Integrators, differentiators, and simple filters

1. Objectives

- Analyze and measure characteristics of inverting and non-inverting amplifiers and other op amp circuits in the time and frequency domain.
- Design and test circuits with opamps.
- Plot gain vs. frequency to understand circuit operations.
- Use SPICE to verify circuit designs.

2. Reference

The opamp characteristics and circuits are covered in the textbook. Make sure you know how to analyze circuits using the simple ideal opamp model.

Review the usage of the dual power supply that can set two tracked supply values (e.g. +10V and –10V). The convention is to use V_{CC} to denote the positive supply and V_{EE} to denote the negative supply.

The only knowledge required for this experiment is: time-domain analysis, basic opamp circuit analysis, and impedance of components. The technique to plot gain as function of frequency is described in the Discussion section below. No knowledge of filters is required.

3. Circuits

You will analyze, design, and simulate the circuits below in various parts of this laboratory.

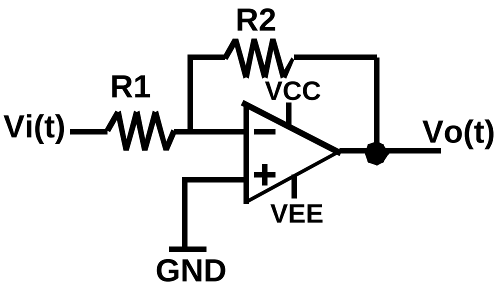

Figure 1. Inverting amplifier.

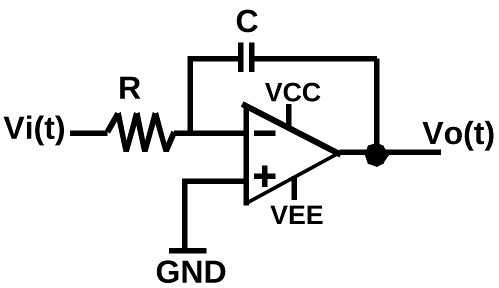

Figure 2. Simple integrator.

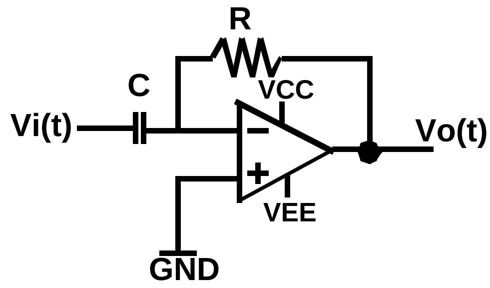

Figure 3. Simple differentiator.

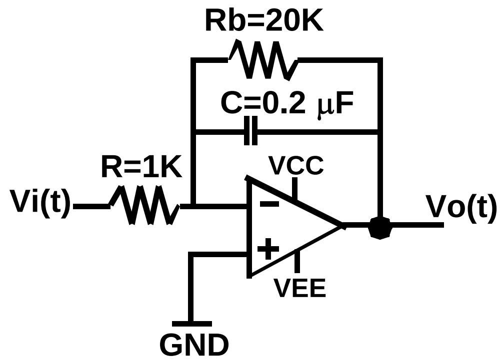

Figure 4. Integrator with shunt resistor.

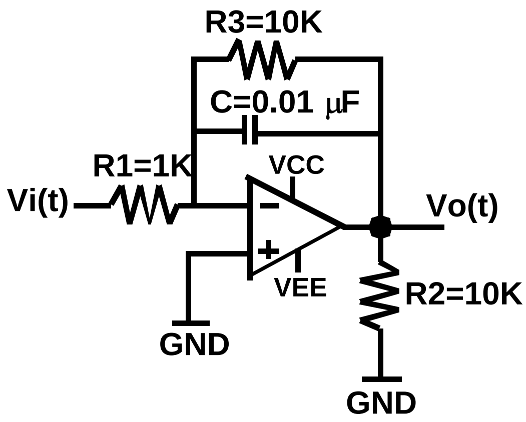

Figure 5. A low-pass filter.

4. Components and specifications

Quantity	*Description*	*Comments*
3	MC 4741C opamp	or equivalent
3	1 KΩ resistor	Measure the exact value you use
3	10 KΩ resistor	Measure the exact value you use
1	20 KΩ resistor	Measure the exact value you use

3	0.01 μF capacitor	Measure the exact value you use, using the the RLC multimeter in the EE stores window
1	0.2 μF capacitor	Measure the exact value you use, using the the RLC multimeter in the EE stores window

Opamp specifications are available from the laboratory web site of this course or manufacturers' web sites. Check your component and download the appropriate specifications.

5. Discussion

In many circuits whose signals are sine or cosine, the gain (ratio of output amplitude versus input amplitude) varies as function of the signal frequency, e.g. as shown in Table 1 below. Figure 6 shows a plot of the gain versus frequency from this table, using the linear scales on both axes. Since the frequency varies over a very wide range, the plot compresses the frequency axis and does not show much about the circuit gain characteristics above the frequency 10000 Hz.

Table 1. Gain versus frequency data.

Frequency	**Vo amplitude**	**Vi amplitude**	**G= |Vo|/|Vi|**
(Hz)	**(V)**	**(V)**	**(ratio)**
10	9.998	10	0.999
20	9.993	10	0.999
50	9.960	10	0.996
100	9.845	10	0.984
200	9.406	10	0.941
500	7.172	10	0.717
1000	3.880	10	0.388
2000	1.368	10	0.137
5000	0.247	10	0.025
10000	0.063	10	0.006
20000	0.016	10	0.002
50000	0.002	10	0.0002

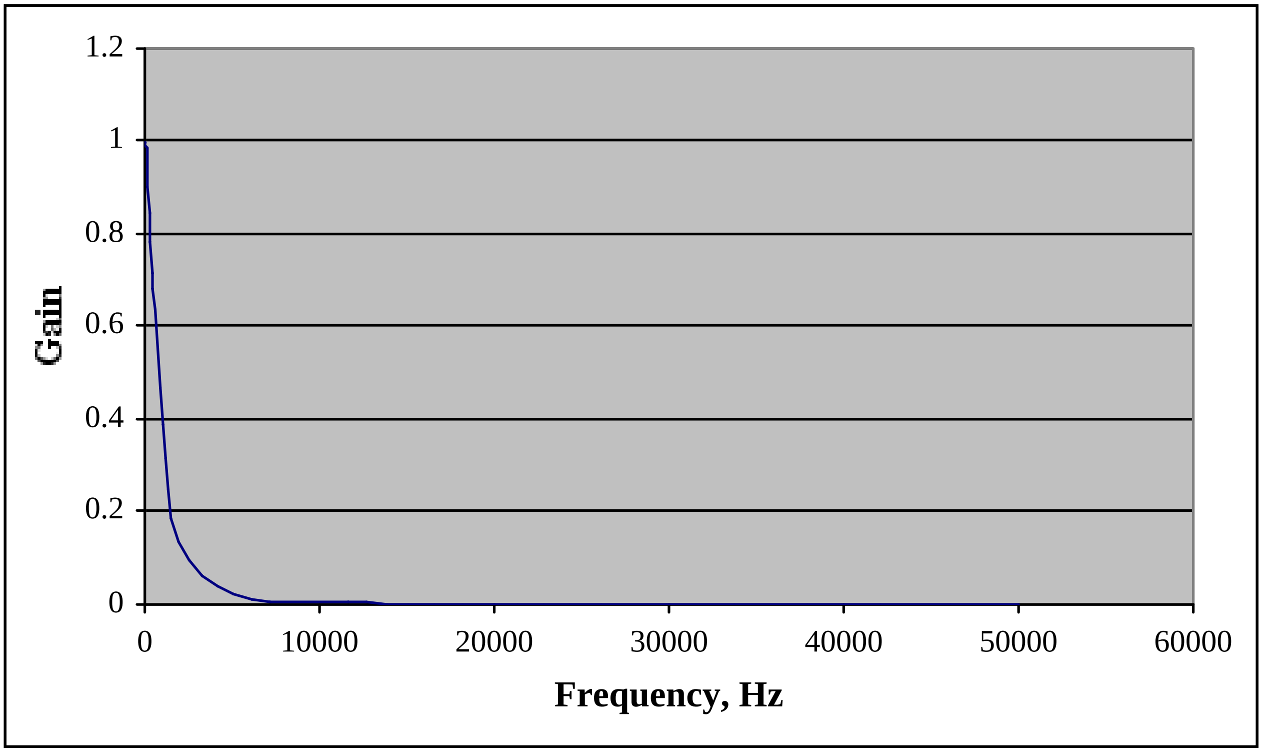

Figure 6. Linear plot of gain versus frequency.

Another way to plot is to use logarithmic scale. First, the gain values are converted to dB (decibel) using the formula:

$$G\ (dB) = 20 \log G$$

As an example, a gain G=100 is converted to G(dB)=40 dB; a gain G=0.1 is converted to G(dB)=-20 dB. Table 2 is produced from Table 1 using this conversion technique.

Table 2. Gain (dB) versus frequency data.

Frequency	G	G
(Hz)	**(ratio)**	**(dB)**
10	0.999	-0.0087
20	0.999	-0.0087
50	0.996	-0.0348
100	0.984	-0.1401
200	0.941	-0.5282
500	0.717	-2.8896
1000	0.388	-8.2234

2000	0.137	-17.2656
5000	0.025	-32.0412
10000	0.006	-44.4370
20000	0.002	-53.9794
50000	0.0002	-73.9794

Next, a *semilog* plot is constructed. The horizontal axis is the frequency, in logarithmic scale. The vertical axis is the gain in dB, in linear scale. The semilog plot of the data in Table 2 is shown in figure 7. Once the raw data of gain versus frequency (e.g. as in Table 1) is available from measurements or computations, the dB conversion and the semilog plot can be produced using software tools such as Matlab, Excel, etc.

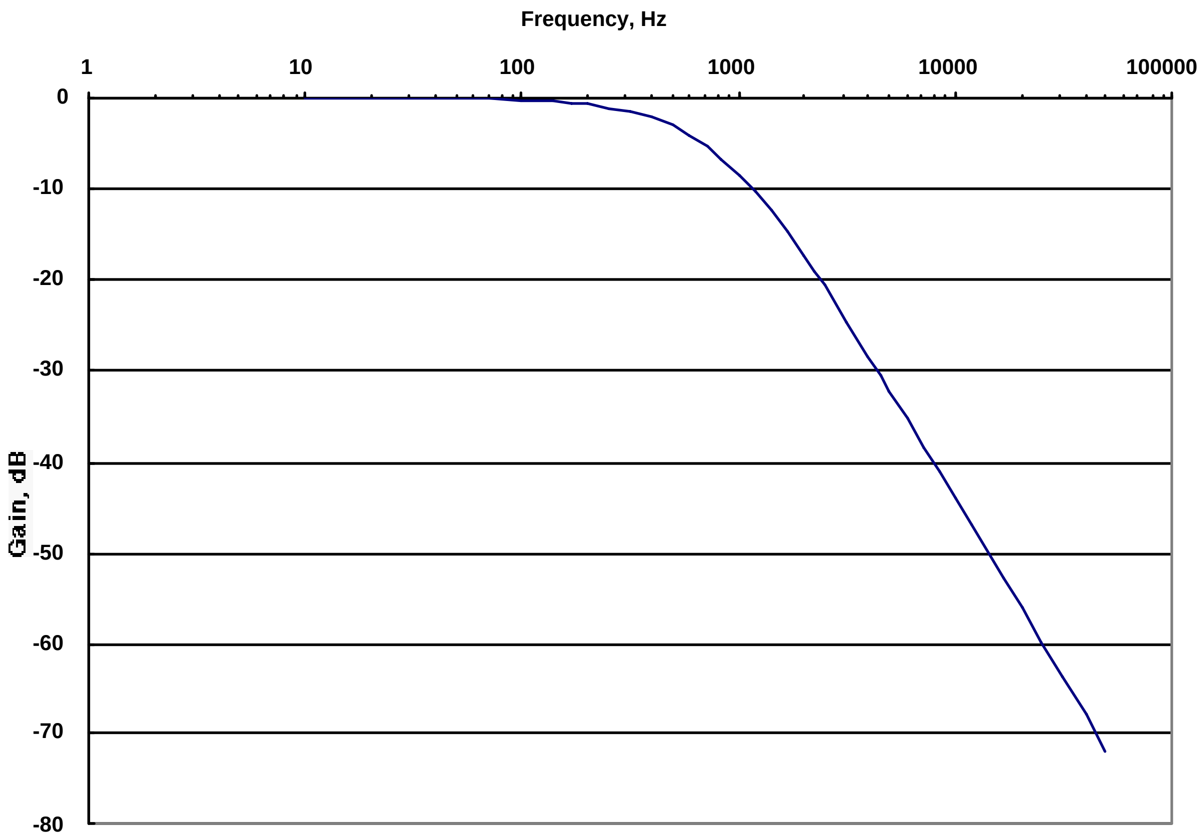

Figure 7. Semilog plot of dB gain versus frequency.

Figure 7 shows better these characteristics of the circuit: the dB gain is relatively fixed at low frequencies, the dB gain drops off linearly (with log of frequency) at high frequencies, and there is a “corner” in the gain plot between these two frequency ranges. This type of plots is regularly used to study circuit characteristics as functions of frequency.

6. Pre-lab

6.1 Designs of simple amplifiers

For the circuit in figure 1,assuming the opamp is ideal, answer the following questions:

1. Design an inverting amplifier using one opamp and two or more resistors. Design it such that it has a gain of -10 (this gain is termed “inverting”). Pick resistor values that you have in the lab kit. Include a schematic of this circuit with the component values labeled with your completed pre-lab assignment.

2. Simulate this inverting amplifier circuit with SPICE to make sure the circuit works as designed. Print out a plot of the simulation.

In the textbook find a non-inverting amplifier circuit using one opamp and two or more resistors. Then assuming the opamp is ideal, answer the following questions:

3. Design a non-inverting amplifier such that it has a gain of +11 (this gain is termed “non-inverting”). Pick resistor values that you have in the lab kit. Include a schematic of this circuit with the component values labeled with your completed pre-lab assignment.

4. Simulate this non-inverting amplifier circuit with SPICE to make sure the circuit works as designed. Print out a plot of the simulation.

6.2 Analysis of integrators and differentiators

For the circuit in Figure 2,assuming the opamp is ideal, answer the following question:

1. Derive the time-domain equation for $V_o(t)$ in terms of $V_i(t)$. Show that the circuit performs the function of an integrator. Don’t forget the initial condition.

For the circuit in Figure 3 with power supplies V_{CC} = 12 V, V_{EE} = -12 V, and assuming the opamp is ideal, answer the following question:

2. Derive the time-domain equation for $V_o(t)$ in terms of $V_i(t)$. Show that the circuit performs the function of a differentiator.

For the circuit in Figure 4, assuming the opamp is ideal, answer the following questions:

3. What is the low-frequency gain of this circuit? Hint: consider what is the gain when a constant input ($\omega \to 0$) is applied?

4. For frequencies $\omega >> 1/(R_bC)$, show that the circuit performs the function of an integrator. Compare the gain for the integrator circuit side by side with your result.

5. Use SPICE transient analysis to simulate this circuit in the time domain using a sine wave input with amplitude 1 V and frequency 1kHz. In one sentence describe how you can tell that it is indeed acting as an integrator. Include the transient SPICE results over 5 periods.

6. Explain the function of the resistor R_b in this circuit. Using SPICE AC analysis to determine the gain as a function of frequency from (f=0 to f=10kHz) with and without Rb. Overlay the two plots and combine the data into a clearly labeled plot.

7. In an ideal op-amp the voltage at the non-inverting terminal equals that at the inverting terminal. In a real opamp, this is not the case, and the error is called “input offset voltage.” Find a typical or maximum value in the op-amp datasheet.

6.3 Analysis and simulation of an active low-pass filter

For the circuit in Figure 5 assume the opamp is ideal. The input signal is:

$$V_i(t) = A\cos(\omega t)$$

where the input amplitude A is small to avoid slew-rate limitations. **Determine how small A must be so as not to run into slew rate limitations based on your results from the previous lab.**

Answer the following questions:

1. Use phasor analysis to derive the equation for the amplitude of $V_o(t)$ in terms of the input amplitude A, input frequency ω, and circuit components R and C. Do **not** use any numerical values. Note: there is **no** need to derive the output phase equation.

2. From item 1 above, determine the circuit gain $|V_o(j\omega)/V_i(j\omega)|$ in terms of input frequency ω and circuit components R and C? Do **not** use any numerical values.

3. Using the numerical values of R and C as shown in Figure 5, plot the circuit gain ($|V_o(j\omega)/V_i(j\omega)|$) using the technique described in the Discussion section. Plot the gain over the frequency range 1Hz to 100KHz. Be sure to plot vs Hz, not rad/sec.

4. Use SPICE AC analysis to simulate this circuit and generate the gain plot vs. log scale frequency. Overlay the analytical and SPICE gain plots. Explain any differences.

5. If the input signal has a low frequency ($(\omega \to 0)$, what is the expected gain from these plots in items 3 and 4 above? If the input signal has a high frequency, what is the expected gain from these plots? Based on these gain values, explain why the circuit is named “low-pass” filter.

6. Note that this circuit is topologically similar to the integrator circuit in Figure 4. Explain why these two types of circuits are similar. Compare the phasor domain circuit gain magnitudes ($|V_o(j\omega)/V_i(j\omega)|$) of these two circuits side-by-side.

7. Experimental procedures

7.1 Instruments needed for this experiment

The instruments needed for this experiment are: a power supply, a function generator, a multimeter, and an oscilloscope.

7.2 Inverting amplifiers

Build the circuit in Figure 1 using power supplies ±12 V and the resistor values from your design in the Pre-lab, section 6.1 item 1.

1. Use a sine wave input with small amplitude so that the output is not affected by the slew rate in this part. Show, with some calculations, that this amplitude is small enough so that at the highest frequencies, you do not run into slew rate limitation.

From the starting input frequency of 10 Hz and varying it using 1-2-5 sequence up to 2 MHz (i.e. set input frequency to 10 Hz, 20 Hz, 50 Hz, 100 Hz, 200 Hz, … up to 1 MHz), measure the experimental values of the gain of this circuit at each frequency. Record them in a table for later data analysis.

2. Get a hardcopy output from the scope display with both waveforms at the frequency 100 KHz. Turn this hardcopy in as part of your lab report.

7.3 Integrators

1. Build the circuit in Figure 4 with power supplies ±12 V. Apply a sine wave input signal with amplitude 300 mVpp (-150mV to 150mV) and frequency 300 Hz. Display the input signal on channel 1 of the oscilloscope.

2. Display V_o on Channel 2 and adjust the timebase to display 2 to 3 complete cycles of the signals.

3. Get a hardcopy output from the scope display with both waveforms to confirm that the circuit is an integrator. Turn this hardcopy in as part of your lab report.

4. Change the input signal to a *square wave* with the same parameters: 300 mVpp (-150mV to 150mV) and frequency 300 Hz. Get a hardcopy output from the scope display with both waveforms to confirm that the circuit is still an integrator. Turn this hardcopy in as part of your lab report.

5. Repeat item 4 above using a triangular input signal with same amplitude and frequency. Turn this hardcopy in as part of your lab report.

6. Now change the input back to a sine wave as in item 1. Remove the resistor R_b. What happens to the output signal? Explain the phenomenon you observe on the oscilloscope. Re-insert the resistor R_b and verify that the circuit functions as designed. Be sure the coupling mode for the channel used to measure the output signal is set to DC coupling mode.

7.4 Low-pass filters

1. Build the circuit in Figure 5 with power supplies ±12 V. Use a sine wave of amplitude 100 mV as an input signal (see item 2 below for frequency) and display both the input and output signals on the oscilloscope (2 to 3 complete cycles).

2. Vary the input signal frequency in 1-2-5 sequence from 10 Hz to 100 KHz. At each frequency, measure the gain of the circuit, using the data from the oscilloscope display. Keep this data in a table for later plotting.

8. Data analysis

8.1 Inverting amplifiers

1. Compare the experimental gain measured in section 7.2 item 1 with the calculated gain in the pre-lab and with the gain as simulated by SPICE. Overlay the three results. Explain any difference between these values.

2. From the table of data in section 7.2 item 1, plot the gain of this circuit as dB versus frequency, using the technique described in the Discussion section.

8.2 Integrators

1. Explain any difference between the SPICE output in section 6.2 item 5 and the experimental data in section 7.3 item 3. Overlay the experimental and the analytical results

2. With the experimental observation in section 7.3 item 6, explain the function of the resistor R_b. Use the plot in 6.2.6 to assist in the explanation.

8.3 Low-pass filters

1. From the data in section 7.4 item 2, plot the gain (in dB) of the circuit as function of frequency (Hz) in log-scale (using the technique described in the Discussion section) and compare it with the plots in section 6.3 item 3 and in section 6.3 item 4 (SPICE plot). Overlay the experimental, SPICE, and analytical solution to form a single plot. Explain any differences between these 3 plots.

9. Further research

Opamp-based active filters are very popular in a wide range of applications and you can look up books with titles such as “Opamp cookbooks” to try out more circuits. Build one or two of these circuits (try a high-pass filter or a notch filter) and test them out. Push the performance limits (slew rate, frequency, different input signal types, etc.) and see what the circuits do when the opamp no longer operates in the ideal region. Measure the waveforms in the lab and correlate with the opamp characteristics.

Check your stereo system specifications and see if you can interpret many of these specifications now using your understanding of circuit gain as function of frequency. Try to read the specifications for a graphics equalizer or a stereo pre-amplifier as an example.

10. Self-test

1. Apply a large signal to any of the circuit in this lab and see when the slew rate limitation begins to show up.

2. Use the digital multimeter in the RMS measurement mode and see if you can measure the gain of an amplifier as function of frequency. Set the input frequency and measure the output RMS value at that frequency. From this RMS output value, calculate the gain, then from the data points (frequency, gain) as the frequency varies, reproduce the gain plot. How is this plot compared to the plots obtained from hand calculation, SPICE simulation, and scope data?