Homework 3 Solutions

1. J&B P3.83

When the Zener diode is regulating, the voltage across it will be $v_R = 9$ V. Thus, the voltage across the 15 $k\Omega$ resistor will be 30-9=21 V. The current flowing through that resistor is then

$$i = \frac{V}{R} = \frac{21 V}{15 k\Omega} = 1.4 \text{ mA}$$

The current flowing backwards through the Zener diode is equal to this current minus the load current. In order to keep the Zener diode in the regulating region (i.e. in the Zener breakdown region) the load current must not exceed the above value of 1.4 mA.

The minimum value of R_L that can be used is set by the above criteria. The minimum value of R_L thus be

$$R_{Lmin} = \frac{9 \text{ V}}{1.4 \text{ mA}} = 6.43 \text{ k}\Omega$$

2.J&B P3.77

Diodes are numbered from left to right.

a) Assume D₁ is ON, D₂ and D₃ are OFF.

$$I_{D1} = \frac{10 - 0.65 - (-5)}{(8.2 + 12 + 10) * 10^{3}} = 0.47 \, mA$$

$$V_{D2} = 5 - (10 - 0.65 - 8200 * I_{D1}) = -0.50 \, V$$

$$V_{D3} = 0 - 5 + 10000 I_{D1} = 0.23 \, V < 0.6 \, V$$

Given $I_s = 10 \text{ pA}$

$$I_{D1} = I_S \left[\exp\left(\frac{V_{D1}}{nV_T}\right) - 1 \right]$$

$$1 * 0.025 * \ln\left[\frac{0.47 * 10^{-3} - 10 * 10^{-12}}{10 * 10^{-12}}\right] = V_{D1}$$

$$V_{D1} = 0.44 V$$

Operating Points:

D1: (0.47 mA, 0.44V) D2: (0 A, -0.5V) D3: (0 A, 0.23 V)

b) Assume D_1 , D_2 and D_3 are all ON

$$I_{D1} = \frac{(10 - 0.65) - (5 - 0.65)}{3.3 * 10^{3}} = 1.51 \, mA > 0 \, (our \, assumption \, is \, right)$$

$$I_{6.8} = \frac{\left((5 - 0.65) - (0 - 0.65)\right)}{6.8 * 10^{3}} = 0.73 \, mA$$

$$I_{D2} = I_{6.8} - I_{D1} = -0.78 \, mA < 0 \, (our \, assumption \, is \, wrong)$$

$$I_{2.4} = \frac{(0 - 0.65) - (-5)}{2.4 * 10^{3}} = 1.8 \, mA$$

 $I_{D3} = I_{2.4} - I_{6.8} = 1.07 \, mA > 0 \, (our \, assumption \, is \, right)$

Assume D_1 , D_3 are ON, D_2 is OFF.

$$I_{D1} = \frac{10 - 0.65 - (-0.65)}{(3.3 + 6.8) * 10^{3}} = 0.99 \, mA$$

$$I_{D3} + I_{D1} = \frac{-0.65 - (-5)}{2.4 * 10^{3}}$$

$$I_{D3} = 0.823 \, mA$$

$$V_{D2} = 5 - (10 - 0.65 - 3300I_{D1}) = -1.08 \, V$$

$$I_{D1} = I_{S} \left[\exp\left(\frac{V_{D1}}{nV_{T}}\right) - 1 \right]$$

$$1 * 0.025 * \ln\left[\frac{0.99 * 10^{-3} - 10 * 10^{-12}}{10 * 10^{-12}}\right] = V_{D1}$$

$$V_{D1} = 0.46 \, V$$

$$I_{D3} = I_S \left[\exp\left(\frac{V_{D3}}{nV_T}\right) - 1 \right]$$

$$1 * 0.025 * \ln\left[\frac{0.82 * 10^{-3} - 10 * 10^{-12}}{10 * 10^{-12}}\right] = V_{D1}$$

$$V_{D3} = 0.45 V$$

Operating Points:

D1: (0.99 mA, 0.46V) D2: (0 mA, -1.08 V) D3: (0.82 mA, 0.45 V)

c) Assume D₁, D₂ are OFF, D₃ is ON.

$$I_{D3} = \frac{11.35 - (-5)}{4.7 * 3 * 10^{3}} = 1.16 \, mA$$

$$V_{D1} = 0 - (-5 + 4700I_{D3}) = -0.45 \, V$$

$$V_{D2} = 5 - (11.35 - 4700I_{D3}) = -0.9 \, V$$

$$I_{D3} = I_{S} \left[\exp\left(\frac{V_{D3}}{nV_{T}}\right) - 1 \right]$$

$$1 * 0.025 * \ln\left[\frac{1.16 * 10^{-3} - 10 * 10^{-12}}{10 * 10^{-12}}\right] = V_{D1}$$

$$V_{D3} = 0.46 \, V$$

Operating Points:

D1: (0 mA, -0.45V) D2: (0 mA, -0.9V) D3: (1.16 mA, 0.46 V)

d) Assume D_1 , D_2 , D_3 are ON.

$$I_{D1} = \frac{-0.65 - (-9.4)}{8.2 * 10^{3}} = 1.07 \, mA$$

$$I_{12K} = \frac{-0.65 - 1.4}{12 * 10^{3}} = -0.17 \, mA$$

$$I_{D2} = I_{D1} + I_{12K} = 0.96 \, mA$$

$$I_{10k} = \frac{1.4 - (-5)}{10 * 10^{3}} = 0.64 \, mA$$

$$I_{D3} = I_{10k} - I_{12k} = 0.81 \, mA$$

$$I_{D1} = I_{S} \left[\exp\left(\frac{V_{D1}}{nV_{T}}\right) - 1 \right]$$

$$1 * 0.025 * \ln\left[\frac{1.07 * 10^{-3} - 10 * 10^{-12}}{10 * 10^{-12}}\right] = V_{D1}$$

$$V_{D1} = 0.46 V$$

$$I_{D2} = I_S \left[\exp\left(\frac{V_{D2}}{nV_T}\right) - 1 \right]$$

$$1 * 0.025 * \ln\left[\frac{0.96 * 10^{-3} - 10 * 10^{-12}}{10 * 10^{-12}}\right] = V_{D1}$$

$$V_{D2} = 0.46 V$$

$$I_{D3} = I_S \left[\exp\left(\frac{V_{D3}}{nV_T}\right) - 1 \right]$$

$$1 * 0.025 * \ln\left[\frac{0.81 * 10^{-3} - 10 * 10^{-12}}{10 * 10^{-12}}\right] = V_{D1}$$

$$V_{D3} = 0.45 V$$

Operating Points:

D1: (1.07 mA, 0.46V) D2: (0.96 mA, 0.46V) D3: (0.81 mA, 0.45 V)

4. J&B P3.87

Nominal value for current is given by:

$$I_z = \frac{V_S - V_Z}{R_S} - \frac{V_Z}{R_L}$$

substituting

$$\frac{50 - 15}{150} - \frac{15}{100} = 83.3 \ mA$$

Nominal Power is given by:

$$P_z = I_z * V_z$$

= 83.3 * 10⁻³ * 15
= 1.25 W

Worst cases:

For I_z and P_z to be maximum, V_z and R_s should be 90% of nominal values and V_s and R_L should be 110% of nominal values (because tolerance is +/-10%):

$$I_{zmax} = \frac{V_s(1.1) - V_z(0.9)}{R_s(0.9)} - \frac{V_z(0.9)}{R_L(1.1)}$$
$$= 184.6 \text{ mA}$$
$$P_{zmax} = I_{zmax} * V_z(0.9)$$
$$= 24.9 \text{ W}$$

5. J&B P3.95

(a) The circuit corresponds to a half-wave rectifier with negative output voltage. Peak voltage is $V_P = \sqrt{2}V_{RMS} = 8.91 \text{ V}$. DC output voltage is then

$$V_{dc} = -(V_P - V_{on}) = -7.91 V$$

(b) Total charge dissipated through the load resistor R in a cycle T is

$$\Delta Q = I_{dc}T = \frac{V_{dc}}{R}T$$

This causes a voltage drop on the capacitor, which is just the ripple voltage V_r

$$\Delta Q = C\Delta V = CV_r$$

$$C = \frac{|V_{dc}|}{RV_r}T = 1.055 F$$

(c) The maximum reverse bias occurs when v_I reaches the maximum value V_P . The peak inverse voltage rating is then

$$PIV \ge V_P - V_{dc} = 2V_P - V_{on} \approx 2V_P = 17.8 V$$

(d) Initial surge current:

$$i_d(t) \approx i_c(t) = C \frac{d}{dt} (V_P sin\omega t) = \omega C V_P cos\omega t$$

The peak value of the surge current is given by

$$I_{SC} = \omega CV_P = 3.54 \times 10^3 A$$

(e) The conduction interval of the circuit is

$$\Delta T = \frac{1}{\omega} \sqrt{\frac{2V_r}{V_P}} = 0.628 \, ms$$

The charge lost from the filter capacitance during the complete period is

$$Q = I_{dc}T$$

This should be equal to the charge supplied through the diode during the short conduction (ΔT). Applying the triangular approximation to the diode current pulse, we get:

$$Q = I_P \frac{\Delta T}{2}$$

Which gives

$$I_P = I_{dc} \frac{2T}{\Delta T} = 840 A$$

6. J&B P3.104

(a)

$$V_{dc} = V_{on} - V_P = -24.5 V$$

(b)

$$C_{min} = \frac{V_P - V_{on}}{V_{rmax}} \frac{T}{2R} = \frac{V_P - V_{on}}{f V_{rmax}} \frac{1}{2R} = 1.63 F$$

(c)

$$PIV = 2V_P = 50.9 V$$

(d)

$$I_{SC} = \omega C V_P = 15.6 \times 10^3 A$$

(e)

$$I_P = I_{dc} \frac{T}{\Delta T} = \frac{V_{dc}}{R} \frac{1}{f \frac{1}{\omega} \sqrt{\frac{2V_r}{V_P}}} = 2.2 \times 10^3 A$$

7. J&B P3.110

Refer to Figure 3.63 or Figure 3.66 from J&B Page 125 for circuit.

(a) Note : V_{dc} will be positive if circuit is similar to Figure 3.63 and V_{dc} will be negative if circuit is similar to Figure 3.66.

$$V_{dc} = 2V_{on} - V_P = -23.5 V \text{ or } 23.5 V$$

(b)

$$C_{min} = \frac{V_P - 2V_{on}}{V_{rmax}} \frac{T}{2R} = \frac{V_P - V_{on}}{fV_{rmax}} \frac{1}{2R} = 1.56 F$$

(c)

$$PIV = V_P = 25.45 V$$

(d)

$$I_{SC} = \omega CV_P = 15.01 \times 10^3 A$$

(e)

$$I_P = I_{dc} \frac{T}{\Delta T} = \frac{V_{dc}}{R} \frac{1}{f \frac{1}{\omega} \sqrt{\frac{2V_r}{V_P}}} = 2.1 \times 10^3 A$$

The bridge rectifier eliminated the need of a center tapped transformer. Also we can see that the value of capacitance has been reduced from 1.63F to 1.56F. The PIV rating has halved which is important in high voltage circuits.

8. J&B P3.119

(a)

$$i_D(t) = \frac{v_1(t) - v_D(t)}{1.0 \, k\Omega}$$
$$i_D(0^+) = i_{Dmax} = \frac{5 \, V}{1.0 \, k\Omega} = 5 \, mA$$

(b) I use $V_{on} = 0.6 \, V$, $V_{on} = 0.7 \, V$ is also good, just show how to solve this problem

$$I_F = \frac{5 - 0.6}{1.0 \, k\Omega} = 4.4 \, mA$$

$$I_R = \frac{-3 - 0.6}{1.0 \, k\Omega} = -3.6 \, mA$$

$$\tau_S = \tau_T ln \left[1 - \frac{I_F}{I_R} \right] = 5.59 \, ns$$

9. J&B P3.123

Given:

Output of diode

$$I_c = 1 - 10^{-15} [\exp(40V_c) - 1] A$$

Short circuit current is I_c when $V_c = 0$

$$I_{sc} = 1 - 10^{-15} [e^0 - 1] = 1 A$$

Open circuit voltage is V_c when $I_c = 0$

$$\frac{1}{40} \ln \left[1 + \frac{1}{10^{-15}} \right] = V_{oc}$$

$$V_{oc} = 0.86 V$$

Power

$$P = I_c * V_c = V_c [1 - 10^{-15} (\exp(40V_c) - 1)]$$

 P_{max} is when first derivative of P goes to 0

$$\frac{dP}{dV_c} = 1 - 10^{-15} [\exp(40Vc) - 1] - 40 * 10^{-15} V_c \exp(40V_c) = 0$$

Solving the equation we get $V_c\!=\!0.77~V$, by substituting we get $I_c\!=\!0.97~A$ and $P_{max}\!=\!7.53~W.$

Q point corresponding to P_{max} is (0.97 A, 0.77 V).