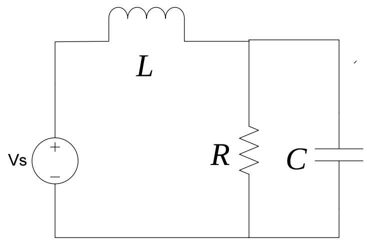
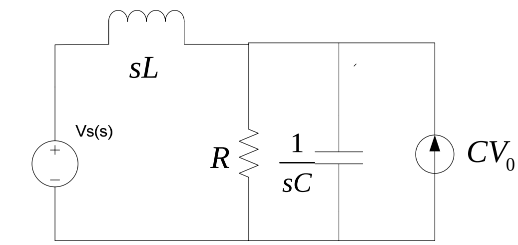
3. Determine $V_c(s)$ for the following circuit with $v_s(t) = 2\,t\,u(t)$. Include initial conditions for the capacitor, but assume that the inductor has no stored energy at $t=0^-$. (25)



Transform the circuit into the Laplace domain:



$$KCL\,@\,Vc$$

$$\frac{V_c - V_s}{sL} + \frac{V_c}{R} + \frac{V_c}{1/sC} - CV_0 = 0$$

$$\left(\frac{1}{sL} + \frac{1}{R} + sC\right)V_c - \frac{1}{sL}V_s - CV_0 = 0$$

$$V_c = \frac{\frac{1}{sL}V_s + CV_0}{\left(\frac{1}{sL} + \frac{1}{R} + sC\right)}$$

$$v_s(t) = 2t \cdot u(t) \leftrightarrow V_s(s) = 2/s^2$$

$$V_c = \frac{\frac{2}{s^3 L} + CV_0}{\left(\frac{1}{sL} + \frac{1}{R} + sC\right)} = \frac{\frac{2}{s^2 LC} + sV_0}{\left(s^2 + s\frac{1}{RC} + \frac{1}{LC}\right)}$$

3b) Find the inverse Laplace transform of $\frac{3s+4}{s^2+2s+2}$:

First perform the partial fraction expansion by factoring the denominator into its roots, in this case complex roots. Next solve for the coefficients A and A*. Perform the inverse transform by inspection with the aid of a Laplace transform table.

$$F(s)=\frac{3s+4}{s^2+2s+2}=\frac{3s+4}{(s+1-j)(s+1+j)}=\frac{A}{(s+1-j)}+\frac{A^*}{(s+1+j)}$$

$$3s+4=A(s+1+j)+A^*(s+1-j)$$

if s=-1+j

$$3(-1+j)+4=A(-1+j+1+j)$$

$$1+j3=Aj2$$

$$A=\frac{1+j3}{j2}=1.5-j0.5=1.58\angle -18.43^\circ$$

$$\therefore \frac{3s+4}{s^2+2s+2}=\frac{1.5-j0.5}{(s+1-j)}+\frac{1.5+j0.5}{(s+1+j)}$$

By table lookup:

$$f(t)=\left[(1.5-j0.5)e^{-(1-j)t}+(1.5+j0.5)e^{-(1+j)t}\right]u(t)$$

$$=\left[3.16e^{-t}\cos(1t-18.43^\circ)\right]u(t)$$

Be careful that the phase sign comes from the “positive” pole, marked in red.