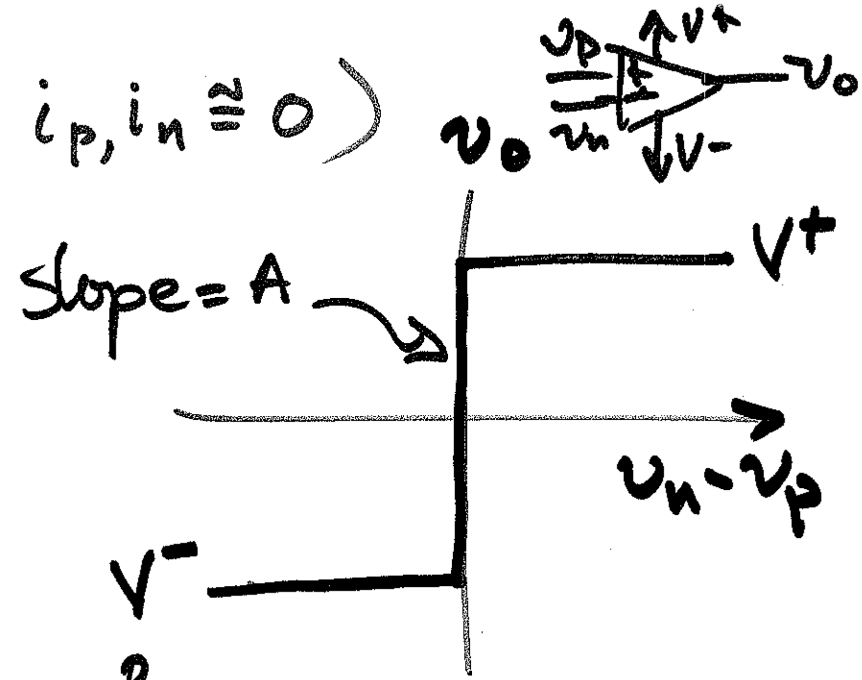
Ideal Op-amp

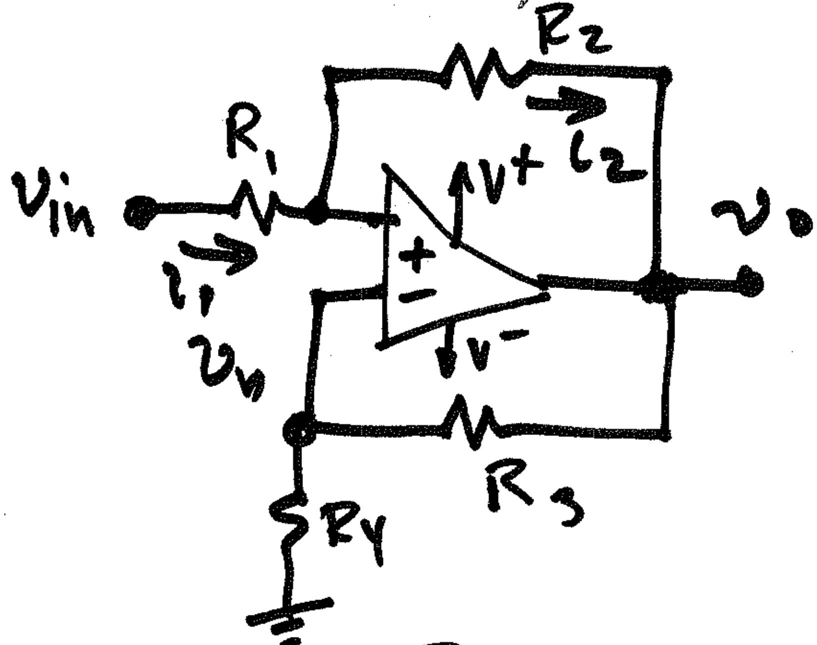
$(A \to \infty,\ i_p, i_n \cong 0)$



If $V^- < v_o < V^+$,

then $\underline{v_p \cong v_n}$

Assume true, then check.



$$v_n = \frac{R_4}{R_3+R_4} v_o = v_p$$

$$i_1 = i_2 = \frac{v_p - v_o}{R_2}$$

$$= \frac{R_4 v_o/(R_3+R_4) - v_o}{R_2}$$

$$i_1 = i_2 = \frac{-R_3}{R_2(R_3+R_4)} v_o$$

$$v_{in} = v_p + i_1 R_1$$

$$= \frac{R_4 v_o}{R_3+R_4} - \frac{R_1 R_3 v_o}{R_2(R_3+R_4)}$$

$$v_{in} = \frac{R_2 R_4 - R_1 R_3}{R_2 (R_3 + R_4)} v_{o}$$

$$v_{out} = \left[\frac{R_2 (R_3 + R_4)}{R_2 R_4 - R_1 R_3} \right] v_{in}$$

$$R_2 = 2\Omega \quad R_3 = 3\Omega$$

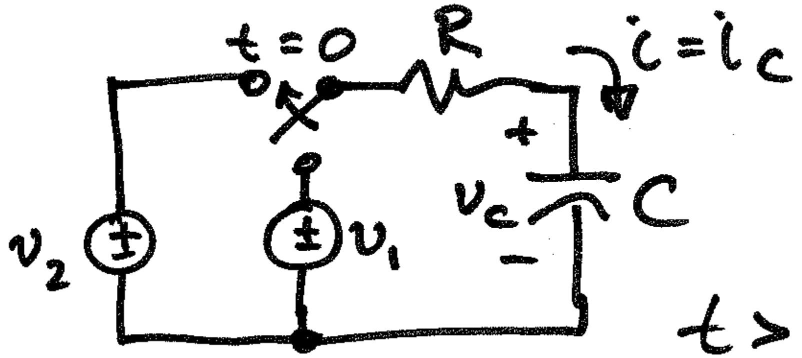
$$R_4 = 4 \quad , R_1 = 1$$

$$v_{out} = \left[\frac{2(3+4)}{8 \pm 3} \right] v_{in} = \frac{14}{5} v_{in}$$

$V^+ = 7V$ $\qquad$ $v_{in} = 5V$

max(v_{in}) $2.5V$ $\qquad$ $v_{out} = \cancel{14V}$ $7V$

w/o sat



$i_c = C\frac{dv_c}{dt}$

$t < 0$

$v_c(0^-) = v_1$

$t > 0$

$$i = \frac{v_2 - v_c}{R} = C\frac{dv_c}{dt}$$

$$\underbrace{C\frac{dv_c}{dt} + \frac{v_c}{R}}_{\text{depend on } v_c} = \frac{v_2}{R}$$

General Sol'n to linear DE

1) Gen'l Sol'n to homogeneous eqn. $\leftarrow 0$

\+ 2) Specific solution to full eqn.

$$C\frac{dv_c}{dt} + \frac{v_c}{R} = 0$$

Characteristic eqn. $Cs + \frac{1}{R} = 0 \Rightarrow s = -\frac{1}{RC}$

$$v_c^{GH} = Ae^{-t/RC}$$

Specific sol'n to full: $v_c^{ss} =$ ~~cst~~ constant $= B$

$$C(0) + \frac{B}{R} = \frac{v_2}{R} \Rightarrow B = v_2$$

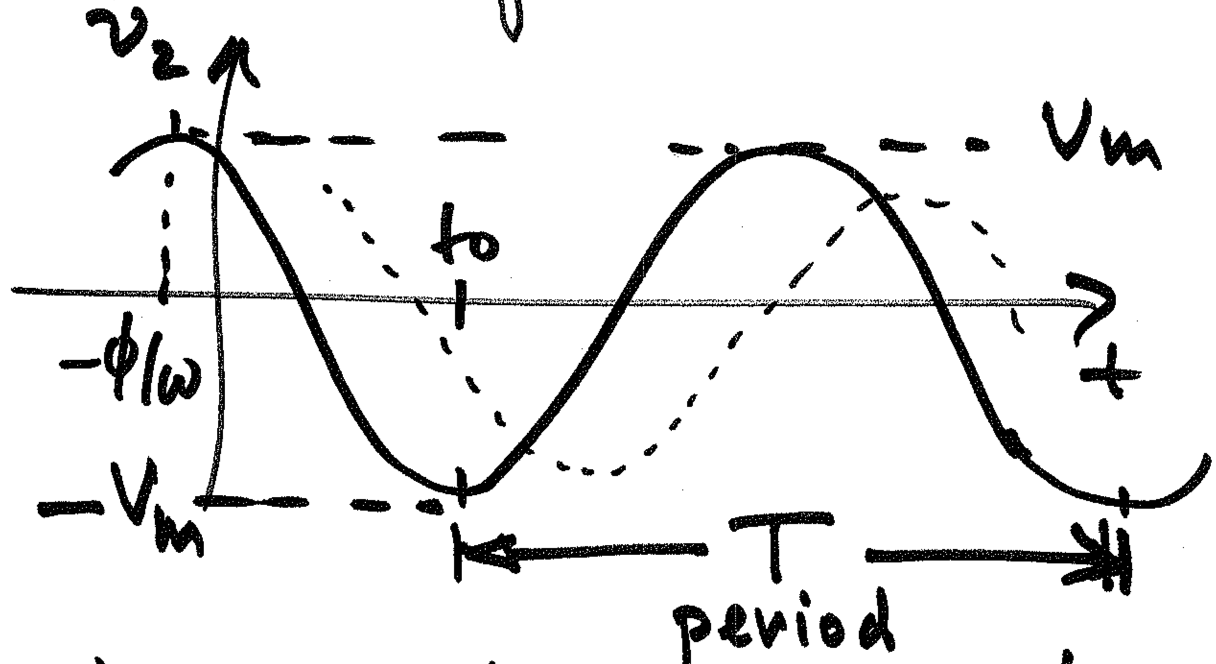
$$v_c(t) = \underbrace{A e^{-t/RC}}_{v_c^{GH}} + \underbrace{v_2}_{v_c^{SS}}$$

$$v_c(0^+) = v_c(0^-) = v_1 = v_2 + A \cancel{e^{-0/RC}}^{\,1}$$

$$A = v_1 - v_2$$

$$v_c(t) = v_2 + (v_1 - v_2) e^{-t/RC}$$

Now ... what if $v_2 = v_2(t) = V_m \cos(\omega t + \underline{\underline{\phi}})$



$\omega = 2\pi f$

$\uparrow$ $\frac{rad}{s}$ $\qquad$ $\uparrow$ Hz or $\frac{1}{s}$

$f = \frac{1}{T}$

$\phi > 0$ shift to left $\qquad \phi$ = phase angle

ϕ in in radians $(0 \to 2\pi)$

or degrees $(0 \to 360°)$

$$V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \phi)\, dt}$$

$$v_c^{GH} = Ae^{-t/RC}$$

Specific Solution: $B\cos(\omega t + \delta)$

$$F\cos(\omega t) + G\sin(\omega t)$$

Page 817: $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$$F = B\cos\delta$$

redefine $t = t' - \phi/\omega$

$$v_2 = V_m\cos(\omega t')$$

$$C\frac{dv_c}{dt'} + \frac{v_c}{R} = \frac{V_m}{R}\cos(\omega t')$$

$$v_c^{ss} = F\cos(\omega t') + G\sin(\omega t')$$

$$C\left[-\underline{F\omega}\sin(\omega t') + \underline{\underline{G\omega}}\cos(\omega t')\right] + \frac{1}{R}\left[\underline{\underline{F\cos}}(\omega t') + \underline{G}\sin(\omega t')\right] = \frac{V_m}{R}\underline{\underline{\cos}}(\omega t')$$

$$\sin:\ -CF\omega + \frac{G}{R} = 0 \Rightarrow G = CFR\omega$$

$$\cos:\ CG\omega + \frac{F}{R} = \frac{V_m}{R} \Rightarrow C^2FR\omega^2 + \frac{F}{R} = \frac{V_m}{R}$$

$$F = \frac{V_m}{R^2C^2\omega^2+1}\ ;\ G = \frac{\omega RCV_m}{\omega^2R^2C^2+1}.$$

$$v_c^{ss} = \frac{V_m}{1+(\omega RC)^2}\left[\cos(\omega t') + \omega RC\sin(\omega t')\right]$$

$$= B\cos(\omega t' + \delta)$$

note

$$\tan\delta = -\omega RC$$

$$\delta = \arctan(-\omega RC)$$

$$\tan^{-1}(-\omega RC)$$

$$(B\cos\delta)^2 + (B\sin\delta)^2 = B^2$$

$$B = \frac{V_m}{1+(\omega RC)^2}\left[1+(\omega RC)^2\right]^{1/2}$$

Whew! That was hard !!

Step I skipped:

$$B\cos(\omega t' + \delta) = B\left[\cos(\omega t')\cos(\delta) - \sin(\omega t')\sin(\delta)\right]$$

$$= \frac{V_m}{1+(\omega RC)^2}\left[\cos(\omega t') + \omega RC \sin(\omega t')\right)$$

$$B\cos\delta = \frac{V_m}{1+(\omega RC)^2} \; ; \quad B\sin\delta = \frac{-\omega RC\, V_m}{1+(\omega RC)^2}$$

$$\frac{B\sin\delta}{B\cos\delta} = \tan\delta = -\omega RC \longrightarrow \delta = \tan^{-1}(-\omega RC)$$

$$(B\cos\delta)^2 + (B\sin\delta)^2 = \left[\frac{V_m}{1+(\omega RC)^2}\right]^2 \left(1+(\omega RC)^2\right)$$

$$B^2 \overset{1}{\left(\cancel{\cos^2\delta + \sin^2\delta}\right)} = V_m^2 / \left[1+(\omega RC)^2\right] \Rightarrow B = \frac{V_m}{\left[1+(\omega RC)^2\right]^{1/2}}$$