13.60 a) Given $y(t) = h(t) * x(t)$, find $y(t)$ when $h(t)$ and $x(t)$ are the rectangular pulses shown in Fig. P13.60(a).

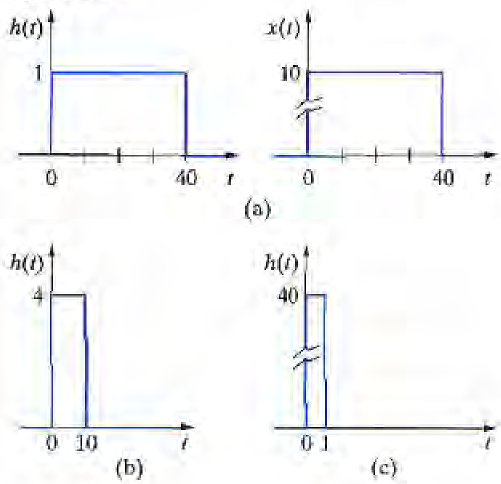
b) Repeat (a) when $h(t)$ changes to the rectangular pulse shown in Fig. P13.60(b).

c) Repeat (a) when $h(t)$ changes to the rectangular pulse shown in Fig. P13.60(c).

d) Sketch $y(t)$ versus t for (a)–(c) on a single graph.

e) Do the sketches in (d) make sense? Explain.

Figure P13.60



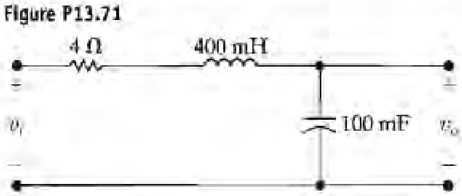
13.71 The input voltage in the circuit seen in Fig. P13.71 is

$$v_i = 10[u(t) - u(t - 0.1)]\ \text{V}.$$

a) Use the convolution integral to find v_o.

b) Sketch v_o for $0 \leq t \leq 1$ s.

Figure P13.71



13.76 The op amp in the circuit seen in Fig. P13.78 is ideal.

PSPICE

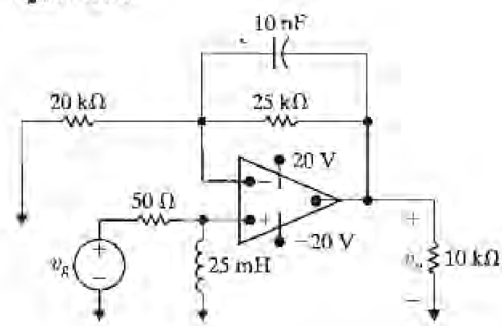
a) Find the transfer function V_o/V_g.

b) Find v_o if $v_g = 10u(t)$ V.

c) Find the steady-state expression for v_o if $v_g = 8 \cos 2000t$ V.

d) Check your answer to (c) with PSpice.

Figure P13.76



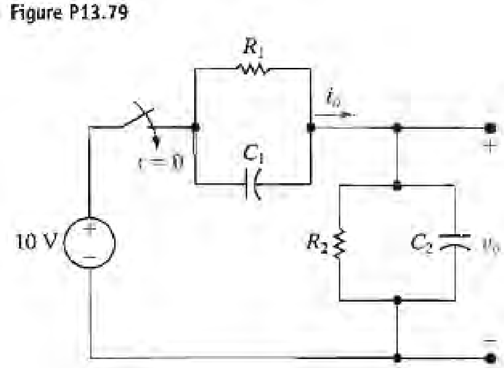
13.79 The parallel combination of R_2 and C_2 in the circuit shown in Fig. P13.79 represents the input circuit to a cathode-ray oscilloscope (CRO). The parallel combination of R_1 and C_1 is a circuit model of a compensating lead that is used to connect the CRO to the source. There is no energy stored in C_1 or C_2 at the time when the 10 V source is connected to the CRO via the compensating lead. The circuit values are $C_1 = 5$ pF, $C_2 = 20$ pF, $R_1 = 1$ MΩ, and $R_2 = 4$ MΩ.

a) Find v_o.

b) Find i_o.

c) Repeat (a) and (b) given C_1 is changed to 80 pF.

Figure P13.79

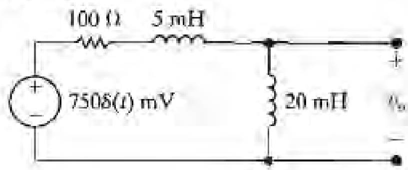


13.85 There is no energy stored in the circuit in Fig. P13.85 at the time the impulsive voltage is applied.

a) Find $v_o(t)$ for $t \geq 0$.

b) Does your solution make sense in terms of known circuit behavior? Explain.

Figure P13.85



14.5 A resistor denoted as R_L is connected in parallel with the capacitor in the circuit in Fig. 14.7. The loaded low-pass filter circuit is shown in Fig. P14.5.

a) Derive the expression for the voltage transfer function V_o/V_i.

b) At what frequency will the magnitude of $H(j\omega)$ be maximum?

c) What is the maximum value of the magnitude of $H(j\omega)$?

d) At what frequency will the magnitude of $H(j\omega)$ equal its maximum value divided by $\sqrt{2}$?

e) Assume a resistance of 300 kΩ is added in parallel with the 4 nF capacitor in the circuit in Fig. P14.4. Find ω_c, $H(j0)$, $H(j\omega_c)$, $H(j0.2\omega_c)$, and $H(j8\omega_c)$.

Figure P14.5

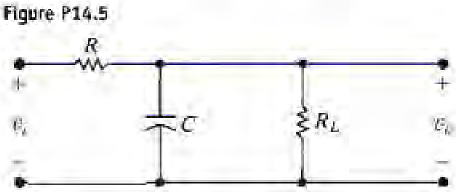
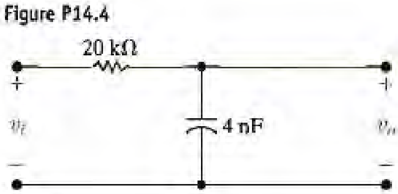


Figure P14.4



14.13 Using a 25 mH inductor, design a high-pass, RL, passive filter with a cutoff frequency of 160 krad/s.

DESIGN PROBLEM

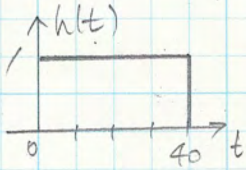
PSPICE

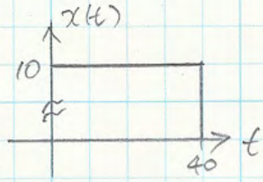
a) Specify the value of the resistance.

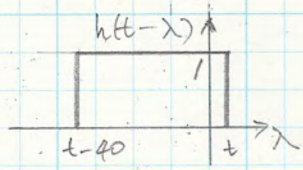
b) Assume the filter is connected to a pure resistive load. The cutoff frequency is not to drop below 150 krad/s. What is the smallest load resistor that can be connected across the output terminals of the filter?

13.60

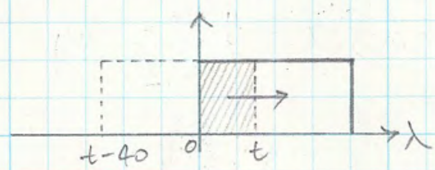
a)





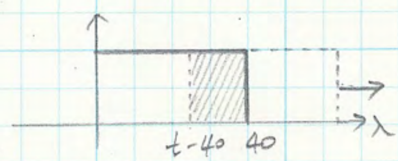


① $0 \le t \le 40$; Area increases.



$$y = h(t-\lambda)\,x(\lambda)$$
$$= \int_0^t h(t-\lambda)\,x(\tau)\,d\lambda$$
$$= \int_0^t 1 \times 10\,d\lambda$$
$$= 10\lambda \Big|_0^t = \underline{10t}$$

② $40 \le t \le 80$; Area decreases.

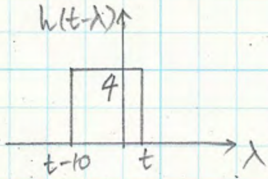


$$y = \int_{t-40}^{40} h(t-\lambda)\,x(\lambda)\,d\lambda$$
$$= 40\lambda \Big|_{t-40}^{40}$$
$$= 40\left[40-(t-40)\right]$$
$$= \underline{40(80-t)}$$

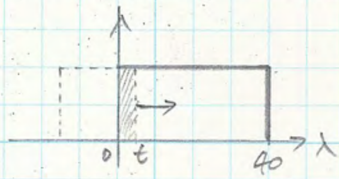
③ $40 \le t$; Area $= 0$

$y = \underline{0}$.

(b)

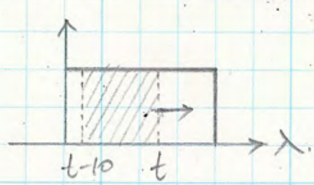


① $0 \le t \le 10$; Area increases



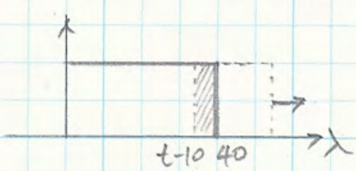
$$y = \int_0^t h(t-\lambda)\, x(\lambda)\, d\lambda$$
$$= 4 \times 10\, \lambda \Big|_0^t$$
$$= \underline{40t}$$

② $10 \le t \le 40$; Area constant.



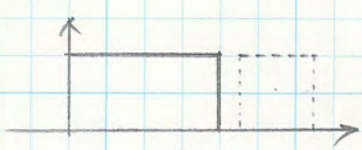
$$y = \int_{t-10}^{t} h(t-\lambda)\, x(\lambda)\, d\lambda$$
$$= 40\lambda \Big|_{t-10}^{t}$$
$$= 40\left[t-(t-10)\right]$$
$$= \underline{400}$$

③ $40 \le t \le 50$; Area decreases



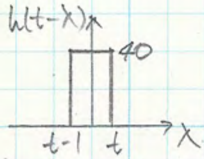
$$y = \int_{t-10}^{40} h(t-\lambda)\, x(\lambda)\, d\lambda$$
$$= 40\lambda \Big|_{t-10}^{40}$$
$$= 40\left[40-(t-10)\right]$$
$$= \underline{40(50-t)}$$

④ $50 \le t$; Area $= 0$.

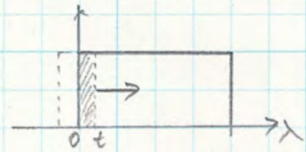


$$y = \underline{0}$$

(c)

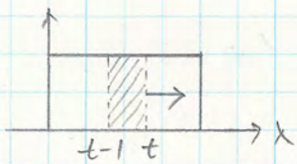


① $0 \le t \le 1$; Area increases



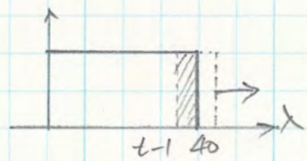
$$y = \int_0^t h(t-\lambda)\,x(\lambda)\,d\lambda$$
$$= \underline{400t}$$

② $1 \le t \le 40$; Area constant



$$y = \int_{t-1}^{t} h(t-\lambda)\,x(\lambda)\,d\lambda$$
$$= 400[t-(t-1)]$$
$$= \underline{400}$$

③ $40 \le t \le 41$; Area decreases

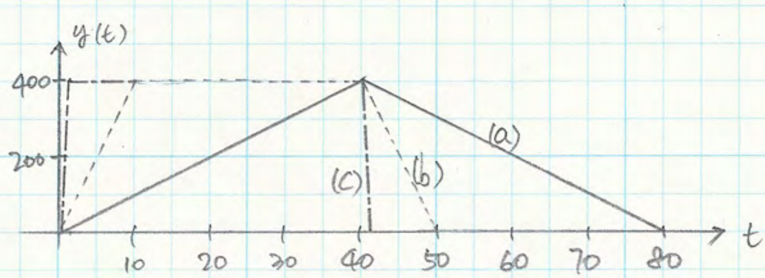


$$y = \int_{t-1}^{40} h(t-\lambda)\,x(\lambda)\,d\lambda$$
$$= 400[40-(t-1)]$$
$$= 400(41-t)$$

④ $41 \le t$; Area $=0$

$$y = \underline{0}$$

(d)



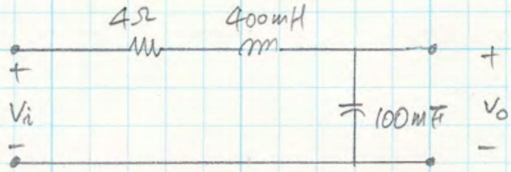
(e) Yes, when $h(t)$ is approaching $40\delta(t)$, $y(t)$ must approach $40x(t)$. $((a) \to (c))$

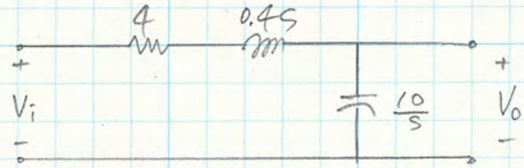
$$y(t) = \int_0^t h(t-\lambda)\,x(\lambda)\,d\lambda \to \int_0^t 40\delta(h-\lambda)\,x(\lambda)\,d\lambda \to 40x(t)$$

This can be seen in plot (c). $y(t) \approx 40x(t)$

13.71

$v_i = 10\left[u(t) - u(t-0.1)\right]$ V





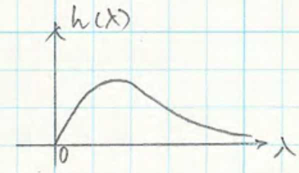
(a) $H(s) = \dfrac{V_o}{V_i} = \dfrac{10/s}{4 + 0.4s + 10/s}$

$$= \frac{10}{0.4s^2 + 4s + 10}$$

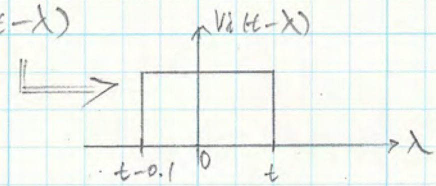
$$= \frac{25}{s^2 + 10s + 25}$$

$$= \frac{25}{(s+5)^2}$$

$\therefore\ h(\lambda) = 25\lambda e^{-5\lambda} u(\lambda) \Longrightarrow$



For convolution, we need $v_i(t-\lambda)$



i) $0 \le t \le 0.1$: Area increasing.

$(fg)' = f'g - fg'$
$f'g = (fg)' + fg'$

$$V_o = \int_0^t (25\lambda e^{-5\lambda}) \cdot 10 \, d\lambda$$

$$= 250 \int_0^t \underbrace{\lambda}_{f} \underbrace{e^{-5\lambda}}_{g'} d\lambda = 250\left[-\frac{1}{5}te^{-5t} - \frac{1}{25}e^{-5t} + \frac{1}{25}\right]$$

$$= -10e^{-5t}(1+5t) + 10.$$

$*$ let $f = \lambda$, $g' = e^{-5\lambda}$.

$\rightarrow f' = 1$, $g = -\frac{1}{5}e^{-5\lambda}$

from $(f \cdot g)' = f'g + fg'$

$fg' = (fg)' - f'g$

$$\int_0^t fg' \, d\lambda = [fg]_0^t - \int_0^t f'g \, d\lambda.$$

$$= \left[\lambda\left(-\frac{1}{5}e^{-5\lambda}\right)\right]_0^t - \int_0^t \left(-\frac{1}{5}\right)e^{-5\lambda} d\lambda$$

$$= \left[-\frac{1}{5}\lambda e^{-5\lambda} - \frac{1}{25}e^{-5\lambda}\right]_0^t$$

ii) $0.1 \le t \le \infty$

$$V_o = \int_{t-0.1}^{t} (25\lambda e^{-5\lambda}) 10\, d\lambda$$

$$= \left[-\frac{1}{5}\lambda e^{-5\lambda} - \frac{1}{25} e^{-5\lambda}\right]_{t-0.1}^{t} \times 250$$

$$= \frac{-1}{25}\left[e^{-5\lambda} + 5\lambda e^{-5\lambda}\right]_{t-0.1}^{t} \times 250$$

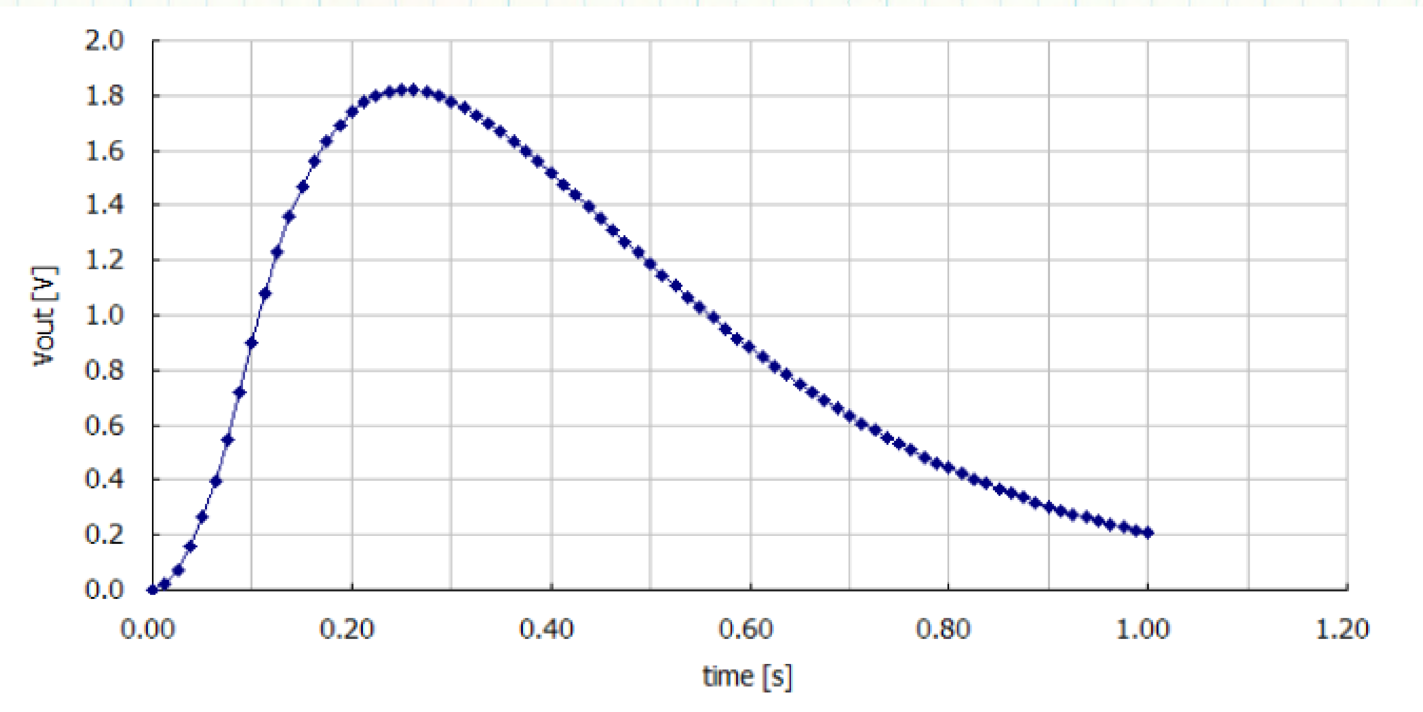
$$= -10\left[e^{-5t} + 5te^{-5t} - e^{-5t+0.5} - 5(t-0.1)e^{-5t+0.5}\right]$$

$$= -10e^{-5t}\left[1+5t + e^{0.5}(-1-5t+0.5)\right]$$

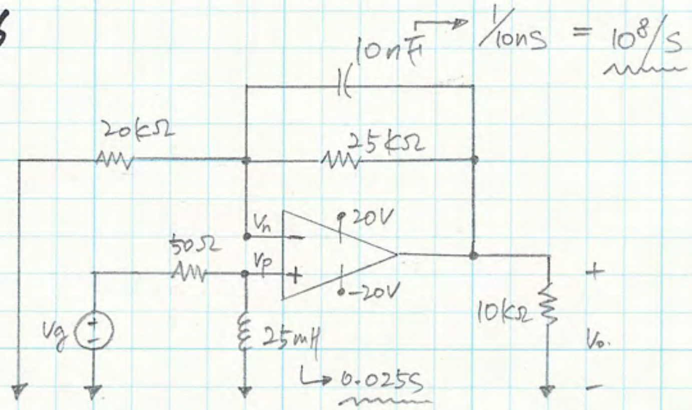
$$= -10e^{-5t}\left[1+5t - e^{0.5}(5t+0.5)\right]$$

(b) sketch. $0 \le t \le 1 \rightarrow$ $0 \le t \le 0.1$; $-10e^{-5t}(1+5t)+10$

$0.1 \le t \le 1.0$; $-10e^{-5t}\left[(1+5t) - e^{0.5}(5t+0.5)\right]$



13.76



(a) $V_p = V_n = V_g \cdot \left(\frac{0.025S}{50+0.025S}\right) = \frac{S}{S+2000} V_g$

KCL at Vn node,

$$\frac{V_n}{20\times10^3} + \frac{V_n - V_o}{25\times10^3} + \frac{V_n - V_o}{10^8/S} = 0$$

$$V_n\left(\frac{1}{20} + \frac{1}{25} + \frac{S}{10^5}\right) = V_o\left(\frac{1}{25} + \frac{S}{10^5}\right)$$

$$V_n(5000 + 4000 + S) = V_o(4000 + S)$$

$$\frac{S}{(S+2000)}(9000+S)V_g = V_o(S+4000) \rightarrow \frac{V_o}{V_g} = \frac{S(S+9000)}{(S+2000)(S+4000)} = H(S)$$

(b) $V_g = 10\,u(t) \rightarrow V_g = 10/S$

$$\therefore V_o = \frac{\cancel{S}(S+9000)}{(S+2000)(S+4000)} \cdot \frac{10}{\cancel{S}} = \frac{10(S+9000)}{(S+2000)(S+4000)}$$

$$= \frac{k_1}{S+2000} + \frac{k_2}{S+4000}$$

$$k_1 = \left.\frac{10(S+9000)}{(S+4000)}\right|_{S=-2000} = \frac{10(7000)}{2000} = 35$$

$$k_2 = \left.\frac{10(S+9000)}{(S+2000)}\right|_{S=-4000} = \frac{10(5000)}{-2000} = -25$$

$$\therefore V_o(t) = [35e^{-2000t} - 25e^{-4000t}]u(t)$$

(c) $V_g = 8\cos 2000t$

$$S = j\omega = j2000$$

$$H(j\omega) = \frac{j2000\,(9000 + j2000)}{(2000 + j2000)\,(4000 + j2000)} = \frac{j2\,(9 + j2)}{(2 + j2)\,(4 + j2)}$$

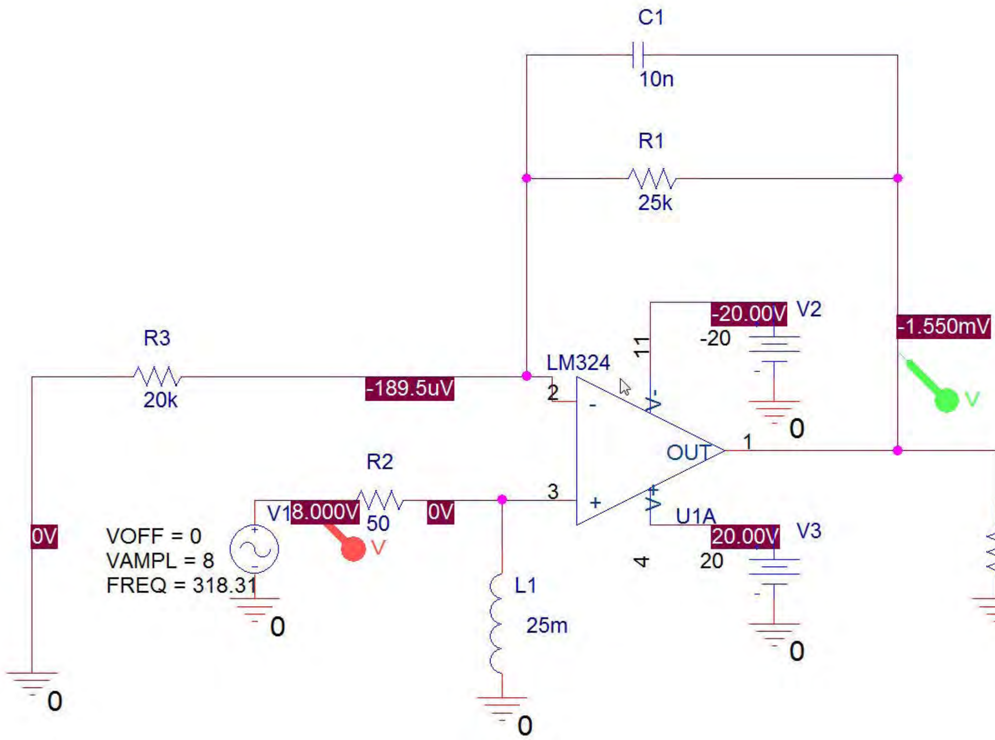
$$= \frac{(2\angle 90^\circ)\,\sqrt{9^2+2^2}\,\angle 12.521^\circ}{(\sqrt{2^2+2^2}\,\angle 45^\circ)(\sqrt{4^2+2^2}\,\angle 26.565^\circ)}$$

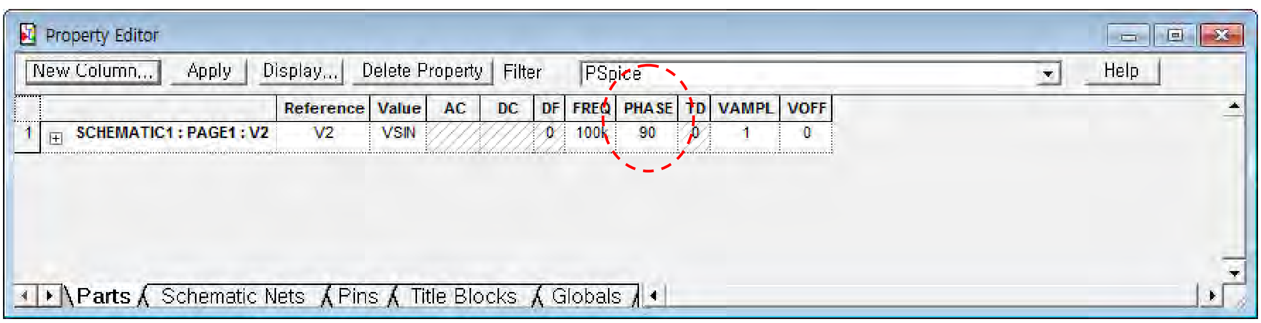
$$= 1.458\,\angle(90^\circ + 12.521^\circ - 45^\circ - 26.565^\circ)$$

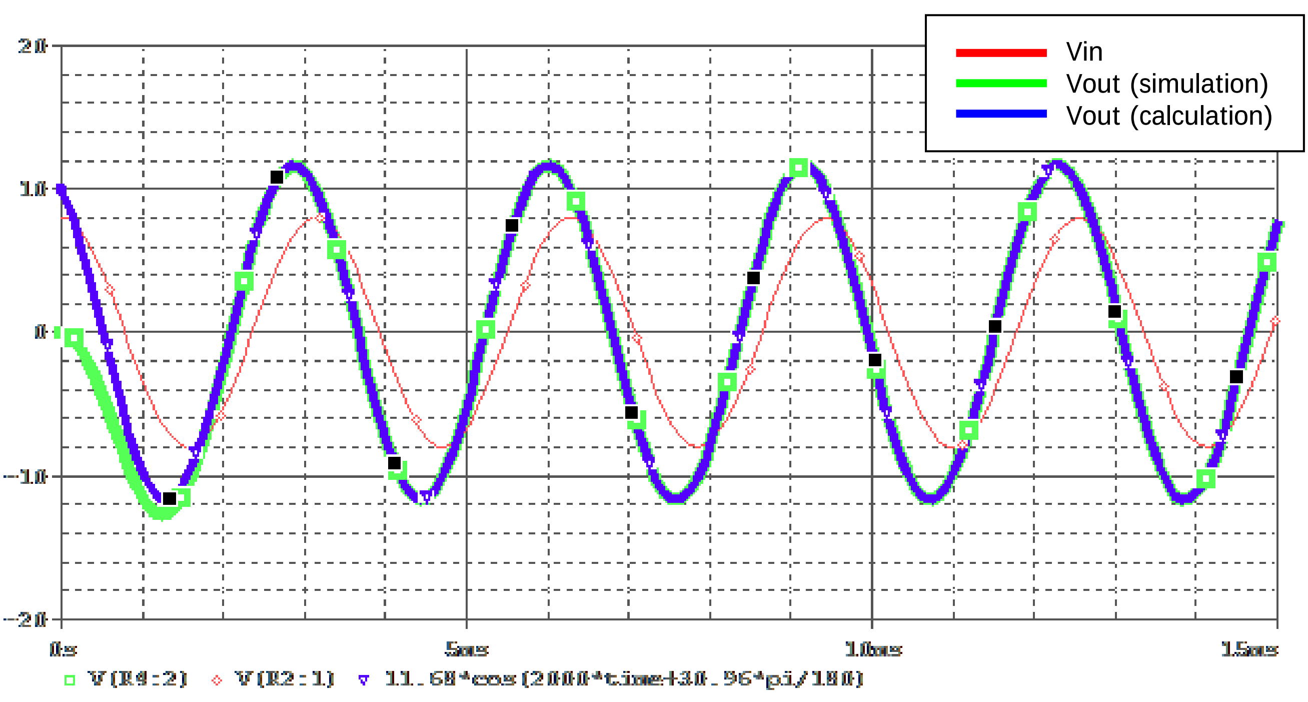
$$= 1.458\,\angle 30.956^\circ$$

$$\therefore V_{o\text{-steady state}} = 8 \times 1.456\cos(2000t + 30.956^\circ)$$

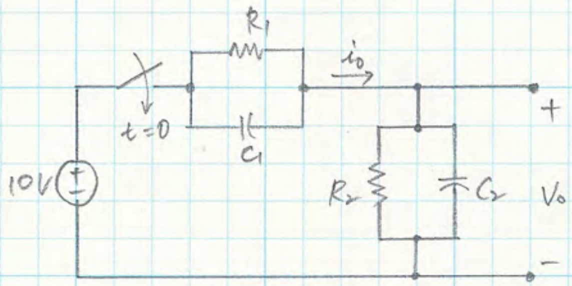
$$= \underline{11.68\cos(2000t + 30.956^\circ)}$$


C1
10n
R1
25k
R3
20k
LM324
-189.5uV
-20.00V
V2
-20
-1.550mV
OUT
U1A
R2
50
V1
8.000V
0V
VOFF = 0
VAMPL = 8
FREQ = 318.31
L1
25m
20.00V
V3
20
R4
20k
0


Property Editor
New Column...
Apply
Display...
Delete Property
Filter
PSpice
Help
Reference
Value
AC
DC
DF
FREQ
PHASE
TD
VAMPL
VOFF
1
SCHEMATIC1 : PAGE1 : V2
V2
VSIN
0
100k
90
0
1
0
Parts
Schematic Nets
Pins
Title Blocks
Globals


Vin
Vout (simulation)
Vout (calculation)
2.0
1.0
0
-1.0
-2.0
0s
5ms
10ms
15ms
V(R4:2)
V(R2:1)
11.68*cos(2000*time+30.96*pi/180)
Time

13.79



$R_1 = 1\,M\Omega$
$R_2 = 4\,M\Omega$
$C_1 = 5\,pF$
$C_2 = 20\,pF$

no energy stored in C_1, C_2 at $t=0$

(a)

let $Z_1 = R_1 \,//\, C_1$
$Z_2 = R_2 \,//\, C_2$

$$Z_1 = \frac{R_1/sC_1}{R_1 + 1/sC_1} = \frac{1/C_1}{s + 1/R_1C_1} = \frac{20\times10^{10}}{s+20\times10^4}\ \Omega$$

$$Z_2 = \frac{1/C_2}{s + 1/R_2C_2} = \frac{5\times10^{10}}{s+12500}\ \Omega$$

$$\therefore V_0 = \frac{10}{s}\,\frac{Z_2}{Z_1+Z_2} = \frac{10\cdot\frac{5\times10^{10}}{s+12500}}{\left(\frac{20\times10^{10}}{s+20\times10^4} + \frac{5\times10^{10}}{s+12500}\right)s}$$

$$= \frac{50(s+20\times10^4)}{[20(s+12500) + 5(s+20\times10^4)]s}$$

$$= \frac{50(s+20\times10^4)}{s(25s+1250000)} = \frac{2(s+20\times10^4)}{s(s+5\times10^4)}$$

$$= \frac{k_1}{s} + \frac{k_2}{s+5\times10^4}$$

$$k_1 = \left.\frac{2(s+20\times10^4)}{s+5\times10^4}\right|_{s=0} = \frac{40\times10^4}{5\times10^4} = 8$$

$$k_2 = \left.\frac{2(s+20\times10^4)}{s}\right|_{s=-5\times10^4} = \frac{2(15\times10^4)}{-5\times10^4} = -6$$

$$\therefore V_0 = [8 - 6e^{-50000t}]\,u(t)\ V$$

(b)

$$I_0 = \frac{V_0}{Z_2} = \frac{2(s+20\times10^4)}{s(s+5\times10^4)} \times \frac{s+12500}{5\times10^{10}} = \frac{2}{5\times10^{10}}\left[\frac{s^2+212500s+25\times10^8}{s^2+50000s}\right]$$

$$= 4\times10^{-11}\left[1 + \frac{162500s+25\times10^8}{s^2+50000s}\right]$$

$$= 4\times10^{-11}\left[1 + \frac{162500s+25\times10^8}{s(s+50000)}\right]$$

$$= 40\times10^{-12}\left(1 + \frac{k_1}{s} + \frac{k_2}{s+50000}\right)$$

$$k_1 = \left.\frac{162500s + 25\times10^8}{s+50000}\right|_{s=0} = \frac{25\times10^8}{5000} = \underline{50000}$$

$$k_2 = \left.\frac{162500s + 25\times10^8}{s}\right|_{s=-50000} = \frac{5625\times10^6}{50000} = \underline{112500}$$

$$\therefore i_o = \underline{\left[40\,\delta(t) + 2\times10^6 + 4.5\times10^6 e^{-50000t}\right]u(t) \ pA}$$

(c) Repeat (a), (b) if $C_1 = 80pF$.

$$Z_1 = \frac{R_1/sC_1}{R_1 + 1/sC_1} = \frac{1/C_1}{s + 1/R_1C_1} = \frac{125\times10^8}{s+12500}\ \Omega$$

$$V_0 = \frac{10}{s}\cdot\frac{Z_2}{Z_1+Z_2} = \frac{10\cdot\frac{5\times10^{10}}{s+12500}}{\left(\frac{125\times10^8}{s+12500} + \frac{5\times10^{10}}{s+12500}\right)s}$$

$$= \frac{50\times10^{10}}{s\,(125\times10^8 + 5\times10^6)}$$

$$= \frac{8}{s}$$

$$\therefore v_0 = \underline{8\,u(t)\ V}$$

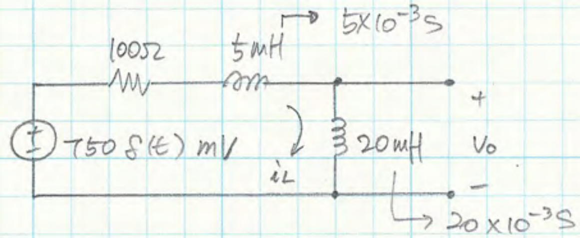
$$I_0 = \frac{V_0}{Z_2} = \frac{8}{s}\cdot\frac{s+12500}{5\times10^{10}} = 160\times10^{-12}\left(\frac{s+12500}{s}\right)$$

$$= 160\times10^{-12}\left(1 + \frac{12500}{s}\right)$$

$$\therefore i_o = 160\,\delta(t) + 160\times12500\,u(t)\ pA$$

$$= \underline{160\,\delta(t) + 2\times10^6\,u(t)\ pA}$$

13.85



(a) $$V_o = \frac{20\times10^{-3}s}{100 + 5\times10^{-3}s + 20\times10^{-3}s} \times 750\times10^{-3}$$

$$= \frac{15000\times10^{-6}s}{100 + 25\times10^{-3}s} = \frac{0.6s}{s+4000} = 0.6 + \frac{-2400}{s+4000}$$

$$\therefore V_o = 0.6\,\delta(t) - 2400e^{-4000t}u(t)\ V.$$

$v = L\frac{di}{dt}$.

(b) At $t=0$, the impulsive voltage establishes a current in the inductors.

$$i_L(0) = \frac{1}{(5+20)\times10^{-3}}\int_{0^-}^{0^+} 0.75\,\delta(t)\,dt \qquad (\leftarrow \text{from } V_L = L\frac{di}{dt})$$

$$= 400\times0.75 = 30A.$$

$$\rightarrow \frac{di_L(0)}{dt} = 30\,\delta(t)$$

$$\therefore V_o(0) = (20\times10^{-3})(30\,\delta(t)) = 0.6\,\delta(t)\ V$$

This agrees with the solution to (b).//

At $t=0^+$ $$\frac{V}{R+SL} = V\cdot\frac{1}{s+R/L} \Rightarrow V\cdot e^{-R/L\,t}$$

For RL circuit,

$$i_L(t) = i_L(0)\cdot e^{-(R/L)t} \qquad \left(\leftarrow \frac{V}{R+SL} = \frac{V/L}{s+R/L} \rightarrow \frac{V}{L}e^{-(R/L)t}\right)$$

$$R = 100\Omega,\ L = (20+5)\times10^{-3}\ H.$$

$$\therefore R/L = 4000$$

$$\rightarrow i_L(t) = 30\cdot e^{-4000t}\ A. \quad (t\geq0^+)$$

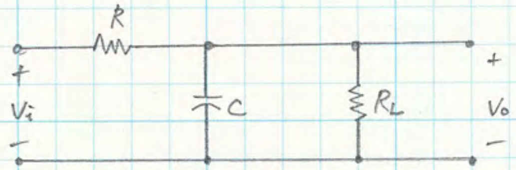
$$\frac{di_L(t)}{dt} = -120000\,e^{-4000t}$$

$$\therefore V_o(t) = (20\times10^{-3})(-120000\,e^{-4000t})$$

$$= -2400\,e^{-4000t}\ V.$$

This agrees with the solution to (b).//

14.5



(a) $V_o = \dfrac{1/sC \parallel R_L}{R + 1/sC \parallel R_L} V_i$ $\qquad 1/sC \parallel R_L = \dfrac{\frac{R_L}{sC}}{\frac{1}{sC} + R_L} = \dfrac{R_L}{1 + sR_LC}$

$$\frac{V_o}{V_i} = \frac{\frac{R_L}{1+sR_LC}}{R + \frac{R_L}{1+sR_LC}} = \frac{R_L}{R_L + R + sRR_LC}$$

$$= \frac{1/RC}{s + 1/RC + 1/R_LC}$$

(b) $$|H(j\omega)| = \frac{1/RC}{\sqrt{\omega^2 + (1/RC + 1/R_LC)^2}}$$

(c) at $\omega = 0 \rightarrow |H(j\omega)|$ is maximum.

$$\therefore |H(j\omega)|_{max} = \frac{1/RC}{1/RC + 1/R_LC} = \frac{R_L}{R_L + R}$$

(d). at $\omega = \omega_c \rightarrow |H(j\omega_c)| = |H(j\omega)|_{max} / \sqrt{2}$

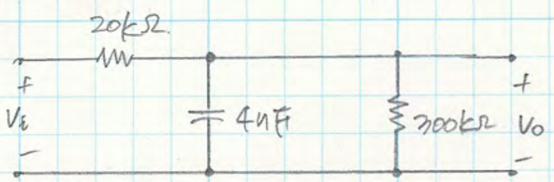
$$\frac{1/RC}{\sqrt{\omega_c^2 + (1/RC + 1/R_LC)^2}} = \frac{1/RC}{1/RC + 1/R_LC} \cdot \frac{1}{\sqrt{2}}$$

$$\therefore \omega_c^2 + (1/RC + 1/R_LC)^2 = (1/RC + 1/R_LC)^2 \cdot 2$$

$$\omega_c^2 = \left(\frac{1}{RC} + \frac{1}{R_LC}\right)^2 = \left(\frac{R + R_L}{RR_LC}\right)^2 = \left(\frac{1}{RC}\left(1 + \frac{R}{R_L}\right)\right)^2$$

$$\therefore \omega_c = \frac{1}{RC}\left(1 + \frac{R}{R_L}\right)$$

(e)



$$\omega_c = \frac{1}{RC}\left(1 + \frac{R}{R_L}\right)$$

$$= 12500\,(1 + 20/300) = 13.333 \text{ krad/s}$$

$$H(j0) = \frac{300}{20 + 300} = 0.9375$$

Ce

$$H(j\omega_c) = \left.\frac{1/RC}{s + \dfrac{R+R_L}{RR_LC}}\right|_{s=j\omega_c = j13333.33}$$

$$= \frac{1.25\times10^4}{j13333.333 + 13333.333}$$

$$= \frac{1.25\times10^4}{1.8856\angle 45^\circ} = \underline{0.6629\angle -45^\circ}$$

$$H(j0.2\omega_c) = \frac{1.25\times10^4}{j0.2\times 13333.333 + 13333.333}$$

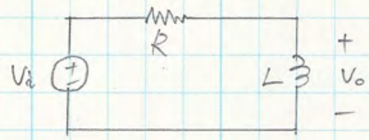
$$= \frac{1.25\times10^4}{13.579\times10^4\angle 11.31^\circ} = \underline{0.9193\angle -11.31^\circ}$$

$$H(j0.8\omega_c) = \frac{1.25\times10^4}{j8\times 13333.333 + 13333.333}$$

$$= \frac{1.25\times10^4}{10.750\times10^4\angle 82.875^\circ} = \underline{0.1163\angle -82.875^\circ}$$

14.13 Design a high-pass, RL passive filter.

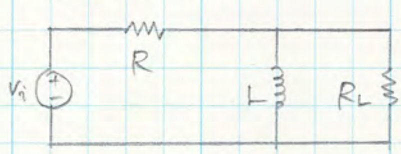
cutoff frequency = 160 krad/s. $L = 25\ mH$



$$H(s) = \frac{sL}{R+sL} = \frac{s}{s+R/L}$$

(a) $\omega_c = R/L \quad \therefore R = \omega_c L = 160\times10^3 \times 25\times10^{-3} = 4000 = \underline{4\ k\Omega}$

(b) with R_L



$$H(s) = \frac{sL/\!/R}{R + sL/\!/R_L} = \frac{\dfrac{sLR_L}{sL+R_L}}{R + \dfrac{sLR_L}{sL+R_L}}$$

$$= \frac{sLR_L}{sLR + RR_L + sLR_L} = \frac{sLR_L}{s(LR+LR_L)+RR_L}$$

$$\therefore \frac{RR_L}{LR+LR_L} = 150\times10^3$$

$$RR_L = 150\times10^3\,(LR + LR_L)$$

$$R_L(R - 150\times10^3 L) = 150\times10^3 LR$$

$$\therefore R_L = \frac{150\times10^3\times100}{2500}$$

$$= \underline{60\ k\Omega}$$