Homework 1 Solutions

1 J&B P2.5(a)

$$R = \frac{\rho L}{A} = \frac{1.66\ \mu\Omega \cdot 2\sqrt{2}\ \text{cm}}{5 \cdot 10^{-8}\ cm^2} = 93.9\Omega$$

2 J&B P2.6

Intrinsic carrier concentration is given by

$$n_i^2 = BT^3 e^{-\frac{E_G}{kT}}$$

for germanium, $E_G = 0.66\ eV$, $B = 2.31 \times 10^{30}\ K^{-3}\text{cm}^{-6}$

(a) At 77 K, $n_i^2 = 2.63 \times 10^{-4}\ \text{cm}^{-3}$

(b) At 300 K, $n_i^2 = 2.27 \times 10^{13}\ \text{cm}^{-3}$

(a) At 500 K, $n_i^2 = 8.04 \times 10^{15}\ cm^{-3}$

3 J&B P2.16

Maximum hole current density is

$$j_p = qpv_p = 1.60 \times 10^{-19}\ \text{C} \cdot 10^{19}\ \text{cm}^{-3} \cdot 10^{7}\ \text{cm/s} = 1.60 \times 10^{7}\ \text{A/cm}^2$$

The maximum current is

$$I = j_p A = 1.60 \times 10^{7}\ \text{A/cm}^2 \cdot 2.5 \times 10^{-7}\ \text{cm}^2 = 4\ A$$

4 J&B P2.22

In has 3 outer electrons, P has 5 outer electrons. Ge has 4 outer electrons.

(a) Germanium has one more electron than indium. If a germanium atom replaces an indium atom, it can donate the extra electron for conduction. Thus it behaves as a donor impurity.

(b) Germanium has one less electron than phosphorus. If a germanium atom replaces a phosphorus atom, it can accept an electron from neighboring bonds, creating a free hole. Thus it behaves as an acceptor impurity.

5 J&B P2.27

$$N_A = 6 \times 10^{18}\ \text{boron atoms/cm}^3$$

(a) This is p-type silicon; holes are the majority carrier; electrons are the minority carrier.

(b,c) For this problem, we need the intrinsic carrier concentration at two different temperatures. The more precise formula for n_i in Eq. (2.1) will be used to compute this:

$$n_i = \sqrt{BT^3 e^{-\frac{E_G}{kT}}}$$

$$n_i(300K) = 1.0 \times 10^{10}\ \text{cm}^{-3}$$

$$n_i(200K) = 1.1 \times 10^{5}\ \text{cm}^{-3}$$

For both 300K and 200K, the majority carrier hole concentration is just equal to

$$N_A = 6 \times 10^{18}\ \text{cm}^{-3}$$

However, the minority carrier electron concentrations depends upon $n_i(T)$:

$$n_{300K} = \frac{\left(n_i(300K)\right)^2}{N_A} = 16.7\ \text{cm}^{-3}$$

$$n_{200K} = \frac{\left(n_i(200K)\right)^2}{N_A} = 2.02 \times 10^{-9}\ \text{cm}^{-3}$$

6 J&B P2.32

$$N_D > N_A$$

Then

$$n = \frac{(N_D - N_A) + \sqrt{(N_D - N_A)^2 + 4n_i^2}}{2} = 1.62 \times 10^{17}\ \text{cm}^{-3}$$

$$p = \frac{n_i^2}{n} = 6.17 \times 10^{16}\ \text{cm}^{-3}$$

We can see that

$$n = 1.62 \times 10^{17}\ \text{cm}^{-3} \neq N_D - N_A = 1 \times 10^{17}\ \text{cm}^{-3}$$

This is because

$$N_D - N_A \gg n_i$$

does **not** hold in this case ($N_D - N_A = n_i = 1 \times 10^{17}\ \text{cm}^{-3}$)

7 J&B P2.35

Indium is an acceptor impurity. The material is p-type.

At 300 K, the intrinsic carrier concentration for silicon is $n_i = 1.0 \times 10^{10}\ \text{cm}^{-3}$.

Acceptor doping concentration is $N_A = 8 \times 10^{19}\ \mathrm{cm}^{-3}$. Thus $N_A \gg n_i$, and we have

$$p = N_A = 8 \times 10^{19}\ \mathrm{cm}^{-3}$$

$$n = \frac{n_i^2}{p} = 1.25\ \mathrm{cm}^{-3}$$

Using total impurity concentration of $N_T = N_A = 8 \times 10^{19}\ \mathrm{cm}^{-3}$, we can extract the electron and hole mobilities by either reading them out from Figure 2.8, or plugging in the value of N_T in the approximated mobility equations:

$$\mu_n = 96\ \mathrm{cm}^2/(\mathrm{V} \cdot \mathrm{s})$$

$$\mu_p = 50\ \mathrm{cm}^2/(\mathrm{V} \cdot \mathrm{s})$$

Conductivity

$$\sigma = q(n\mu_n + p\mu_p)$$

Since

$$p \gg n$$

The electron part can be neglected, and thus

$$\sigma = qp\mu_p = 640\ (\Omega \cdot \mathrm{cm})^{-1}$$

Resistivity

$$\rho = \frac{1}{\sigma} = 1.56 \times 10^{-3}\ \Omega \cdot \mathrm{cm}$$

8 J&B P2.50

Diffusion current density of holes is

$$j_p^{diff} = -qD_p \frac{dp}{dx}$$

The concentration profile for holes is

$$p(x) = 10^5 + 10^{19} e^{\left(-\frac{x}{L_p}\right)}\ \mathrm{cm}^{-3}$$

Then

$$j_p^{diff} = 10^{19} q \frac{D_p}{L_p} e^{\left(-\frac{x}{L_p}\right)} cm^{-3} = 1.2 \times 10^5 e^{\left(-\frac{x}{2\times10^{-4}\ cm}\right)}\ \mathrm{A/cm}^2$$

At $x = 0$, diffusion current density is

$$j_p^{diff}(x=0) = 1.2 \times 10^5\ \mathrm{A/cm^2}$$

Diffusion current is

$$I_p^{diff}(x=0) = j_p^{diff}(x=0) \cdot A = 1.2 \times 10^{-2}\ \mathrm{A}$$

7 J&B P2.52

Note that the positive x direction is to the right, and the direction of the electric field is to the left, thus $E = -20\ \mathrm{V/cm}$. (The signs of $E, \frac{dn}{dx}, \frac{dp}{dx}, j$ are very important in this exercise!) The concentration profiles are linear, thus the gradients of concentrations are

$$\frac{dp}{dx} = \frac{\Delta p}{\Delta x} = -5 \times 10^{19}\ \mathrm{cm^{-4}}$$

$$\frac{dn}{dx} = \frac{\Delta n}{\Delta x} = -5 \times 10^{19}\ \mathrm{cm^{-4}}$$

Mobilities of electrons and holes are $\mu_n = 350\ \mathrm{cm^2/(V \cdot s)}$, $\mu_p = 150\ \mathrm{cm^2/(V \cdot s)}$. From Einstein's relationship

$$\frac{D}{\mu} = \frac{kT}{q} = V_T$$

The diffusivities of electrons and holes are (assuming room temperature, $V_T = 25.9\ mV$)

$$D_n = \mu_n V_T = 9.1\ \mathrm{cm^2/s}$$

$$D_p = \mu_p V_T = 3.9\ \mathrm{cm^2/s}$$

At $x = 0$, $n = 10^{16}\ \mathrm{cm^{-3}}$, $p = 1.01 \times 10^{18}\ \mathrm{cm^{-3}}$

Electron diffusion current density:

$$j_n^{diff} = -qD_n \frac{dn}{dx} = -72\ \mathrm{A/cm^2}$$

Hole diffusion current density:

$$j_p^{diff} = -qD_p \frac{dp}{dx} = 31\ \mathrm{A/cm^2}$$

Electron drift current density:

$$j_n^{drift} = qn\mu_n E = -11.2\ \mathrm{A/cm^2}$$

Hole drift current density:

$$j_p^{drift} = qp\mu_p E = -4.85 \times 10^2 \text{ A/cm}^2$$

Total current density:

$$j^{tot} = j_n^{diff} + j_p^{diff} + j_n^{drift} + j_p^{drift} = -5.4 \times 10^2 \text{ A/cm}^2$$

Since the profiles are linear, at $x = 1\ \mu$m (mid-point)

$$n = \frac{(10^{16} + 10^4)}{2} = 5 \times 10^{15} \text{ cm}^{-3}$$

$$p = \frac{(1.01 \times 10^{18} + 10^{18})}{2} = 1.005 \times 10^{18} \text{ cm}^{-3}$$

Electron and hole diffusion currents are the same as derived above:

$$j_n^{diff} = -qD_n \frac{dn}{dx} = -72 \text{ A/cm}^2$$

$$j_p^{diff} = -qD_p \frac{dp}{dx} = 31 \text{ A/cm}^2$$

Electron drift current density:

$$j_n^{drift} = qn\mu_n E = -5.6 \text{ A/cm}^2$$

Hole drift current density:

$$j_p^{drift} = qp\mu_p E = -4.8 \times 10^2 \text{ A/cm}^2$$