EE 331 Spring 2014 Homework #0 solutions

1.24)

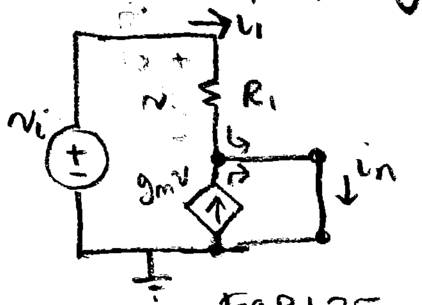
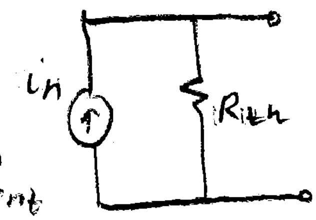


Fig.P1.25

want Norton Equivalent ⟶

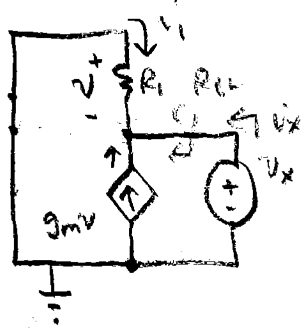


Kinda Like example 1.2, do lots of KCL/KVL:

$$i_n = i_1 + g_m v \quad , \; i_1 = G_1 v_i$$

$$i_n = v_i(G_1 + g_m) = \left(\frac{1}{10k} + 0.002\right) v_i = 0.0021 v_i$$

R_{th}:



$$R_{th} = \frac{v_x}{i_x}$$

KCL again:

$$-i_1 - g_m v - i_x = 0$$

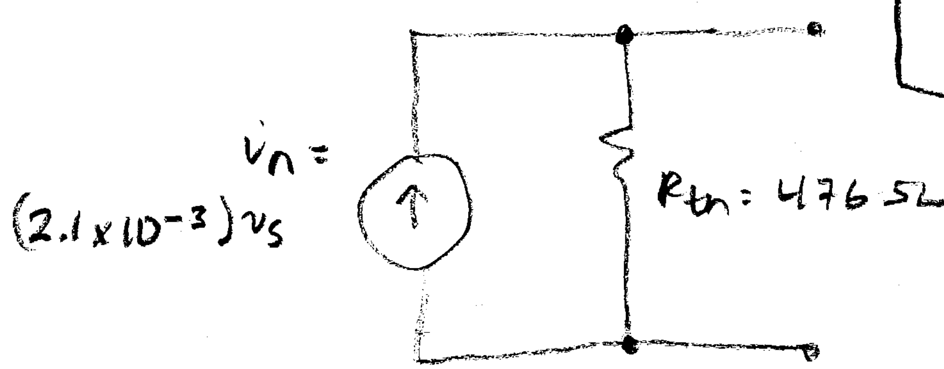
$$i_x = -\left(\frac{v}{R_1}\right) - g_m(-v_x) \quad \{ v_x = -v$$

$$i_x = v_x\left(\frac{1}{R_1} + g_m\right)$$

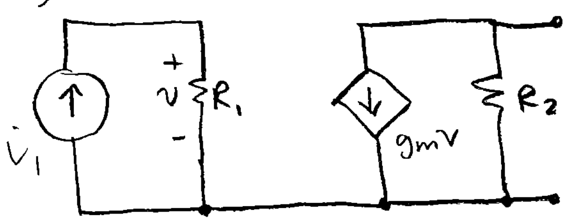
$$\therefore \frac{v_x}{i_x} = R_{th} = \frac{1}{\left(\frac{1}{R_1} + g_m\right)} = \frac{1}{\left(\frac{1}{10k} + 0.002\right)}$$

$$\approx 476\,\Omega$$

∴ solution



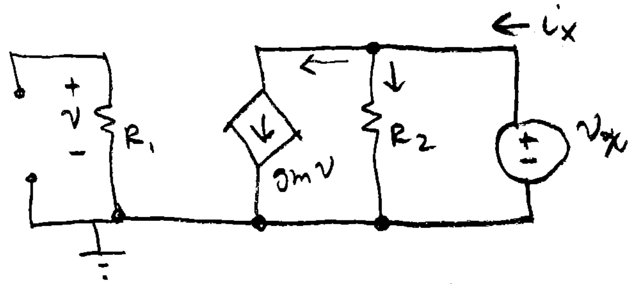
1.29)



$g_m = 0.0025\ S$
$R_1 = 200k\ \Omega$
$R_2 = 1.5\ M\Omega$

Find R_{th}:

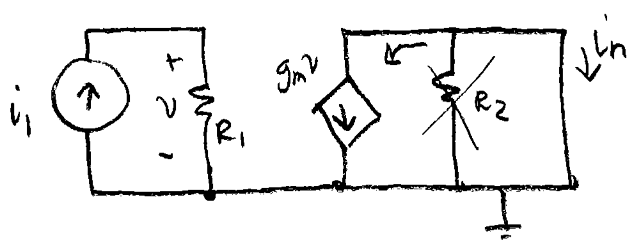
$R_{th} = \frac{v_x}{i_x}$



KCL @ v_x: $-i_x + \frac{v_x}{R_2} + g_m v = 0$ ($v = 0V$)

$\therefore \frac{v_x}{i_x} = R_{th} = R_2 = 1.5\ M\Omega$

short output to find i_n:



(R_2 shorted out)

KCL: $i_n = -g_m v$ $v = i_1 R_1$ (substitute)

$i_n = -g_m R_1 i_1$

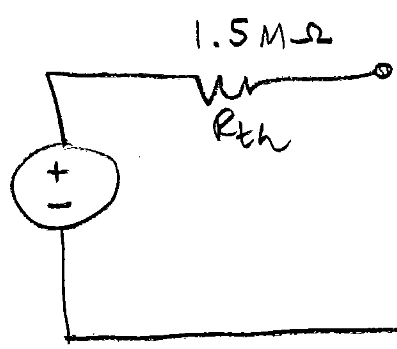
$\therefore v_{th} = i_n R_{th} = (-g_m R_1 i_1)(R_{th})$ ($R_1 i_1 = v_s$)

$= -(0.0025)(1.5M)\, v_s$

$= -3750\, v_s$

Solution

$v_{th} = -3750\, v_s$



problem 1 & 2 here have wrong #'s but use other methods to solve FYI

Ee331, spring 2012

Solution to HW0

Problem 1

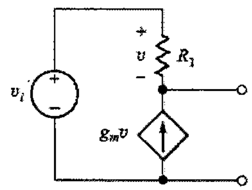
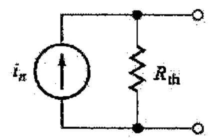
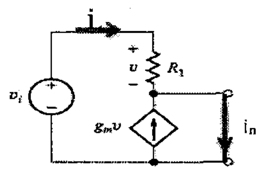


Figure P1.25



=>

i_n represents the current when output circuit is short.



$i_1 + g_m(i_1 R_1) = i_n$

$i_1 = v_i / R_1$

$=> i_n = (1+g_m R_1) i_1 = \frac{(1+g_m R_1)}{R_1} v_i = \frac{251}{10k\Omega} v_i$

R_{th} represents the equivalent resistance present at the output terminals with all independent sources set at zero.

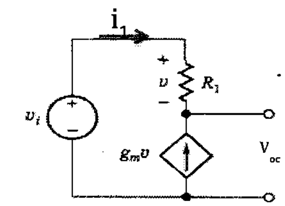
There are three methods to solve R_{th}.

Method 1: open circuit voltage divided by short circuit current

$$R_{th} = \frac{V_{oc}}{I_{sc}}$$

I_{sc} is the output current when output circuit is short, same with i_n

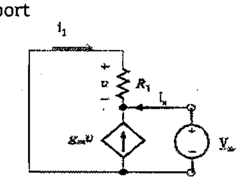
V_{oc} is the output voltage when output circuit is open.



$i_1 = -g_m(i_1 R_1) => i_1 = 0,\ V_{oc} = v_i$

$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{v_i}{(1+g_m R_1)v_i / R_1} = \frac{R_1}{1+g_m R_1} = 39.84\ \Omega$

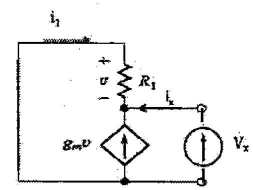
Method 2: Set voltage source to zero, add an external voltage source at the output port



$R_{th} = \frac{V_x}{i_x},\ i_x = -(i_1 + g_m i_1 R_1),\ V_x = -i_1 R_1$

⇨ $R_{th} = \frac{R_1}{1+g_m R_1} = 39.84\ \Omega$

Method 3: Set voltage source to zero, add an external current source at the output port



$i_x = -(i_1 + g_m i_1 R_1),\ V_x = -i_1 R_1$

$R_{th} = \frac{V_x}{i_x} = \frac{-i_1 R_1}{-(i_1 + g_m i_1 R_1)} = \frac{R_1}{i_1 + g_m i_1 R_1} = 39.84\Omega$

Problem2:

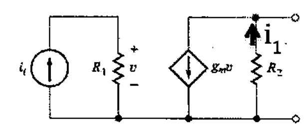
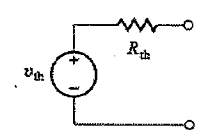


Figure P1.29

=>



v_{th} represents the equivalent voltage when output is open.

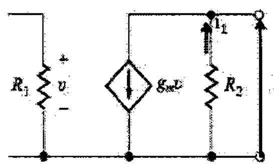
$$v = i_i R_1$$
$$i_1 = g_m v = i_i R_1$$
$$v_{th} = i_1 R_2 = R_1 R_2 g_m i_i$$

R_{th} represents the equivalent resistance when all independent sources are zero.

Three ways to solve R_{th}

Method 1: open circuit voltage divided by short circuit current

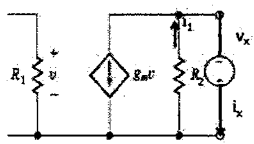
$$R_{th} = \frac{V_{oc}}{I_{sc}}$$
$$V_{oc} = v_{th} = R_1 R_2 g_m i_i$$



I_{sc} is the output current when output circuit is short

$$i_{sc} = -g_m v = -g_m i_i R_1$$
$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{R_1 R_2 g_m i_i}{g_m i_i R_1} = R_2 = 1.5\text{M}\Omega$$

Method 2: Set current source to zero, add an external voltage source at output

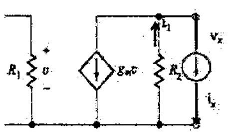


i_i=0, v=0, g_mv=0 i_x= i_1-g_mv=i_1

i_1=v_x/R_2

=> R_{th}=v_x/i_x=i_1R_2/i_1=R_2=1.5MΩ

Method 3: set current source to zero, add an external current source at output



$$i_x = i_1$$
$$v_x = i_1 R_2$$

R_{th}=v_x/i_x=i_1R_2/i_1=R_2=1.5MΩ

Problem 3

Expressions for V_1, I_2 and I_3 are given by:

$$V_1 = \frac{V}{1+\left(\frac{1}{1/R_3 + 1/R_2}\right)\frac{1}{R_1}}$$
$$I_2 = \frac{V - V_1}{R_2}$$
$$I_3 = \frac{V - V_1}{R_3}$$

Nominal values are:

$$V_1^{nom} = \frac{1}{1+\left(\frac{1}{1/11k + 1/30k}\right)\frac{1}{24k}} \approx 0.75\text{V}$$
$$I_2^{nom} = \frac{1-0.7489}{30k} \approx 8.37\text{uA}$$
$$I_3^{nom} = \frac{1-0.7489}{11k} \approx 22.83\text{uA}$$

According to the expression of V_1, the max and minimum values of V_1 are:

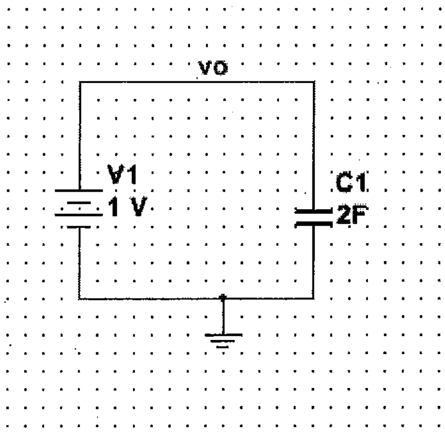
$$V_1^{max} = \frac{1\times 1.05}{1+\frac{1}{1/(11k\times 0.9) + 1/(30k\times 0.9)}\frac{1}{24k\times 1.1}} \approx 0.82V$$
$$V_1^{min} = \frac{1\times 0.95}{1+\frac{1}{1/(11k\times 1.1) + 1/(30k\times 1.1)}\frac{1}{24k\times 0.9}} \approx 0.67V$$

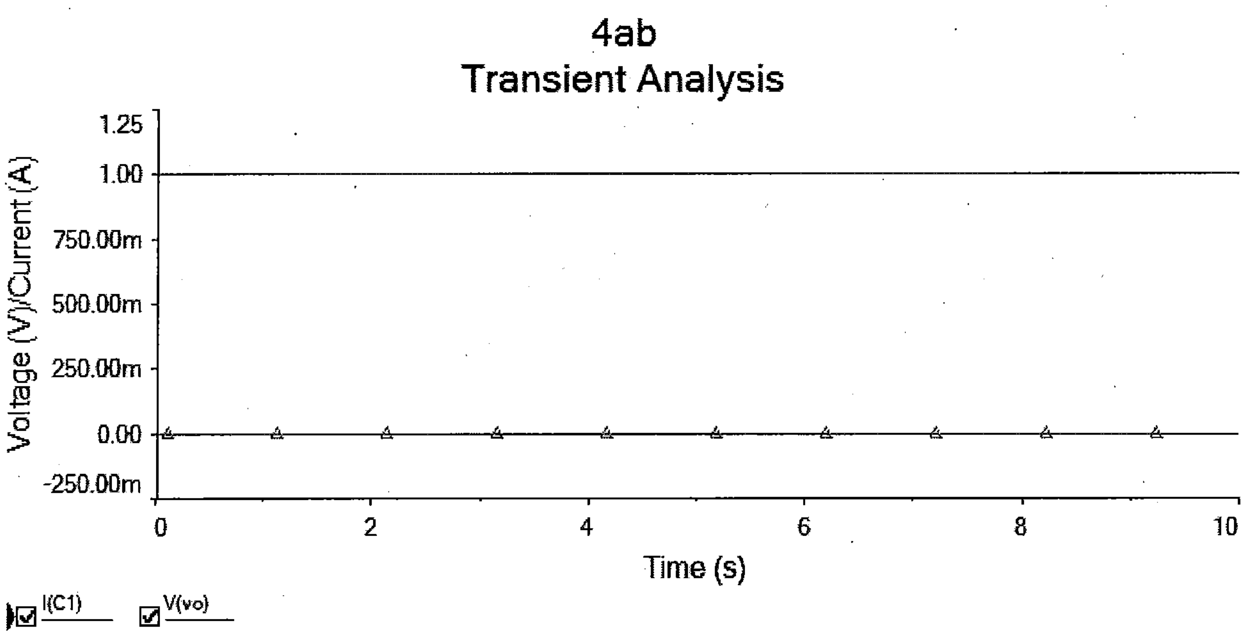
Worst-case values of I_2 and I_3 are:

$$I_2^{max} = \frac{V - V_1^{min}}{R_2^{min}} = \frac{1-0.67}{30k\times 0.9} = 12.22uA$$
$$I_2^{min} = \frac{V - V_1^{max}}{R_2^{max}} = \frac{1-0.82}{30k\times 1.1} = 5.45uA$$
$$I_3^{max} = \frac{V - V_1^{min}}{R_3^{min}} = \frac{1-0.67}{11k\times 0.9} = 33.33uA$$
$$I_3^{min} = \frac{V - V_1^{max}}{R_3^{max}} = \frac{1-0.82}{11k\times 1.1} = 14.88uA$$

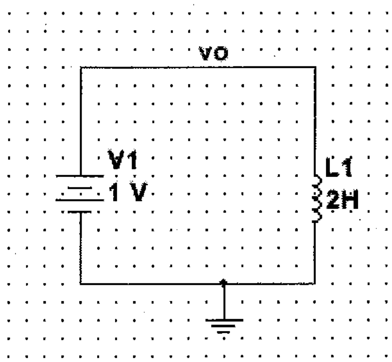
Problem 4 Solution

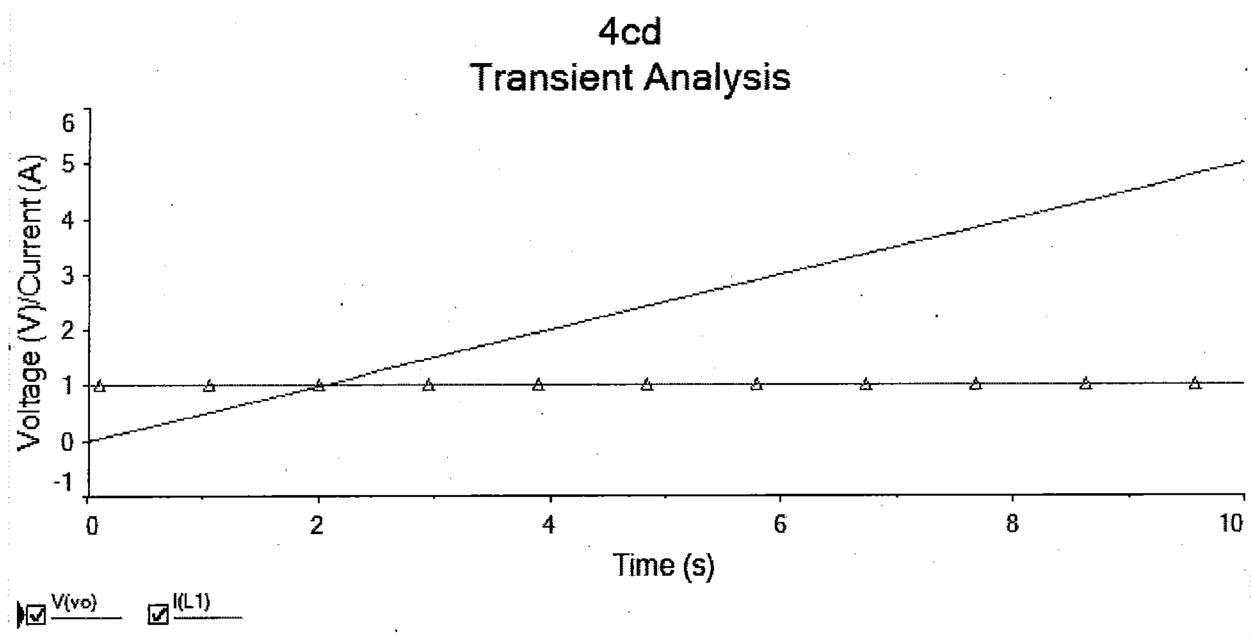
a) and b)





C) and d)


vo
V1
1 V
L1
2H


4cd
Transient Analysis
Voltage (V)/Current (A)
6
5
4
3
2
1
0
-1
0
2
4
6
8
10
Time (s)
V(vo)
I(L1)

~~Problem 4~~: Problem 5

a) The first step in finding the transfer function is to construct the s-domain equivalent circuit. By definition, the transfer function is the ratio of $\frac{V_o}{V_0}$, which can be computed from a single node-voltage equation. Summing the currents away from the upper node generates

$$\frac{V_O - V_0}{1000} + \frac{V_O}{250 + 0.05s} + \frac{V_O s}{10^6} = 0$$

Solve V_O,

$$V_O = \frac{1000(s + 5000)V_0}{s^2 + 6000s + 25 \times 10^6}$$

Hence the transfer function is

$$H(s) = \frac{V_o}{V_0} = \frac{1000(s + 5000)}{s^2 + 6000s + 25 \times 10^6}$$

The poles of $H(s)$ are the roots of the denominator polynomial. Therefore

$$-p_1 = -3000 - j4000$$
$$-p_2 = -3000 + j4000$$

The zeros of $H(s)$ are the roots of the numerator polynomial; thus

$$-z_1 = -5000$$

For the impulse response, let

$$V_o = \frac{1000(s + 5000)}{s^2 + 6000s + 25 \times 10^6} = \frac{K_1}{s + 3000 - j4000} + \frac{K_1^*}{s + 3000 + j4000}$$
$$K_1 = 500 - j250 = 559.017\angle -26.57°$$
$$K_1^* = 500 + j250 = 559.017\angle 26.57°$$

Test the solutions with the zero of $H(s)$

$$H(-5000) = \frac{500 - j250}{-5000 + 3000 - j4000} + \frac{500 + j250}{-5000 + 3000 + j4000} = 0$$

Then, transform back to time domain,

$$y(t) = h(t) = 1118e^{-3000t}\cos(4000t - 26.57°)u(t)$$

For the response of the input source, let

$$V_o = \frac{15000(s + 5000)}{s(s^2 + 6000s + 25 \times 10^6)} = \frac{K_2}{s} + \frac{K_3}{s + 3000 - j4000} + \frac{K_3^*}{s + 3000 + j4000}$$
$$K_2 = 3$$
$$K_3 = -1.5 - j0.75 = 1.6771\angle -153.44°$$
$$K_3^* = -1.5 + j0.75 = 1.6771\angle 153.44°$$

Test the root of zero

$$\frac{3}{-5000} + \frac{-1.5 - j0.75}{-5000 + 3000 - j4000} + \frac{-1.5 + j0.75}{-5000 + 3000 + j4000} = 0$$

Then the response due to the input is

$$y(t) = [3 + 3.35e^{-3000t}\cos(4000t - 153.44°)]u(t)$$

We can see the response includes a DC which is 3 V and a fast decaying cosine wave.

b) Multisim simulation result is shown below:

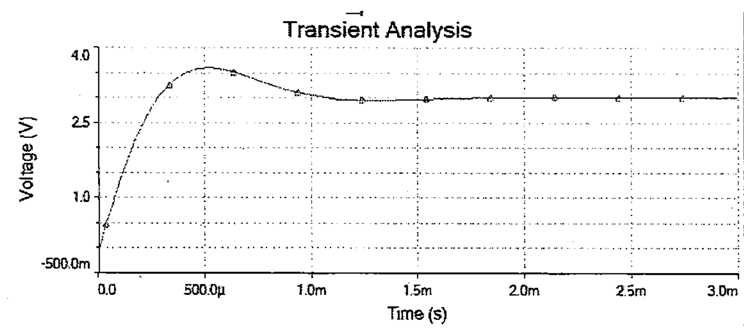


Figure 1 Transient response to v_{dc}

The steady state voltage is 3V, consistent with solution in part a).

~~Problem 5~~: Problem 6

Applying superposition, $v_o = v_{oDC} + v_{oAC}$, v_{oDC} is the response of the DC source v_{dc}, and v_{oAC} is the response of the AC source v_{ac}. Then in s-domain, we have

$$V_O = \frac{1000(s + 5000)(V_0 + V_1)}{s^2 + 6000s + 25 \times 10^6} = V_{OAC} + V_{ODC}$$

We have got V_{ODC} in the last problem and

$$V_{OAC} = \frac{1000(s + 5000)V_0}{s^2 + 6000s + 25 \times 10^6}$$

The frequency of the voltage source is 10^6 rad/s, hence we evaluate $H(s)$ at $H(j10^6)$:

$$H(j10^6) = \frac{1000(j10^6 + 5000)}{(j10^6)^2 + 6000(j10^6) + 25 \times 10^6} \approx 0.001\angle - 90^\circ$$

Then

$$v_{oac} = 0.005cos(10^6 t - 60^\circ)u(t)$$

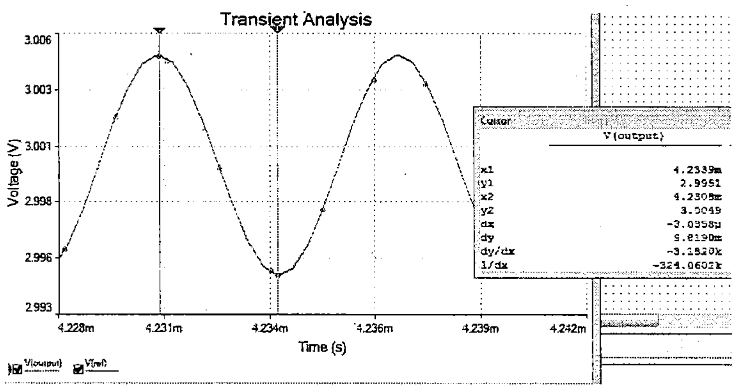
Then

$$v_o = v_{odc} + v_{oac} = [3 + 0.005cos(10^6 t - 60^\circ) + 1.6771e^{-3000t}cos(4000t - 153.44^\circ)]u(t)$$

The steady-state response is let time goes to infinity, thus the fast decaying cosine wave can be ignored, then

$$v_{o\_steady} = [3 + 0.005cos(10^6 t - 60^\circ)]u(t)$$

b) Steady state output simulated by Multisim is shown below:



Frequency $f = \frac{1}{2dx} = \frac{1}{2 \times 3.086 \times 10^{-6}} = 1.62 \times 10^{+5}$, $2\pi f \approx 10^6$

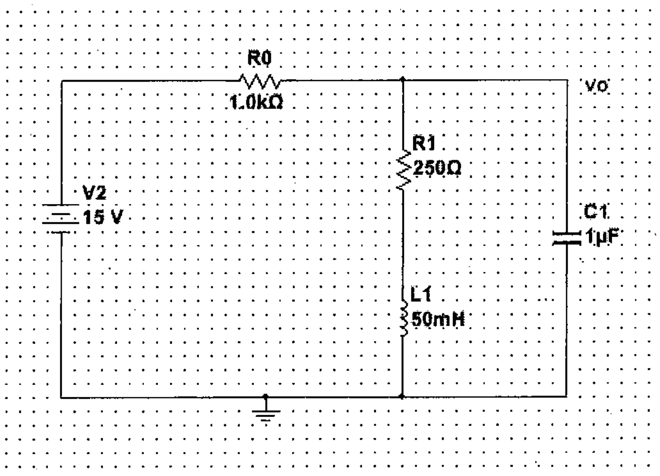
Central voltage level is (2.9951+3.0049/2≈3 V

Amplitude is dy/2=9.8mV/2≈5mV

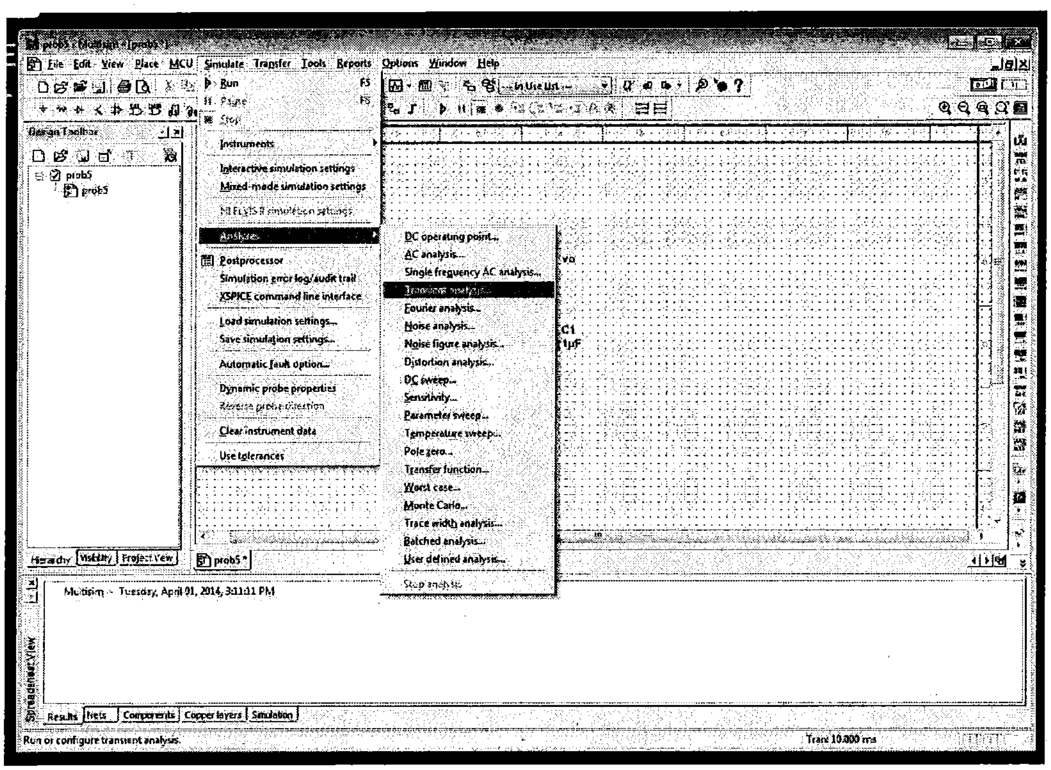
To verify the phase value, the steady output waveform in red is compared with a reference waveform of 3+0.005cos(10^6t-60°) in green. As can be seen, they are completely overlapped in steady state.

Homework #0
Problem 5. b) solution

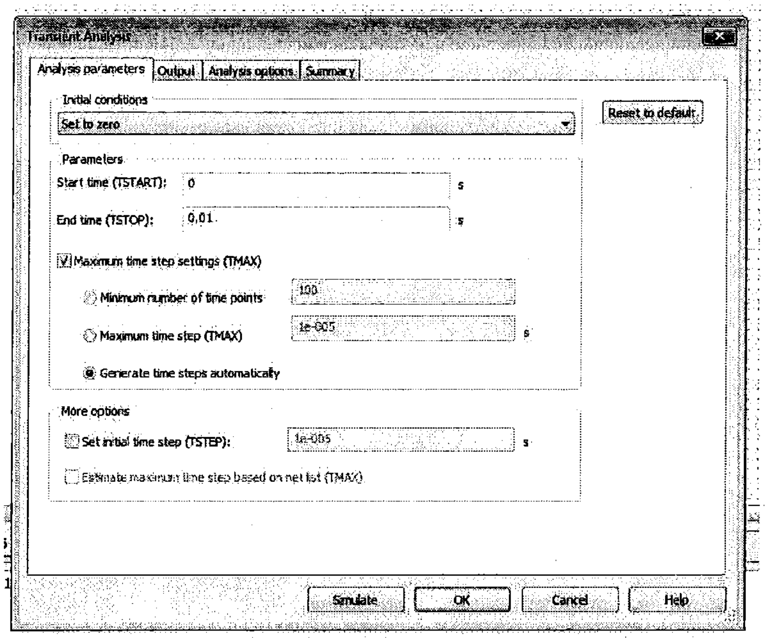
Using Multisim, put together the following circuit. I labeled the node v0 for convenience:



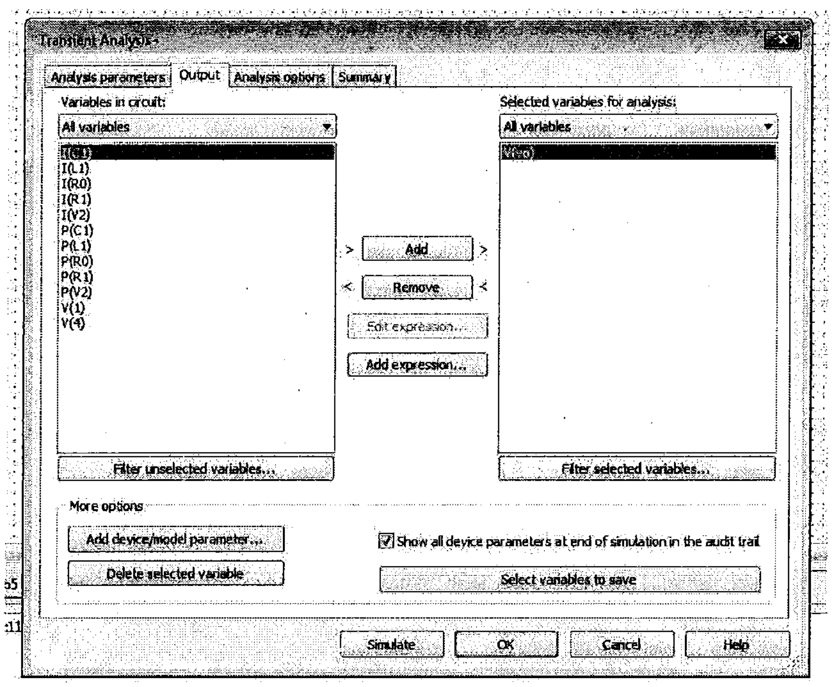
Next, run a Transient Analysis:



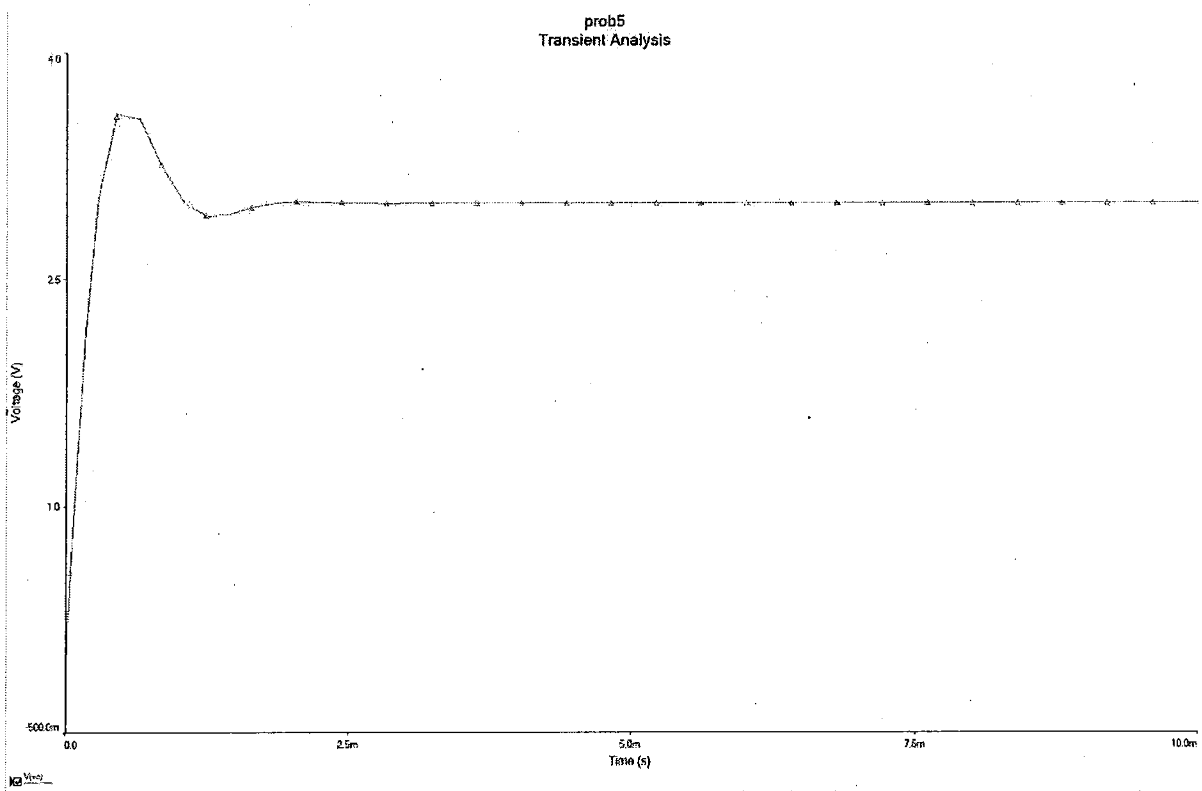
I set the stop time (TSTOP) to 10 mS, but you can pick whatever amount of time makes a good-looking plot. I kept everything else set to default:



Next, I went to the Output tab and selected V(vo) so the graph will display the output voltage we want for this problem:



Last, hit the "Simulate" button at the bottom of the Transient Analysis dialog, and a graph like this should appear showing the Transient Response at the node vo:



Homework #0
Problem 6. b) solution

Solution is similar to Problem 5. b), but notice that the sinusoidal voltage input for Multisim requires RMS voltage = Vamplitude/1.41 = 5/1.41 = 3.54 V, and 10^6 rad/sec frequency is converted to kHz.