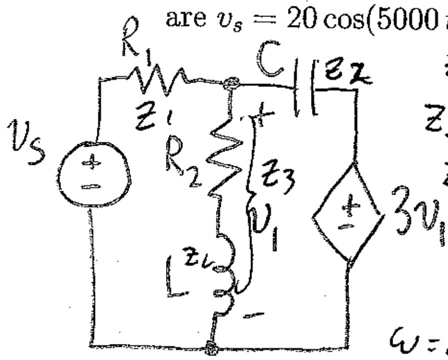
Name Solutions UW ID#: ______ Section: ___ Version A

Midterm — EE 233

Spring 2008

The test is closed book, with one sheet of notes and calculators allowed. Show all work. Be sure to state all assumptions made and **check** them when possible. The number of points per problem are indicated in parentheses. Total of 100 points in 3 problems on 3 pages.

1. Calculate $v_1(t)$ in the circuit below using sinusoidal steady-state analysis. Component values are $v_s = 20\cos(5000\,t - 90^o)$, $R_1 = 5\,\Omega$, $R_2 = 2\,\Omega$, $L = 0.2\,\text{mH}$, and $C = 40\,\mu\text{F}$. (30)



$Z_1 = 5\,\Omega$

$Z_2 = Z_C = \frac{1}{j\omega C} = \frac{1}{j(5000)(4\times10^{-5})} = -j5$

$Z_3 = R_2 + j\omega L$

$= 2 + j(5000)(0.2\times10^{-3})$

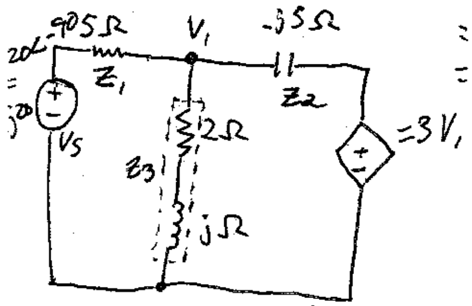
$= 2 + j$

$\omega = 5000$

$V_s : 20\cos(5000t - 90)$

$= 20\angle -90$

$= -j20$



Summing Currents into V_1 using Node Voltage

$$\frac{V_S - V_1}{Z_1} + \frac{-V_1}{Z_3} + \frac{3V_1 - V_1}{Z_2} = 0$$

$$\frac{V_S}{Z_1} - \frac{V_1}{Z_1} + \frac{-V_1}{Z_3} + \frac{2V_1}{Z_2} = 0$$

$$\frac{V_S}{Z_1} = \left(\frac{1}{Z_1} + \frac{1}{Z_3} - \frac{2}{Z_2}\right)V_1$$

$$V_S = \left(1 + \frac{Z_1}{Z_3} - \frac{2Z_1}{Z_2}\right)V_1$$

$$\frac{V_S}{1 + \frac{Z_1}{Z_3} - \frac{2Z_1}{Z_2}} = V_1$$

$$\frac{-j20}{1 + \frac{5}{2+j} - \frac{10}{-j5}} = V_1$$

$$\frac{-j20}{1 + 2 - j - j2} = V_1$$

$$\frac{-j20}{3 - j3} = V_1$$

$$\frac{20\angle -90}{3\sqrt{2}\angle -45} = V_1$$

$$\frac{10\sqrt{2}}{3}\angle -90+45$$

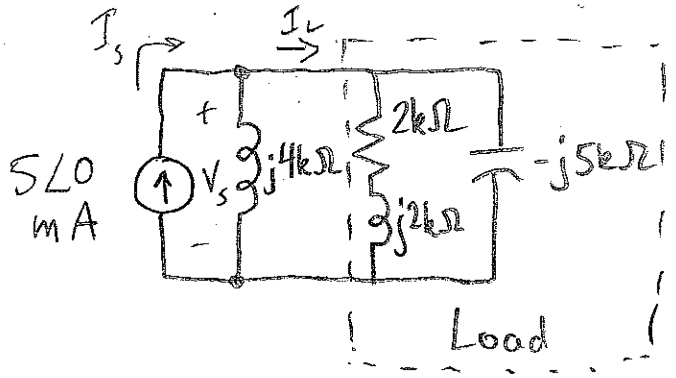
$$4.71\angle -45 = V_1$$

$$V_1(t) = 4.71\cos(5000t - 45^\circ)\ V$$

$\mu = 23.6$

Median $= 26$

2. (a) Calculate the total complex power (include units) delivered by the current source. [Hint: You might want to use admittances.] (25)



$$S_s = \frac{V_s I_s^*}{2} = \frac{|I_s|^2}{2\,Y_{total}}$$

$$Y_{total} = Y_s + Y_L \qquad Y_s = \frac{1}{j4k\Omega} = -j0.25\,mS$$

$$Y_L = \frac{1}{2k+j2k} + \frac{1}{-j5k} = \frac{2k-j2k}{(2k)^2+(2k)^2} + j0.2\,mS$$

$$= 0.25 - j0.25 + j0.2 = 0.25 - j0.05\,mS$$

$$Y_{total} = Y_s + Y_L$$

$$= 0.25 - j0.3\,mS = 0.39\angle{-50^\circ}\,mS$$

$$S_s = \frac{(5)^2(mA)^2}{2(0.39\angle{-50^\circ}\,mS)} = 32\angle 50^\circ\,mVA = 20.6 + j24.5\,VA$$

$$P = 20.6\,mW, \quad Q = 24.5\,mVAR$$

Considering just power to the load (as many of you did):

$$I_L = Y_L V_s = Y_L \frac{I_s}{Y_{total}} \qquad S_L = \frac{1}{2} V_L I_L^* = \frac{1}{2}\left(\frac{I_s}{Y_{total}}\right)\left(Y_L^* \frac{I_s^*}{Y_{total}^*}\right) = \frac{1}{2}\frac{|I_s|^2}{|Y_{total}|^2} Y_L^*$$

$$= \frac{1}{2}\frac{(5mA)^2}{(0.39\,mS)^2}(0.25 + j0.05)\,mS$$

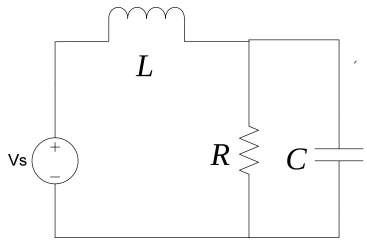
$$= 20.5 + j4.1\,mVA$$

For reference, $V_s = \frac{I_s}{Y_{total}} = 12.8\angle 50^\circ V$ and $I_L = Y_L V_s = 3.26\angle 38.7\,mA$

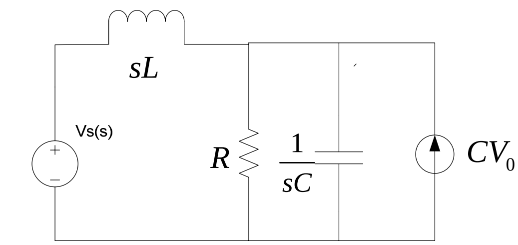
(b) Based on your analysis in part (a), could the power delivered to the load be increased by adding additional capacitance in parallel with the load? Explain. (5)

Yes. The total impedance seen by the source is inductive ($B_{total} < 0$, $X_{total} > 0$, $Q > 0$), so adding additional capacitance ($B > 0, X < 0$) will make load less reactive. (X_L closer to X_s^* or B_L closer to B_s^*), increasing real power to load.

3. Determine $V_c(s)$ for the following circuit with $v_s(t) = 2\,t\,u(t)$. Include initial conditions for the capacitor, but assume that the inductor has no stored energy at $t=0^-$. (25)



Transform the circuit into the Laplace domain:



$$KCL\,@\,Vc$$

$$\frac{V_c - V_s}{sL} + \frac{V_c}{R} + \frac{V_c}{1/sC} - CV_0 = 0$$

$$\left(\frac{1}{sL} + \frac{1}{R} + sC\right)V_c - \frac{1}{sL}V_s - CV_0 = 0$$

$$V_c = \frac{\frac{1}{sL}V_s + CV_0}{\left(\frac{1}{sL} + \frac{1}{R} + sC\right)}$$

$$v_s(t) = 2t \cdot u(t) \leftrightarrow V_s(s) = 2/s^2$$

$$V_c = \frac{\frac{2}{s^3L} + CV_0}{\left(\frac{1}{sL} + \frac{1}{R} + sC\right)} = \frac{\frac{2}{s^2LC} + sV_0}{\left(s^2 + s\frac{1}{RC} + \frac{1}{LC}\right)}$$

3b) Find the inverse Laplace transform of $\dfrac{3s+4}{s^2+2s+2}$:

First perform the partial fraction expansion by factoring the denominator into its roots, in this case complex roots. Next solve for the coefficients A and A*. Perform the inverse transform by inspection with the aid of a Laplace transform table.

$$F(s)=\frac{3s+4}{s^2+2s+2}=\frac{3s+4}{(s+1-j)(s+1+j)}=\frac{A}{(s+1-j)}+\frac{A^*}{(s+1+j)}$$

$$3s+4=A(s+1+j)+A^*(s+1-j)$$

if s=-1+j

$$3(-1+j)+4=A(-1+j+1+j)$$

$$1+j3=Aj2$$

$$A=\frac{1+j3}{j2}=1.5-j0.5=1.58\angle -18.43^\circ$$

$$\therefore \frac{3s+4}{s^2+2s+2}=\frac{1.5-j0.5}{(s+1-j)}+\frac{1.5+j0.5}{(s+1+j)}$$

By table lookup:

$$f(t)=\left[(1.5-j0.5)e^{-(1-j)t}+(1.5+j0.5)e^{-(1+j)t}\right]u(t)$$

$$=\left[3.16e^{-t}\cos(1t-18.43^\circ)\right]u(t)$$

Be careful that the phase sign comes from the “positive” pole, marked in red.