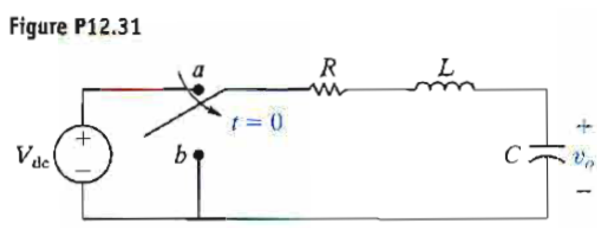
1. Check your answer to Problem 12.37 (5th Problem on HW#3) with SPICE.

2. Problem 12.38.

The circuit parameters in the circuit in Fig. P12.31 are $R = 5000\ \Omega$, $L = 1$ H, and $C = 0.25\ \mu$F. If $V_{dc} = 15$ V, find $v_o(t)$ for $t \geq 0$.

Figure P12.31



3. Find *f(t)* for the following functions:

$$(a)\ F(s) = \frac{s^3 + 8s^2 + 16s + 12}{s^2 + 5s + 6}$$

$$(b)\ F(s) = \frac{2s^2 - 4s + 2}{(s+2)^3}$$

$$(c)\ F(s) = \frac{s+3}{s^2 + 4s + 5}$$

$$(d)\ F(s) = \frac{3s+2}{(s^2 + 2s + 5)^2}$$

4. Apply the initial and final value theorem to each transform pair in Problem 3, if possible.

From HW 3,

$$i_o(t) = \left(133.33e^{-2000t} - 33.33e^{-8000t}\right)u(t) \quad [\mathrm{mA}]$$

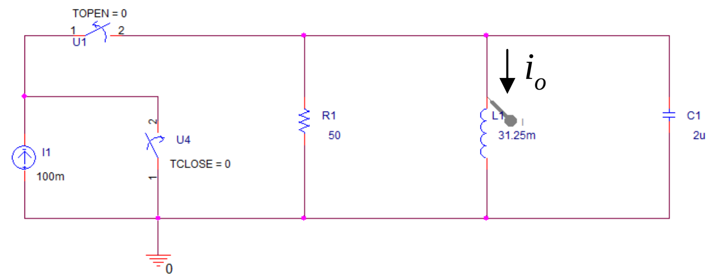
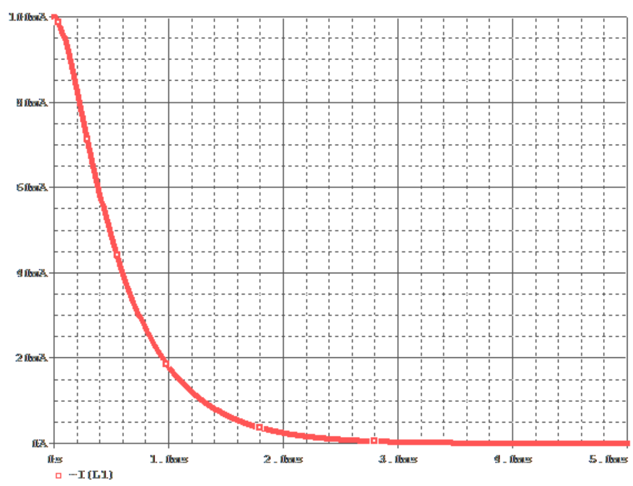


Fig. Schematic for simulation



Time [s]	io [mA]	
	Calculation	Simulation
0.0000	100.000	100.000
0.0005	48.440	48.490
0.0010	18.033	18.053
0.0015	6.638	6.646
0.0020	2.442	2.445
0.0025	0.898	0.900
0.0030	0.330	0.331
0.0035	0.122	0.122
0.0040	0.045	0.045
0.0045	0.016	0.017
0.0050	0.006	0.006

Fig. Simulation result

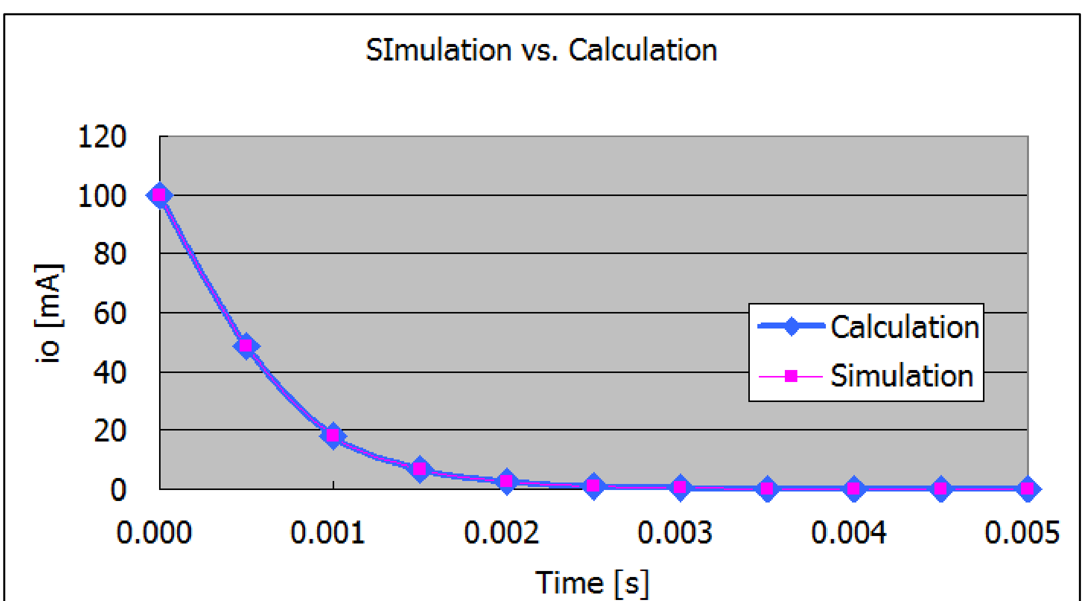
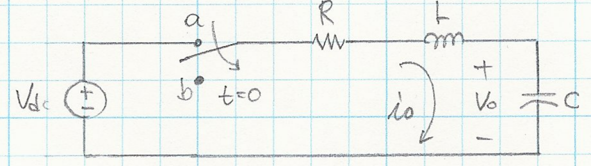


Fig. Simulation and calculation results

12.38



$R = 5000\,\Omega$

$L = 1\,H$

$C = 0.25\,\mu F$

$V_{dc} = 15$

For $t \geq 0^+$

$$R i_o + L\frac{di_o}{dt} + v_o = 0$$

$$i_o = C\frac{dv_o}{dt} \quad \therefore \quad \frac{di_o}{dt} = C\frac{d^2 v_o}{dt^2}$$

$$\therefore\ R\cdot C\frac{dv_o}{dt} + LC\frac{d^2 v_o}{dt^2} + v_o = 0$$

$$\rightarrow \frac{d^2 v_o}{dt} + \frac{R}{L}\frac{dv_o}{dt} + \frac{1}{LC}v_o = 0$$

Laplace transform

$$s^2 V_o(s) - s\,V_o(0) - \cancel{\frac{dV(0)}{dt}} + \frac{R}{L}\left[sV_o(s) - V(0)\right] + \frac{1}{LC}V(s) = 0$$

$$\left(\begin{array}{l} V_o(0) = V_{dc} = 15 \\ \frac{dV_o(0)}{dt} = 0 \end{array}\right)$$

$$\therefore\ V_o(s)\left[s^2 + \frac{R}{L}s + \frac{1}{LC}\right] = V_{dc}\left(s + \frac{R}{L}\right)$$

$$\rightarrow V_o(s) = \frac{V_{dc}(s + R/L)}{s^2 + (R/L)s + (1/LC)}$$

Using circuit parameters,

$$\frac{R}{L} = 5000, \quad \frac{1}{LC} = 4\times10^6$$

$$V_o(s) = \frac{15(s+5000)}{s^2 + 5000s + 4\times10^6}$$

$\leftarrow s_{1,2} = -2500 \pm \sqrt{6.25\times10^6 - 4\times10^6}$

$s_1 = -1000$ rad/s.

$s_2 = -4000$ rad/s.

$$V_o(s) = \frac{15(s+5000)}{(s+1000)(s+4000)}$$

$$= \frac{k_1}{s+1000} + \frac{k_2}{s+4000}$$

$$k_1 = \frac{15(4000)}{3000} = 20V \quad , \quad k_2 = \frac{15(1000)}{-3000} = -5V$$

$$\therefore V_o(s) = \frac{20}{s+1000} - \frac{5}{s+4000}$$

$$\rightarrow V_o(t) = \left[20e^{-1000t} - 5e^{-4000t}\right]u(t)$$

3. (a)

$$F(s) = \frac{s^3+8s^2+16s+12}{s^2+5s+6} = (s+3) - \frac{5s+6}{s^2+5s+6} \quad \leftarrow s_1 = -2, s_2 = -3$$

$$\begin{array}{r|l} & s+3 \\ \hline s^2+5s+6 & s^3+8s^2+16s+12 \\ & s^3+5s^2+6s \\ \hline & 3s^2+10s+12 \\ & 3s^2+15s+18 \\ \hline & -5s-6 \end{array}$$

$$F(s) = (s+3) - \left(\frac{k_1}{s+2} + \frac{k_2}{s+3}\right) \qquad k_1 = \left[\frac{5s+6}{s+3}\right]_{s=-2} = \frac{-10+6}{-2+3} = -4$$

$$= (s+3) - \left(\frac{-4}{s+2} + \frac{9}{s+3}\right) \qquad k_2 = \left[\frac{5s+6}{s+2}\right]_{s=-3} = \frac{-15+6}{-3+2} = 9$$

$$\rightarrow f(t) = \left[\frac{d\delta(t)}{dt} + 3\delta(t) + 4e^{-2t} - 9e^{-3t}\right]u(t)$$

3. (b)

$$F(s) = \frac{2s^2-4s+2}{(s+2)^3} = \frac{k_1}{(s+2)^3} + \frac{k_2}{(s+2)^2} + \frac{k_3}{(s+2)}$$

$$k_1 \quad \frac{2s^2-4s+2}{} = \left[k_1 + k_2(s+2) + k_3(s+2)^2\right]_{s=-2}$$

$$\rightarrow k_1 = 8+8+2 = 18$$

$$k_2; \quad \frac{d}{ds}\left[2s^2-4s+2\right]_{s=-2} = \frac{d}{ds}\left[k_1+k_2(s+2)+k_3(s+2)^2\right]_{s=-2}$$

$$\left[4s-4\right]_{s=-2} = \left[k_2+2k_3(s+2)\right]_{s=-2}$$

$$-12 = k_2$$

$$k_3; \quad \frac{d^2}{ds^2}\left[2s^2-4s+2\right]_{s=-2} = \frac{d^2}{ds^2}\left[k_1+k_2(s+2)+k_3(s+2)^2\right]_{s=-2}$$

$$4 = 2k_3$$

$$k_3 = 2$$

$$\therefore F(s) = \frac{18}{(s+2)^3} - \frac{12}{(s+2)^2} + \frac{2}{(s+2)}$$

$$\rightarrow f(t) = \left[9t^2e^{-2t} - 12te^{-2t} + 2e^{-2t}\right]u(t)$$

3.(c)

$$F(s) = \frac{s+3}{s^2+4s+5} = \frac{s+3}{(s+2+j)(s+2-j)} = \frac{k_1}{s+2+j} + \frac{k_2}{s+2-j}$$

$$k_1 = \left[\frac{(s+3)\cancel{(s+2+j)}}{\cancel{(s+2+j)}(s+2-j)}\right]_{s=-2-j} = \frac{-2-j+3}{-2-j+2-j}$$

$$= \frac{1-j}{-2j} = \frac{1+j}{2} = 0.5+j0.5 = 0.7071\angle 45^\circ$$

$$k_2 = k_1^* = 0.5-j0.5 = 0.7071\angle -45^\circ$$

$$F(s) = \frac{0.7071\angle 45^\circ}{s+2+j} + \frac{0.7071\angle -45^\circ}{s+2-j}$$

$$\therefore f(t) = 2\times 0.7071\, e^{-2t}\cos(t-45^\circ)\,u(t)$$

3. (d)

$$F(s) = \frac{3s+2}{(s^2+2s+5)^2} = \frac{3s+2}{(s+1-j2)^2(s+1+j2)^2}$$

$$= \frac{K_1}{(s+1-j2)^2} + \frac{K_2}{(s+1-j2)} + \frac{K_3}{(s+1+j2)^2} + \frac{K_4}{(s+1+j2)}$$

$$K_1 = \left[\frac{(3s+2)\cancel{(s+1-j2)^2}}{\cancel{(s+1-j2)^2}(s+1+j2)^2}\right]_{s=-1+j2} = \frac{-3+j6+2}{(-1+j2+1+j2)^2}$$

$$= \frac{-1+j6}{-16} = 0.0625 - j0.3750 = \underline{0.3802 \angle -80.534^\circ}$$

$$K_3 = K_1^* = 0.0625 + j0.3750 = \underline{0.3802 \angle 80.534^\circ}$$

$$K_2 = \frac{d}{ds}\left[\frac{(3s+2)\cancel{(s+1-j2)^2}}{\cancel{(s+1-j2)^2}(s+1+j2)^2}\right]_{s=-1+j2} = \frac{d}{ds}\left[\frac{3s+2}{(s+1+j2)^2}\right]_{s=-1+j2}$$

$$= \left[\frac{3(s+1+j2)-2(3s+2)}{(s+1+j2)^3}\right]_{s=-1+j2}$$

$$= \frac{3(\cancel{-1}+j2\cancel{+1}+j2)-2(-3+j6+2)}{(\cancel{-1}+j2\cancel{+1}+j2)^3}$$

$$= \frac{2}{-j64}$$

$$= j0.0313 = \underline{0.0313\angle 90^\circ}$$

$$K_4 = K_2^* = -j0.0313 = \underline{0.0313\angle -90^\circ}$$

$$\therefore F(s) = \frac{0.3802\angle -80.534^\circ}{(s+1-j2)^2} + \frac{0.0313\angle 90^\circ}{(s+1-j2)}$$

$$+ \frac{0.3802\angle +80.534^\circ}{(s+1+j2)^2} + \frac{0.0313\angle -90^\circ}{(s+1+j2)^2}$$

$$\rightarrow f(t) = [2\times 0.3802\, t\, e^{-t}\cos(2t-80.534^\circ)$$

$$+ 2\times 0.0313\, e^{-t}\cos(2t+90^\circ)]\, u(t)$$

$$= \underline{[0.7604\, t\, e^{-t}\cos(2t-80.534^\circ)}$$

$$\underline{-0.0626\, e^{-t}\sin 2t\,]\, u(t)}$$

4. Initial & Final Value theorem.

$\lim_{t \to 0^+} f(t) = \lim_{s \to \infty} sF(s)$; if $f(t)$ contains no impulse function.

$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$; if the poles of $F(s)$, except for a first-order pole at the origin lie in the left half of the s plane.

(a) IVT ; can not be applied $\because f(t)$ contains $\delta(t)$

FVT ; $s = -2, -3$; poles.

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s \left(\frac{s^3 + 8s^2 + 16s + 12}{s^2 + 5s + 6} \right) = 0$$

(check) $\lim_{t \to \infty} f(t) = 0 + 3 \cdot 0 + 4 \cdot 0 - 9 \cdot 0 = 0$

(b) IVT ; $\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s) = \lim_{s \to \infty} \frac{s(2s^2 - 4s + 2)}{(s+2)^3} = 2$

(check) $\lim_{t \to 0} f(t) = 18 \cdot 0 - 12 \cdot 0 + 2 \cdot 1 = 2$

FVT ; $s = -2, -2, -2$

$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s) = \lim_{s \to 0} \frac{s(2s^2 - 4s + 2)}{(s+2)^3} = 0$

(check) $\lim_{t \to \infty} f(t) = 18 \cdot 0 - 12 \cdot 0 + 2 \cdot 0 = 0.$

(c) IVT ; $\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s) = \lim_{s \to \infty} \frac{s(s+3)}{s^2 + 4s + 5} = 1$

(check) $\lim_{t \to 0} f(t) = 2 \times 0.7071 \times 1 \times \cos(-45°)$

$= 2 \times 0.7071 \times 1 \times \frac{1}{\sqrt{2}} = 1.$

FVT ; $\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s) = \lim_{s \to 0} \frac{s(s+3)}{s^2 + 4s + 5} = 0$

(check) $\lim_{t \to \infty} f(t) = 2 \times 0.7071 \times 0 \times \cos(\infty - 45°)$

$= 0.$ $\quad (\because -1 \le \cos(\infty - 45°) \le 1)$

(d) IVT ;

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{s(3s+2)}{(s^2+2s+5)^2} = 0$$

$$\text{(check)} \quad \lim_{t\to 0} f(t) = 0.7604\cdot 0 + 0.0626\cdot 0 = 0.$$

FVT ;

$$s = -1+j2,\ -1+j2,\ -1-j2,\ -1-j2$$

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{s(3s+2)}{(s^2+2s+5)^2} = 0$$

$$\text{(check)} \quad \lim_{t\to\infty} f(t) = 0.7604\cdot 0\cdot \cos(\infty + 80.534^\circ)$$

$$-0.0626\cdot 0\cdot \sin(\infty)$$

$$= 0.$$