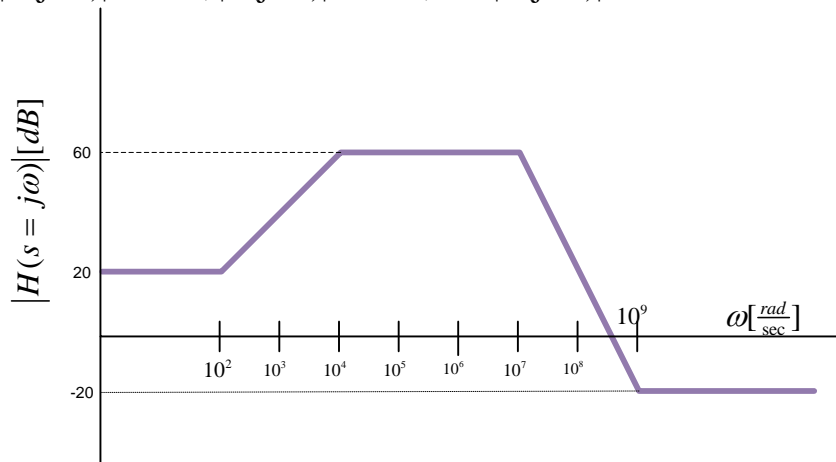


Homework #8
EE233 Spring 2010
Assigned Friday 5/28/08, Due **Friday** 6/4/08 in class

1. Given the following asymptotic magnitude plot,

(a) Determine the transfer function $H(s)$ if the gains at the corner points are $|H(j100)|=23$ dB, $|H(j10^4)|=57$ dB, $|H(j10^7)|=72$ dB, and $|H(j10^9)|=-20$ dB.



(b) Use a math tool such as MatLab (Look under help for “Bode”) to plot the exact magnitude and phase response of the transfer function you determined in (a).

2. Nilsson & Riedel 15.12

3. Nilsson & Riedel 15.44

4. Nilsson & Riedel 15.48

5. Your research group has developed a small, actuated mirror for directing a low-energy laser beam to be used for projecting images directly onto the retina. The mirror is held by a short flexible armature, and is controlled by two small actuators that scan the laser across the eye at 15kHz. Unfortunately there is a resonant mode in the arm at around 2kHz that is excited by noise in the actuator control signal. The result is that even a small amount of noise at this frequency causes the mirror to vibrate severely, and reduces the image quality.

Your job is to implement a narrowband band-reject filter to eliminate noise in the motor control signal. The center frequency of your filter should be 2kHz, and the bandwidth should be 100 Hz.

Use reasonable values for each component. For ease of grading, use $1\mu\text{F}$ capacitors when possible. Assume that you will use ideal op-amps.

6. Your company has developed a new type of loudspeaker system that uses very low power to reproduce sound. Like a home stereo system it uses 3 independent speakers, the woofer, midrange and tweeter, to reproduce the low, mid and high frequencies, respectively. The woofer is designed to reproduce frequencies up to 200Hz. The midrange delivers frequencies between 200 Hz and 1400 Hz. The tweeter is designed for frequencies above 1400 HZ.

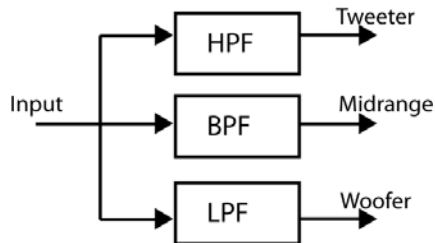


Figure 1: Block diagram of filter network showing three filters

You have been given the job of designing a filter system to separate the signal and deliver only the desired frequencies to each speaker. Others on your team will take care of the rest (buffering of your input signal, level-adjustment to boost the filter output to the appropriate level, line conditioning to prevent RF interference, etc). Also assume, for this exercise, that phase-distortion is not a problem.

Each filter should be designed separately.

For the low frequency speaker use an active low-pass filter with two cascaded first order filters to get sharper roll-off (a steeper cutoff in the frequency plot). The cutoff frequency should be 100Hz and should have unity gain in the passband. Make sure to take into account the shift in cutoff frequency that occurs when you cascade the two sections.

For the midrange circuit implement a band-pass filter by cascading low-pass, high-pass, and inverting op-amp circuits. The resulting filter should pass frequencies from 100Hz to 1400Hz, and have a maximum gain of 10 (20dB).

The tweeters your company developed require more precise frequency selection. Implement a 4th order high-pass Butterworth filter with a cutoff frequency of 1400Hz.

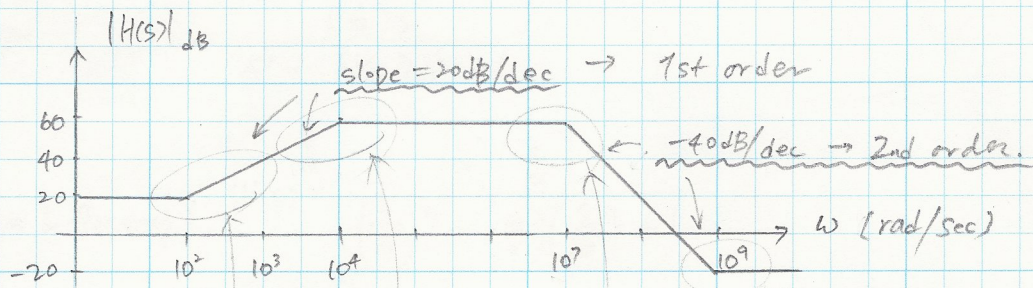
For each of the three filters you should provide:

- A schematic of the op-amp circuit
- Specifications for all components used
- Transfer function, $H(s)$, for your circuit.
- Magnitude only Bode plot of the showing the frequency response of your circuit

Use reasonable values for each component. For ease of grading, use 1 μ F capacitors when possible. Assume that you will use ideal op--amps.

Bonus: Simulate your circuits in PSpice and show the frequency response.

1.



$$H(s) = (s + 10^2) \times \frac{1}{(s + 10^4)} \times \frac{1}{(s^2 + as + 10^{14})} \times \frac{s^2 + bs + 10^{18}}{1} \times A$$

ω_1^2
 ω_2^2
gain

at $\omega = 0$. $|H(j\omega)| = 10 = \frac{10^2}{10^4} \cdot \frac{10^{18}}{10^{14}} \cdot A = 10^2 A$

$$A = 0.1$$

(check at $\omega = \infty$. $|H(j\omega)| = A = 0.1 = -20 \text{ dB}$)

at $\omega = \omega_1 = 10^7$ and $\omega = \omega_2 = 10^9$. $\rightarrow \left| \frac{s + 10^2}{s + 10^4} \right| \approx 1$

i) at $\omega = \omega_1 = 10^7$

$$20 \log |H(j\omega)| = 20 \log \left| 0.1 \times \frac{\omega_2^2 - \omega_1^2 + j\omega_1 b}{j\omega_1 a} \right| = 10^{7.2}$$

$$0.1 \times \frac{\sqrt{(\omega_1^2 - \omega_2^2)^2 + b^2 \omega_1^2}}{a \omega_1} = 10^{7.2/20} = 10^{3.6}$$

$$\sqrt{(\omega_1^2 - \omega_2^2)^2 + b^2 \omega_1^2} = 10^{4.6} a \omega_1$$

$$(\omega_1^2 - \omega_2^2)^2 + b^2 \omega_1^2 = 10^{9.2} a^2 \omega_1^2 \quad \text{--- (1)}$$

ii) at $\omega = \omega_2 = 10^9$

$$20 \log |H(j\omega)| = 20 \log \left| 0.1 \times \frac{j b \omega_2}{\omega_1^2 - \omega_2^2 + j a \omega_2} \right| = -20$$

$$0.1 \times \frac{b \omega_2}{\sqrt{(\omega_1^2 - \omega_2^2)^2 + a^2 \omega_2^2}} = 0.1$$

$$(\omega_1^2 - \omega_2^2)^2 + a^2 \omega_2^2 = b^2 \omega_2^2 \quad \text{--- (2)}$$

$$\textcircled{1} \times \omega_2^2 \rightarrow (\omega_1^2 - \omega_2^2)^2 \omega_2^2 - 10^{9.2} a^2 \omega_1^2 \omega_2^2 + b^2 \omega_1^2 \omega_2^2 = 0.$$

$$\textcircled{2} \times \omega_1^2 \rightarrow (\omega_1^2 - \omega_2^2)^2 \omega_1^2 + a^2 \omega_1^2 \omega_2^2 - b^2 \omega_1^2 \omega_2^2 = 0.$$

$$(\omega_1^2 - \omega_2^2)^2 (\omega_1^2 + \omega_2^2) + a^2 \omega_1^2 \omega_2^2 (1 - 10^{9.2}) = 0.$$

$$a^2 = \frac{(\omega_1^2 - \omega_2^2)^2 (\omega_1^2 + \omega_2^2)}{\omega_1^2 \omega_2^2 (10^{9.2} - 1)} = 6.309 \times 10^{12}$$

$$\underline{a = 2.512 \times 10^6}$$

from $\textcircled{2}$,

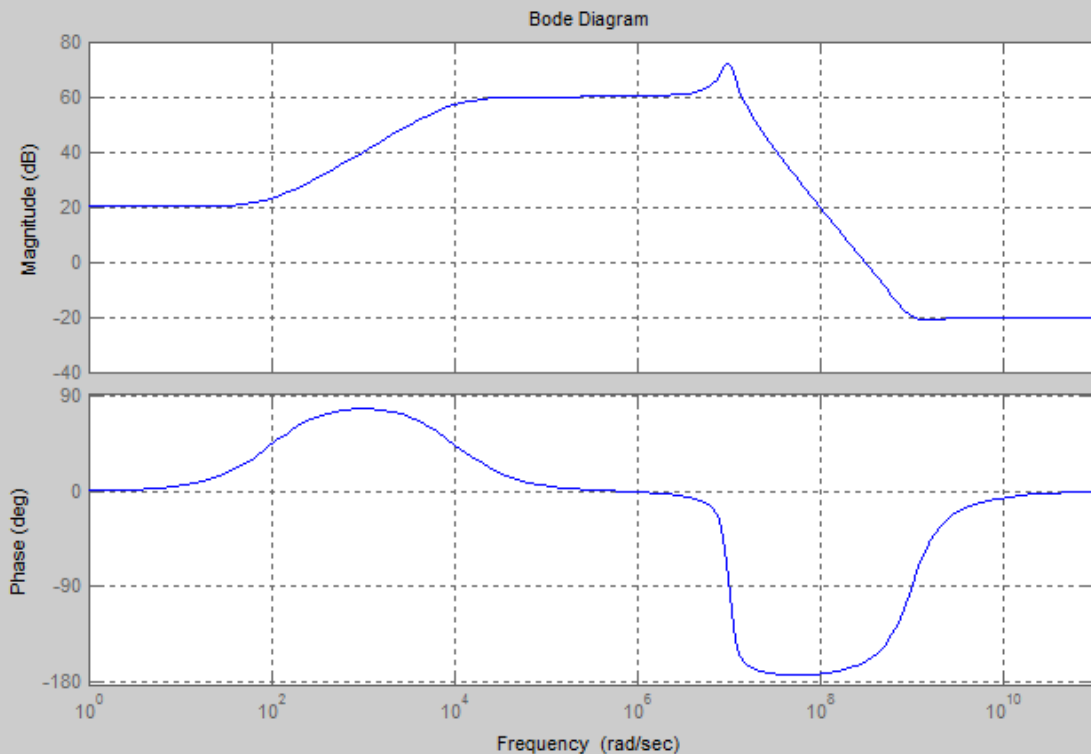
$$(\omega_1^2 - \omega_2^2)^2 + a^2 \omega_2^2 = b^2 \omega_2^2$$

$$b^2 = \frac{(\omega_1^2 - \omega_2^2)^2 + a^2 \omega_2^2}{\omega_2^2} = 9.998 \times 10^{17}$$

$$\underline{b = 9.998 \times 10^8}$$

$$H(s) = 0.1 \times \frac{(s+10^2)(s^2 + 9.998 \times 10^8 s + 10^{18})}{(s+10^9)(s^2 + 2.512 \times 10^6 s + 10^{14})}$$

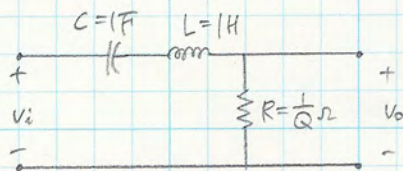
$$= 0.1 \times \frac{s^3 + s^2(10^2 + 9.998 \times 10^8) + s(10^{18} + 9.998 \times 10^{10}) + 10^{20}}{s^3 + s^2(10^9 + 2.512 \times 10^6) + s(10^{14} + 2.512 \times 10^{10}) + 10^{18}}$$



2. (15.12)

Proto type. $H(s) = \frac{(1/Q)s}{s^2 + (1/Q)s + 1}$

Scaled
(Magnitude & Frequency) $H'(s) = \frac{(\frac{1}{Q})(\frac{s}{k_f})}{(\frac{s}{k_f})^2 + (\frac{1}{Q})(\frac{s}{k_f}) + 1}$



Sol. $H(s) = \frac{R}{1/sC + Ls + R} = \frac{RCs}{LCs^2 + RCs + 1}$
 $= \frac{(R/L)s}{s^2 + (R/L)s + (1/LC)} = \frac{\beta s}{s^2 + \beta s + \omega_0^2}$

For the prototype circuit,

$$\omega_0 = 1$$

$$\beta = \omega_0/Q = 1/Q$$

For the scaled circuit,

$$H'(s) = \frac{(R'/L')s}{s^2 + (R'/L')s + (1/L'C')}$$

Where $R' = k_m R$, $L' = (k_m/k_f)L$, $C' = C/k_f k_m$

$$\therefore R'/L' = (k_m R) \left(\frac{k_f}{k_m} \right) \frac{1}{L} = k_f \cdot R/L = \underline{k_f \beta}$$

$$1/L'C' = \left(\frac{k_m}{k_f} L \cdot \frac{C}{k_f k_m} \right)^{-1} = \underline{k_f^2 \omega_0^2}$$

$$\therefore H'(s) = \frac{k_f \beta s}{s^2 + k_f \beta s + k_f^2 \omega_0^2}$$

$$= \frac{\frac{\beta}{k_f} s}{\left(\frac{s}{k_f} \right)^2 + \frac{\beta}{k_f} s + \omega_0^2}$$

$$\beta = 1/Q$$

$$= \frac{(\frac{1}{Q})(\frac{s}{k_f})}{\left(\frac{s}{k_f} \right)^2 + (\frac{1}{Q})(\frac{s}{k_f}) + \omega_0^2}$$

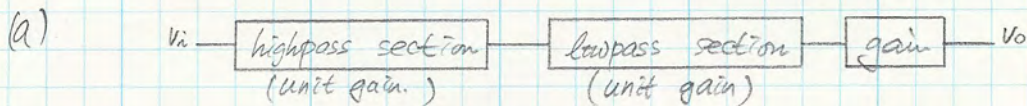
3. (15.44) Design a broadband Butterworth bandpass filter.

$$\omega_{c1} = 1000 \text{ Hz}, \quad \omega_{c2} = 8000 \text{ Hz}$$

$$\text{passband gain} = 10 \text{ dB.}$$

The gain should be down at least 20 dB at 400 Hz. and 20 kHz.

Use 50 nF in the high-pass circuit
5 k Ω in the low-pass circuit.

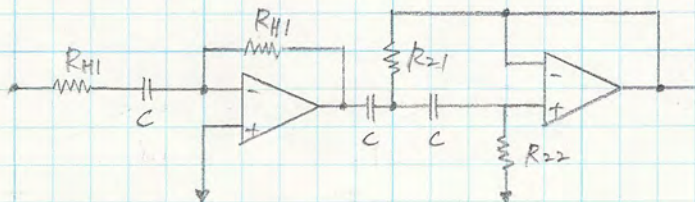


① For the highpass section.

$$n = \frac{-0.05 A_s}{\log_{10}(\omega_s/\omega_p)} = \frac{(-0.05)(-20)}{\log_{10}(1000/400)} = 2.51$$

$$\rightarrow n = 3$$

$$H_{hp}(s) = \frac{s^3}{(s+1)(s^2+s+1)} = \left(\frac{s}{s+1}\right) \left(\frac{s^2}{s^2+s+1}\right) \quad \text{; prototype}$$



For the prototype 1st order section.

$$H_{hp,1}(s) = -\frac{s}{s + \frac{1}{R_{H1}C}} \rightarrow \left. \begin{array}{l} R_{H1} = 1 \Omega \\ C = 1 F. \end{array} \right\} \text{for prototype}$$

$$\therefore \text{scaling factor. } k_f = 2\pi f_{c1} = 2000\pi$$

$$k_m = \frac{1}{C\omega_{c1}} = \frac{1}{50 \times 10^{-9} \times 2000\pi} = 3.183 \times 10^3$$

$$\left(\begin{array}{l} \therefore R_{H1} = k_m \cdot 1 = 3.183 \text{ k}\Omega. \\ C = 50 \text{ nF}. \end{array} \right) \text{for the scaled section.}$$

For the prototype 2nd order section.

$$H_{hp,2}(s) = \frac{s^2}{s^2 + \frac{2}{R_{H2}C}s + \frac{1}{R_{H2}R_{H3}C^2}} \rightarrow \left. \begin{array}{l} R_{H2} = 2 \Omega \\ R_{H3} = 0.5 \Omega \\ C = 1 F. \end{array} \right\} \text{for prototype.}$$

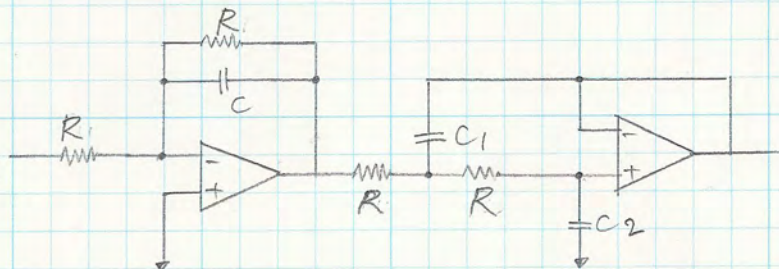
$$\left(\begin{array}{l} \therefore R_{H2} = k_m \cdot 0.5 = 1.592 \text{ k}\Omega. \\ R_{H3} = k_m \cdot 2 = 6.366 \text{ k}\Omega. \\ C = 50 \text{ nF}. \end{array} \right) \text{for the scaled section.}$$

② For the lowpass section.

$$n = \frac{-0.05 A_s}{\log_{10}(W_s/W_p)} = \frac{(-0.05)(-20)}{\log_{10}(20000/8000)} = \underline{\underline{2.51}}$$

$$\rightarrow \underline{\underline{n=3}}$$

$$H_{lp}(s) = \frac{1}{(s+1)(s^2+s+1)} = \left(\frac{1}{s+1}\right) \left(\frac{1}{s^2+s+1}\right) \quad ; \text{prototype.}$$



For the prototype 1st order section

$$H_{lp1}(s) = -\frac{1/RC}{s + 1/RC} \rightarrow \left. \begin{array}{l} R = 1\Omega \\ C = 1F \end{array} \right\} \text{for prototype.}$$

We need to use $5k\Omega$ in the low-pass section.

$$\therefore k_m = 5 \times 10^3.$$

$$k_f = 2\pi(8000) = 50.265 \times 10^3$$

$$\left(\begin{array}{l} \therefore R = 5k\Omega \\ C = \frac{1}{k_m W_{c2}} = \frac{1}{(5000)(2\pi \cdot 8000)} = \underline{\underline{3.98 \mu F.}} \end{array} \right) \text{for the scaled section}$$

For the prototype 2nd order section.

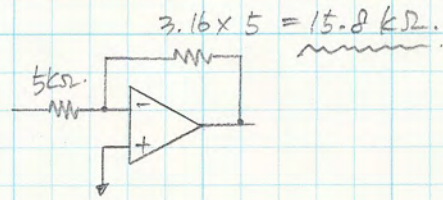
$$H_{lp2}(s) = \frac{1/C_2}{s^2 + \frac{2}{RC_1}s + \frac{1}{R^2 C_1 C_2}} \rightarrow \left. \begin{array}{l} R = 1\Omega \\ C_1 = 2F \\ C_2 = 0.5F \end{array} \right\} \text{for prototype}$$

$$\left(\begin{array}{l} \therefore R = 5k\Omega \\ C_1 = \frac{2}{k_m W_{c2}} = \underline{\underline{7.96 \mu F.}} \\ C_2 = \frac{0.5}{k_m W_{c2}} = \underline{\underline{1.99 \mu F.}} \end{array} \right) \text{for the scaled section.}$$

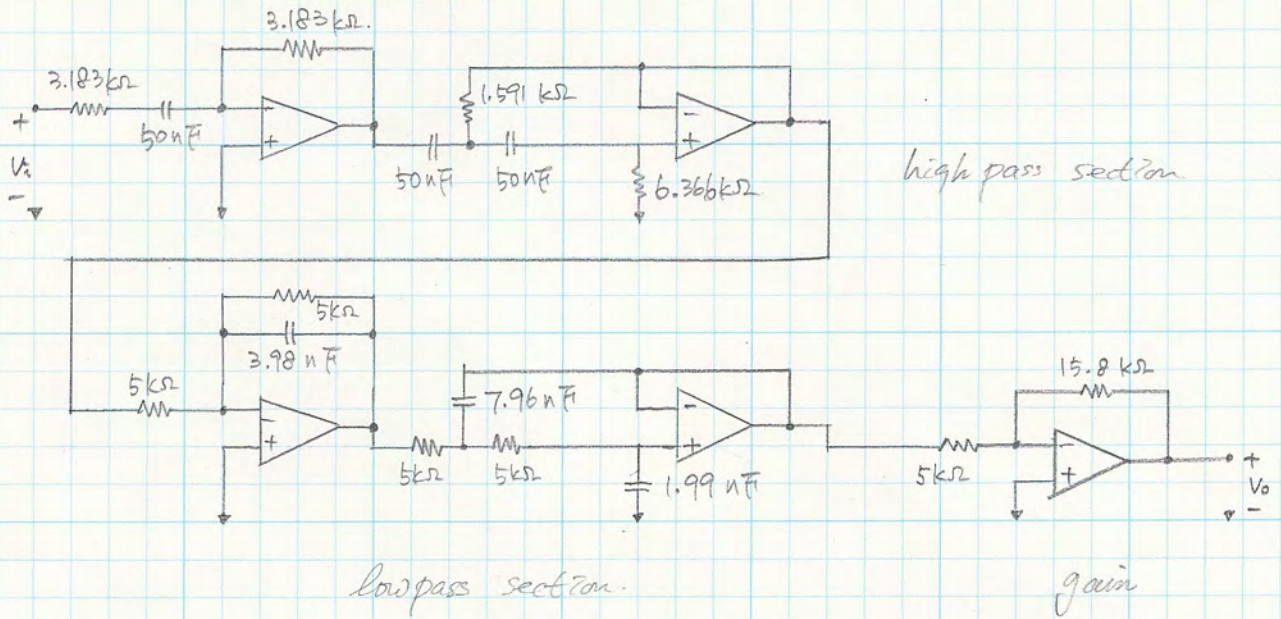
② gain amplifier.

$$20 \log_{10} K = 10 \text{ dB.}$$

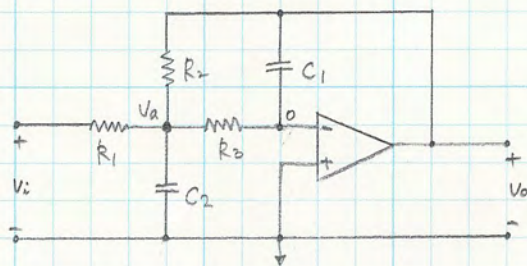
$$\therefore K = 3.16$$



(b)



4. (15.48)



(a) At low frequencies.

the two capacitors are open.
no current in R_3 because the op-amp is ideal.

The circuit behaves as an inverting amplifier with a gain of $-R_2/R_1$.

At very high frequencies.

the capacitors are short.

The output becomes zero.

(b) KCL at V_a : $(V_a - V_i)G_1 + V_a sC_2 + (V_a - V_o)G_2 + V_a G_3 = 0$ ----- ①

KCL at 0: $-V_a G_3 - V_o sC_1 = 0$ ----- ②

$$\hookrightarrow V_a = -\frac{sC_1}{G_3} V_o$$

from ①.

$$V_a (G_1 + G_2 + G_3 + sC_2) - V_o G_2 = G_1 V_i$$

$$-\frac{sC_1}{G_3} (G_1 + G_2 + G_3 + sC_2) - V_o G_2 = G_1 V_i$$

$$\left[(G_1 + G_2 + G_3 + sC_2) sC_1 + G_2 G_3 \right] V_o = -G_1 G_3 V_i$$

$$H(s) = \frac{V_o}{V_i} = \frac{-G_1 G_3}{s^2 C_1 C_2 + (G_1 + G_2 + G_3) C_1 s + G_2 G_3}$$

$$= \frac{-G_1 G_3 / C_1 C_2}{s^2 + \left(\frac{G_1 + G_2 + G_3}{C_2} \right) s + \frac{G_2 G_3}{C_1 C_2}}$$

$$= \frac{-\frac{G_2 G_3}{C_1 C_2} \frac{G_1}{G_2}}{s^2 + \left(\frac{G_1 + G_2 + G_3}{C_2} \right) s + \frac{G_2 G_3}{C_1 C_2}}$$

$$= \frac{-K b_0}{s^2 + b_1 s + b_0}$$

$$K = G_1 / G_2$$

$$b_0 = \frac{G_2 G_3}{C_1 C_2}$$

$$b_1 = \frac{G_1 + G_2 + G_3}{C_2}$$

(c) if $C_2 = 1 \text{ F}$.

$$\rightarrow b_1 = \frac{G_1 + G_2 + G_3}{C_2} = G_1 + G_2 + G_3$$

$$\begin{aligned}\therefore b_1 &= K G_2 + G_2 + G_3 & (\leftarrow G_1 = K G_2) \\ &= K G_2 + G_2 + \frac{b_0 C_1}{G_2} & (\leftarrow G_3 = \frac{b_0}{G_2} C_1) \\ &= G_2 (1+K) + \frac{b_0 C_1}{G_2}\end{aligned}$$

$$G_2^2 (1+K) + b_1 G_2 + b_0 C_1 = 0.$$

$$\begin{aligned}\therefore G_2 &= \frac{b_1}{2(1+K)} \pm \sqrt{\frac{b_1^2 - 4 b_0 C_1 (1+K)}{4(1+K)^2}} \\ &= \frac{b_1 \pm \sqrt{b_1^2 - 4 b_0 (1+K) C_1}}{2(1+K)}\end{aligned}$$

For G_2 to be realizable

$$b_1^2 - 4 b_0 (1+K) C_1 \geq 0$$

$$\therefore C_1 \leq \frac{b_1^2}{4 b_0 (1+K)}$$

(d) 1. Select $C_2 = 1 \text{ F}$.

2. Select C_1 using $C_1 \leq \frac{b_1^2}{4 b_0 (1+K)}$

3. Calculate G_2

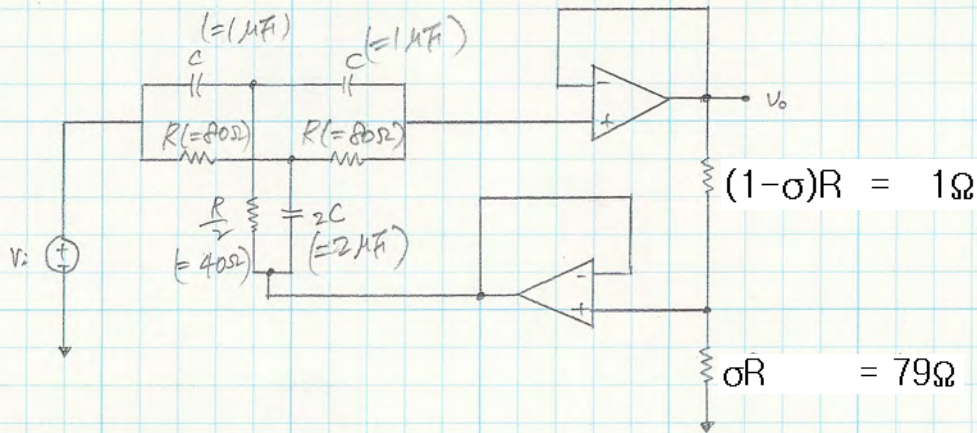
4. Calculate G_1

5. Calculate G_3

5. Band Reject Filter. $\omega_0 = 2 \text{ kHz}$ $\text{BW} = 100 \text{ Hz}$.

Due to narrow $\text{BW} (= 100 \text{ Hz})$, a high- Q bandreject filter can be good solution.

(You can build a bandreject filter using 1st order low pass filter and high pass filter. However, it has poor frequency response.) $Q = 2000/100 = 20$.



From eq. 15.60 ~ 15.64,

$$H(s) = \frac{V_o}{V_i} = \frac{(s^2 + \frac{1}{R^2 C^2})}{s^2 + \frac{4(1-\sigma)}{RC} + \frac{1}{R^2 C^2}}$$

$$\omega_0^2 = \frac{1}{R^2 C^2} \quad \text{--- ①}$$

$$B = \frac{4(1-\sigma)}{RC} \quad \text{--- ②}$$

$$\omega_0 = 2\pi(2000) = \frac{1}{RC} \rightarrow RC = \frac{1}{40000\pi}$$

$$B = \frac{4(1-\sigma)}{RC} = 4(1-\sigma) \cdot 40000\pi = 200\pi \rightarrow \sigma = 0.9875$$

if $C = 1 \mu\text{F}$.	$\rightarrow R = 79.577 \Omega$	$\rightarrow R = 80 \Omega$
	$(1-\sigma)R = 0.995 \Omega$	$(1-\sigma)R = 1 \Omega$
	$\sigma R = 78.583 \Omega$	$\sigma R = 79 \Omega$

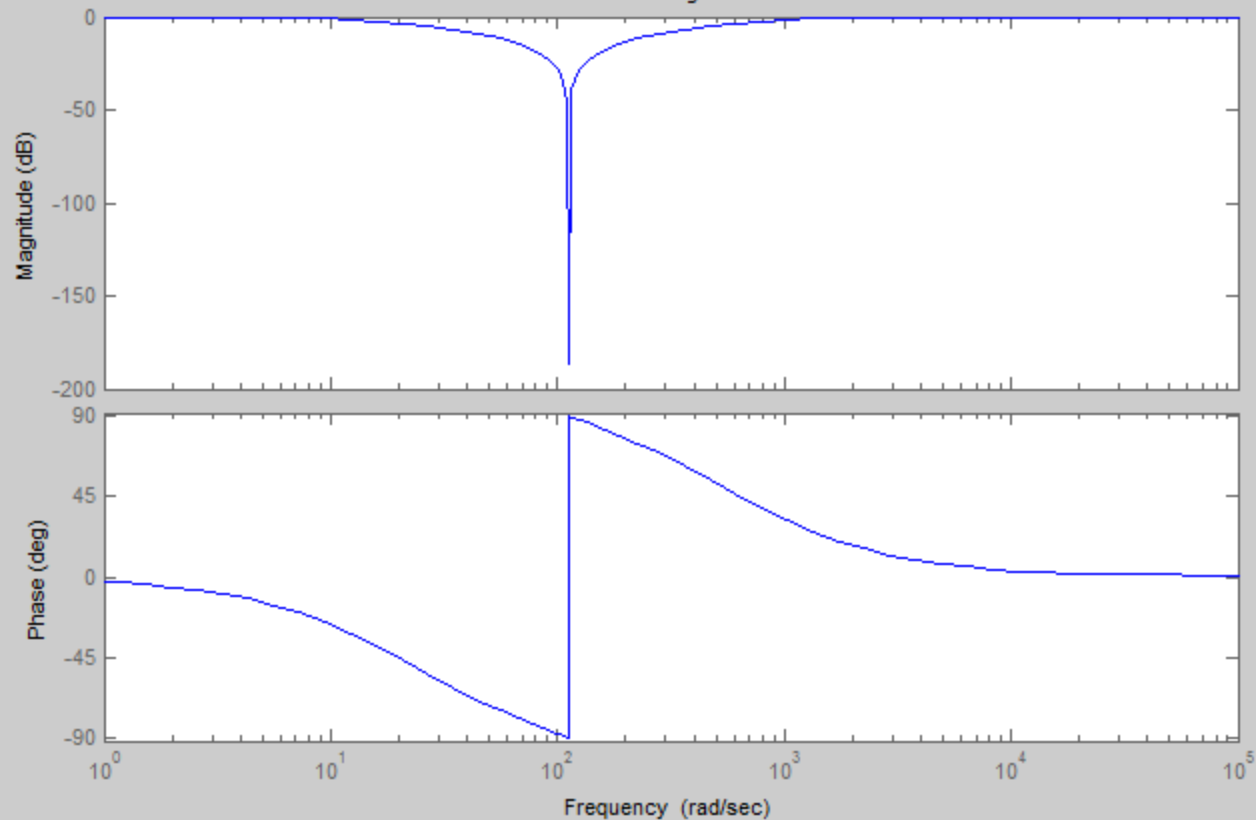
* from Simulation

$$\omega_{c1} = 1.95 \text{ kHz}$$

$$\omega_{c2} = 2.06 \text{ kHz}$$

$$\text{BW} = 110 \text{ Hz.}$$

Bode Diagram



6. (a) HPF. $f_1 = 1400 \text{ Hz}$. ; 4th order high pass Butterworth,

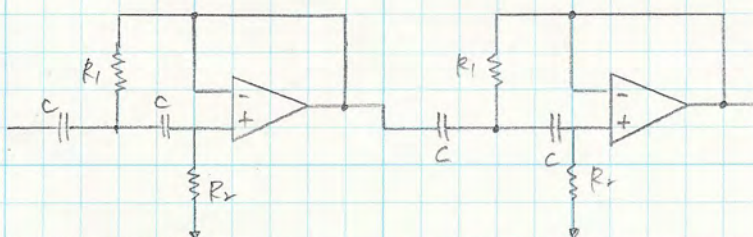
(b) BPF $f_{c1} = 100 \text{ Hz}$. $f_{c2} = 1400 \text{ Hz}$. ; cascading low-pass, high-pass and inverting circuits

(c) LPF. $f_c = 100 \text{ Hz}$. ; two cascaded 1st order filters

a) HPF. (4th order Butterworth).

From the table 15.1,

$$H(s) = \frac{s^4}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$



For each stage

$$H(s) = \frac{s^2}{s^2 + \frac{2}{R_2 C} s + \frac{1}{R_1 R_2 C^2}}$$

$$1 = \frac{1}{R_2 R_1}$$

i) For the prototype of the 1st stage. ($C = 1 \text{ F}$)

$$R_2 = 2/0.765 = 2.614 \Omega$$

$$R_1 = 1/R_2 = 0.3825 \Omega$$

Scaling factors. (We will use $C = 1 \mu\text{F}$)

$$k_f = 2\pi f_c = 2000\pi$$

$$k_m = \frac{1}{k_f \cdot C} = 113.682$$

$$R_2 = k_m \times 2.614 = 297.16 \Omega$$

$$R_1 = k_m \times 0.3825 = 43.48 \Omega$$

$$C = 1 \mu\text{F}$$

$$\rightarrow H_1(s) = \frac{s^2}{s^2 + 6.73 \times 10^3 s + 7.74 \times 10^7}$$

ii) For the prototype of the 2nd stage ($C = 1 \text{ F}$)

$$R_2 = 2/1.848 = 1.0823 \Omega$$

$$R_1 = 1/R_2 = 0.924 \Omega$$

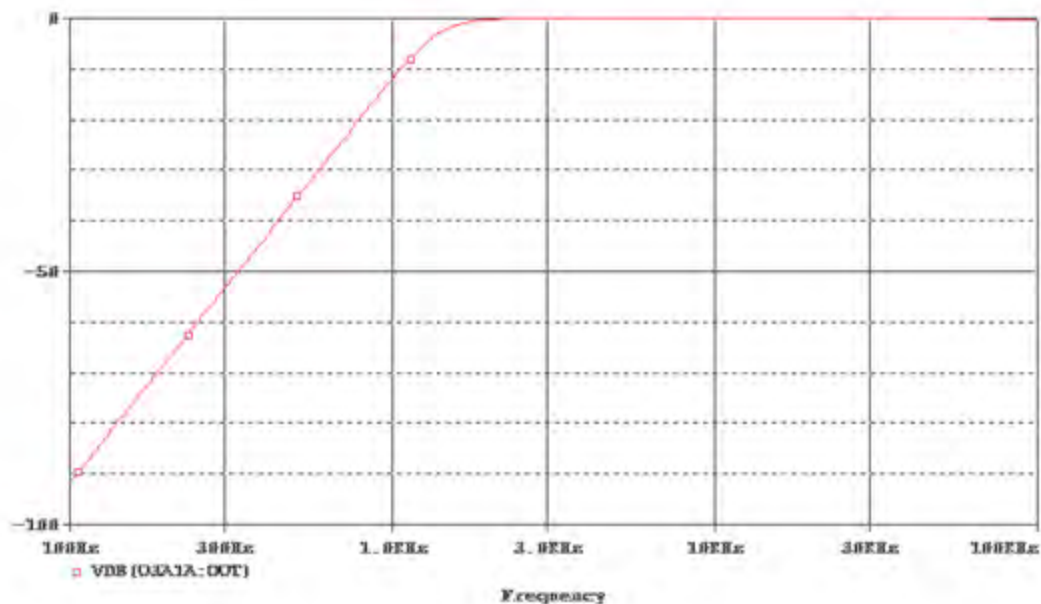
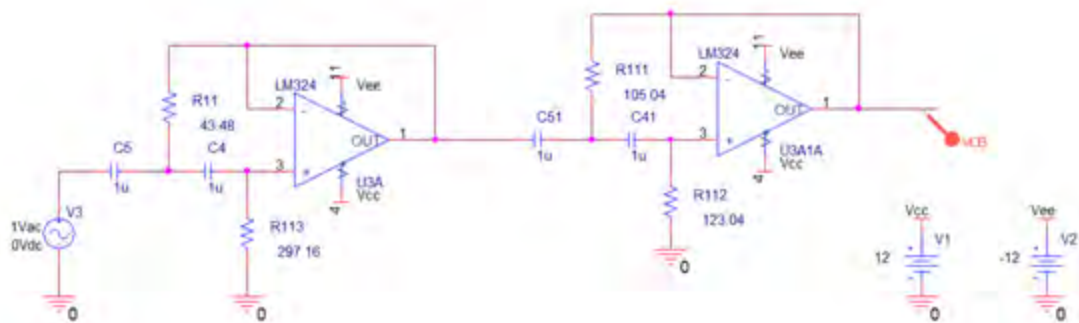
$$R_2 = k_m \cdot 1.0823 = 123.04 \Omega$$

$$R_1 = k_m \cdot 0.924 = 105.04 \Omega$$

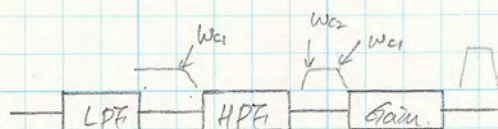
$$C = 1 \mu\text{F}$$

$$\rightarrow H_2(s) = \frac{s^2}{s^2 + 1.63 \times 10^4 s + 7.74 \times 10^7}$$

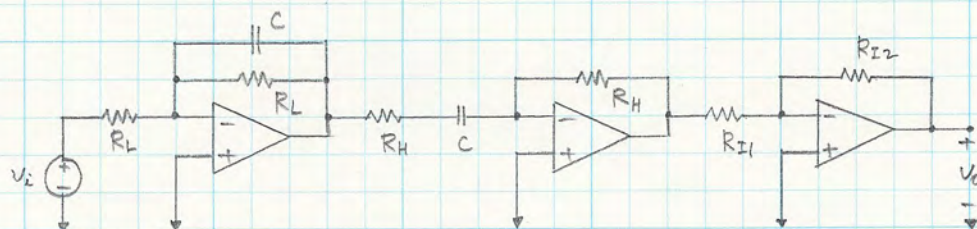
$$\rightarrow H(s) = H_1(s) \cdot H_2(s) = \frac{s^4}{(s^2 + 6.73 \times 10^3 s + 7.74 \times 10^7)(s^2 + 1.63 \times 10^4 s + 7.74 \times 10^7)}$$



(b) BPF



$$\left(\begin{array}{l} \omega_{c1} = 2800\pi \text{ rad/s} \\ \omega_{c2} = 200\pi \text{ rad/s} \end{array} \right)$$



i) For the 1st stage (LPF). (We will use $C = 1\mu\text{F}$)

$$\omega_{c1} = 2\pi \cdot 1400 = \frac{1}{R_L C} \rightarrow R_L = 113.6\Omega \quad C = 1\mu\text{F}$$

ii) For the 2nd stage (HPF)

$$\omega_{c2} = 2\pi \cdot 100 = \frac{1}{R_H C} \rightarrow R_H = 1.592\text{ k}\Omega \quad C = 1\mu\text{F}$$

iii) For the 3rd stage (Inverting amp.) i maximum gain = 10 (= 20dB)

From

$$H(s) = \frac{V_o}{V_i} = \left(\frac{1/R_L C}{s + 1/R_L C} \right) \cdot \left(\frac{s}{s + 1/R_H C} \right) \cdot \frac{R_{12}}{R_{11}} = \frac{\omega_{c1}}{s + \omega_{c1}} \cdot \frac{s}{s + \omega_{c2}} \cdot \frac{R_{12}}{R_{11}}$$

$$= \frac{\omega_{c1} s}{s^2 + (\omega_{c1} + \omega_{c2})s + \omega_{c1}\omega_{c2}} \cdot \frac{R_{12}}{R_{11}}$$

For maximum gain (at $\omega_0 = \sqrt{\omega_{c1}\omega_{c2}} = 2350.95 \text{ rad/s}$)

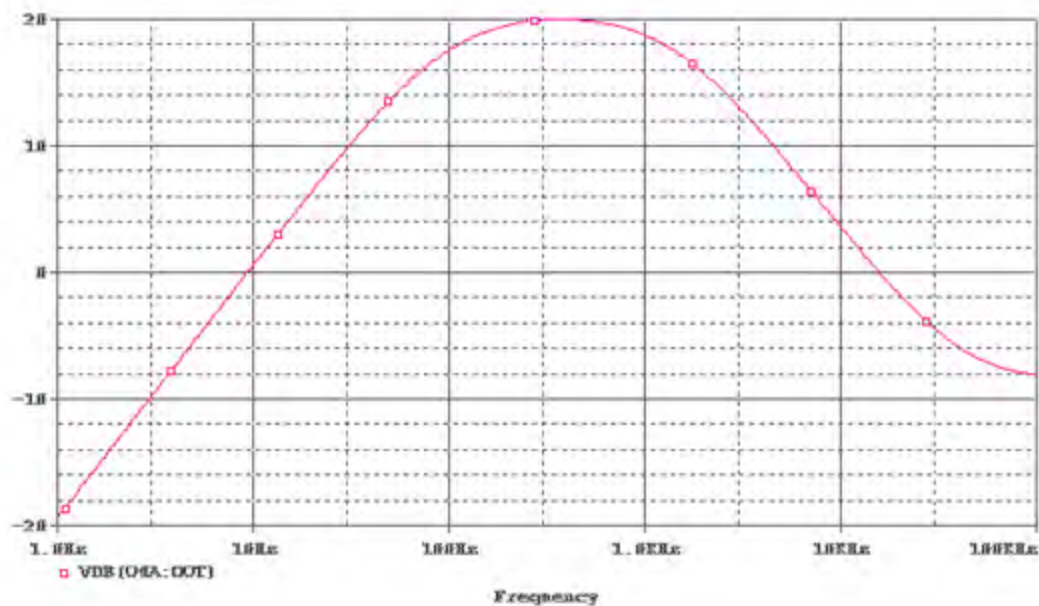
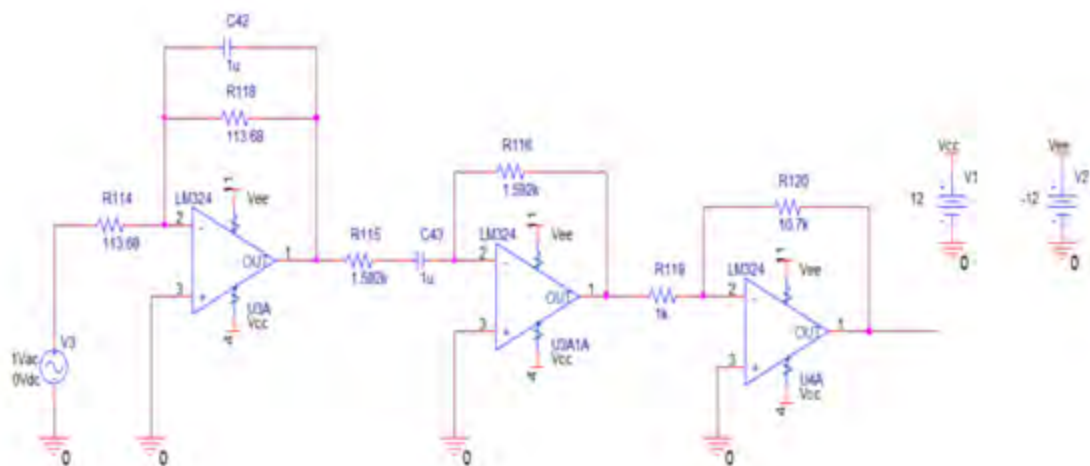
$$|H(s)|_{\max} = |H(j\omega_0)| = \frac{\omega_{c1}\omega_0}{\sqrt{(\omega_{c1}\omega_{c2} - \omega_0^2)^2 + (\omega_{c1} + \omega_{c2})^2\omega_0^2}} \cdot \frac{R_{12}}{R_{11}}$$

$$= \frac{(2800\pi)(2350.95)}{\sqrt{(200\pi \cdot 2800\pi - 2350.95^2)^2 + (200\pi + 2800\pi)^2 2350.95^2}} \cdot \frac{R_{12}}{R_{11}}$$

$$= 0.9333 \frac{R_{12}}{R_{11}} = 10$$

$$\rightarrow R_{12} = 10.7 R_{11} \quad \therefore \text{if } R_{11} = 1\text{ k}\Omega, R_{12} = 10.7\text{ k}\Omega$$

$$H(s) = \frac{8976.46 s \times 10.7}{s^2 + 9424.78 s + 5.527 \times 10^6}$$



(c) LPA : An active LPA with two cascaded 1st order filters.
 $\omega_c = 100 \text{ Hz}$

From eq. 15.21 and 15.22,

For the prototype circuit,

$$H(s) = \frac{1}{(s+1)^2}$$

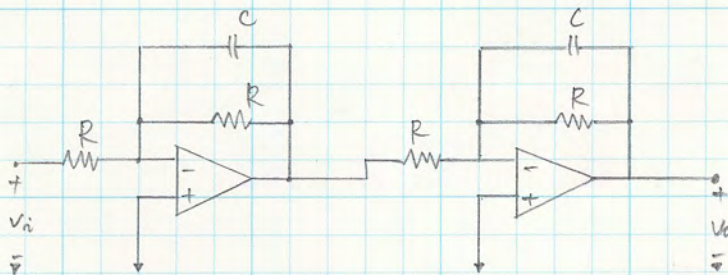
$$\omega_c = \sqrt{2\sqrt{2}-1} = \underline{0.6436 \text{ rad/s.}}$$

scaling factor (We will use $C = 1 \mu\text{F}$)

$$k_f = 2\pi \cdot 100 / 0.6436 = 976.265$$

$$k_m = \frac{1}{k_f C} = 1024.312$$

$$\therefore \begin{aligned} R &= 1024.3 \Omega \\ C &= 1 \mu\text{F} \end{aligned}$$



$$\underline{H(s) = \frac{1}{(s + 1/k_f)^2} = \frac{1}{(s + 976.28)^2}}$$

