2})}{s[(s+a)^2+\omega^2]}$$

$$= \frac{s+a.}{s[(s+a)^2+\omega^2]}$$

12.27



(a) $$\frac{V_o(t) - V_{dc}}{R} + \frac{1}{L}\int_0^t V_o(\tau)\,d\tau + C\frac{dV_o(t)}{dt} = 0$$

$$\therefore\ V_o(t) + \frac{R}{L}\int_0^t V_o(\tau)\,d\tau + RC\frac{dV_o(t)}{dt} = V_{dc}$$

(b) $$V_o(s) + \frac{R}{L}\cdot\frac{V_o(s)}{s} + RC\cdot s\,V_o(s) = \frac{V_{dc}}{s}$$

$$L\cdot s\,V_o(s) + R\,V_o(s) + RLC\,s^2\,V_o(s) = L\,V_{dc}$$

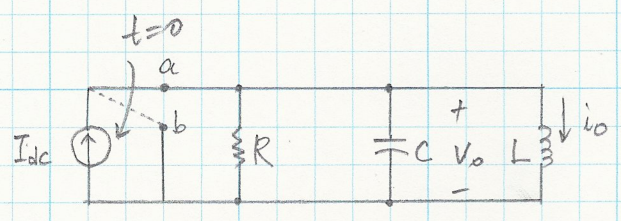
$$V_o(s)\left(RLC\,s^2 + Ls + R\right) = L\,V_{dc}$$

$$\therefore\ V_o(s) = \frac{V_{dc}}{RCs^2 + s + R/L} = \frac{1}{RC}\,\frac{V_{dc}}{s^2 + s/RC + 1/LC}$$

(c) from $i_o = \frac{1}{L}\int_0^t V_o(\tau)\,d\tau$

$$I_o(s) = \frac{1}{L}\,\frac{V_o(s)}{s} = \frac{1}{RLC}\cdot\frac{V_{dc}}{s\left(s^2 + s/RC + 1/LC\right)}$$

12.37



R = 50Ω
L = 31.25 mH
C = 2μF
Idc = 100 mA

find $i_o(t)$ for $t \geq 0$.

sol). For $t \geq 0$,

$$\frac{V_o(t)}{R} + C\frac{dV_o(t)}{dt} + i_o(t) = 0. \quad \text{------------------ ①}$$

$$V_o(t) = L\frac{di_o(t)}{dt} \longrightarrow \frac{dV_o(t)}{dt} = L\cdot\frac{d^2 i_o(t)}{dt^2} \quad \text{--------- ②}$$

from ① and ②,

$$\frac{L}{R}\frac{di_o(t)}{dt} + LC\frac{d^2 i_o(t)}{dt^2} + i_o(t) = 0.$$

$$\frac{d^2 i_o(t)}{dt} + \frac{1}{RC}\frac{di_o(t)}{dt} + \frac{1}{LC} i_o(t) = 0$$

$$\left[s^2 I_o(s) - s\,i_o(0) - i_o'(0)\right] + \frac{1}{RC}\left[sI_o(s) - i_o(0)\right] + \frac{1}{LC}I_o(s) = 0.$$

$\left(\begin{matrix} i_o(0) = I_{dc} \\ i_o'(0) = 0 \end{matrix}\right)$ for inductor.

$$I_o(s)\left[s^2 + \frac{s}{RC} + \frac{1}{LC}\right] = sI_{dc} + \frac{I_{dc}}{RC} = I_{dc}\left(s + \frac{1}{RC}\right)$$

$$\therefore I_o(s) = \frac{I_{dc}(s + 1/RC)}{(s^2 + s/RC + 1/LC)}$$

Using circuit parameters,

$$I_o(s) = \frac{0.1(s + 10^4)}{s^2 + 10^4 s + 16\times10^6} \quad \leftarrow s_1 = -2000,\ s_2 = -8000.$$

$$= \frac{0.1(s + 10000)}{(s+2000)(s+8000)}$$

$$= \frac{K_1}{s+2000} + \frac{K_2}{s+8000}$$

cont.

$$I_o(s) = \frac{k_1(s+8000)+k_2(s+2000)}{(s+2000)(s+8000)} = \frac{0.1s+1000}{(s+2000)(s+8000)}$$

$\therefore k_1 + k_2 = 0.1$ ------- ①

$8000k_1 + 2000k_2 = 1000.$ ----- ②

from ①, ②

$8k_1 + 2k_2 = 1$

$8(0.1 - k_2) + 2k_2 = 1$

$k_2(-8+2) = 1 - 0.8$

$\therefore k_2 = \frac{0.2}{-6} = -0.0333$

$\rightarrow k_1 = 0.1 - k_2 = 0.1 - (-0.0333) = 0.1333$

$$\therefore I_o(s) = \frac{0.1333}{s+2000} - \frac{0.0333}{s+8000}$$

$$\therefore i_o(t) = \mathcal{L}^{-1}[I_o(s)] = [0.1333\,e^{-2000t} - 0.0333\,e^{-8000t}]\,u(t)\ A$$

$$= [133.33\,e^{-2000t} - 33.33\,e^{-8000t}]\,u(t)\ mA.$$

※ using partial fractions,

$k_1 = \frac{0.1(8000)}{6000}$ $(s = -2000)$

$= 0.1333$

$k_2 = \frac{0.1(2000)}{-6000}$ $(s = -8000)$

$= -0.0333$