Homework 6 Solutions

1. J&B P6.39

(a) Resistor load Inverter, $V_H = V_{DD} = 2.5\ V$. When $v_I = V_H$, $v_O = V_L$.

For M_S, in triode region, equate the drain current and solve for the equation

$$\frac{V_{DD} - V_L}{R_D} = K_n' \left(\frac{W}{L}\right)_S \left(V_H - V_{TN} - \frac{1}{2}V_L\right) V_L$$

$$V_L = 0.022\ V$$

$$I_D = 12.4\ \mu A$$

$$P_D = V_{DD} I_D = 30.98\ \mu W$$

(b) Change $R = 400\ k\Omega$ and $\left(\frac{W}{L}\right)_S = 6$, applied same method used in (a)

$$V_L = 0.0055\ V$$

$$I_D = 6.2\ \mu A$$

$$P_D = V_{DD} I_D = 15.59\ \mu W$$

2. J&B P6.50

(a) $I_D = \frac{P_D}{V_{DD}}$ should be same (80 μA), $V_H = 2.5\ V$, $V_L = 0.5\ V$, in triode region

$$I_D = K_n' \left(\frac{W}{L}\right)_S \left(V_H - V_{TN} - \frac{1}{2}V_L\right) V_L$$

$$\left(\frac{W}{L}\right)_S = \frac{0.97}{1}$$

$$R = \frac{V_{DD} - V_L}{I_D} = 25\ k\Omega$$

(b)

$$K_n R = K_n' \left(\frac{W}{L}\right)_S R = 2.43$$

$$NM_H = V_{DD} - V_{TN} + \frac{1}{2K_n R} - 1.63\sqrt{\frac{V_{DD}}{K_n R}} = 0.452\ V$$

$$NM_L = V_{TN} + \frac{1}{K_n R} - \sqrt{\frac{2V_{DD}}{3K_n R}} = 0.183\ V$$

3.

(a) J&B P6.59

When $v_I = V_L$, the capacitor parallel to M_S gets charged until M_L is off, in such case, $V_{GSL} = V_{TNL}$; because $\gamma = 0$, then no body effect, $V_{TNL} = V_{TO}$, $V_{GSL} = V_{TO} = V_{DD} - V_H$, then

$$V_H = V_{DD} - V_{TO} = 2.7\ V$$

When $v_I = V_H$, $v_O = V_L$, current flows through both transistors. $V_{GSL} = V_{DD} - V_L$, $V_{TNL} = V_{TO}$, $V_{GSS} = V_H$, $V_{DSS} = V_L$. Then M_L is in saturation, and M_S is in triode. Equate the drain current and solve for equation

$$\frac{K_n'}{2}\left(\frac{W}{L}\right)_L (V_{DD} - V_L - V_{TO})^2 = K_n'\left(\frac{W}{L}\right)_S \left(V_H - V_{TO} - \frac{1}{2}V_L\right) V_L$$

$$V_L = 0.196\ V$$

$$I_D = 156.8\ \mu A$$

Power dissipation for $v_O = V_L$ is

$$P_D = V_{DD} I_{DD} = 517\ \mu W$$

(b) J&B P6.60

Similar to part (a), except that

$$V_{TNL} = V_{TO} + \gamma\left(\sqrt{V_{SB} + 2\varphi_F} - \sqrt{2\varphi_F}\right)$$

Because $V_{SB} = V_H$

$$V_{DD} - V_H = V_{TO} + \gamma\left(\sqrt{V_H + 2\varphi_F} - \sqrt{2\varphi_F}\right)$$

$$V_H = 2.17\ V$$

To find V_L, solve the equation of

$$K_n'\left(\frac{W}{L}\right)_S \left(V_H - V_{TO} - \frac{1}{2}V_L\right) V_L = \frac{K_n'}{2}\left(\frac{W}{L}\right)_L \left(V_{DD} - V_L - V_{TO} - \gamma\left(\sqrt{V_L + 2\varphi_F} - \sqrt{2\varphi_F}\right)\right)^2$$

$$V_L = 0.243\ V$$

$$I_D = 140.5\ \mu A$$

Power dissipation for $v_o = V_L$ is

$$P_D = V_{DD} I_{DD} = 464\ \mu W$$

4. J&B P6.76

$V_{DD} = 2.5\,V$, $I_{DD} = 80\,\mu A$, $V_L = 0.2\,V$, $V_H = 2.5\,V$. When $v_I = V_H$, $v_O = V_L$, M_L is in triode, and M_S is in triode.

For M_L, $V_{DSL} = V_{DD} - V_L = 2.3\,V$, $V_{GSL} = V_{GG} - V_L = 3.8\,V$. Also $V_{TNL} = V_{TO} + \gamma\left(\sqrt{V_L + 2\varphi_F} - \sqrt{2\varphi_F}\right) = 0.66\,V$.

$$I_D = K_n'\left(\frac{W}{L}\right)_L\left(V_{GSL} - V_{TNL} - \frac{1}{2}V_{DSL}\right)V_{DSL}$$

$$\left(\frac{W}{L}\right)_L = \frac{5.72}{1}$$

For M_S, $V_{TNS} = V_{TO} = 0.6\,V$.

$$I_D = K_n'\left(\frac{W}{L}\right)_S\left(V_H - V_{TNS} - \frac{1}{2}V_L\right)V_L$$

$$\left(\frac{W}{L}\right)_S = \frac{2.22}{1}$$

5. J&B P6.84

$V_H = V_{DD}$ still holds as long as the load device is depletion mode. $V_{SB} = V_H = V_{DD} = 2.5\,V$.

$$V_{TNL} = V_{TO} + \gamma\left(\sqrt{V_{SB} + 2\varphi_F} - \sqrt{2\varphi_F}\right) \le 0$$

$$\gamma \le 1.014\,\sqrt{V}$$

Then the largest γ is $1.014\,\sqrt{V}$.

6. J&B P6.87

The driver device does not have any body bias, but the load device does have a body bias equal to the output voltage

$$I_D = \frac{P_D}{V_{DD}} = 75.8\,\mu A$$

Flowing through both device when the output is LOW.

$$V_{TNL} = V_{TO} + \gamma\left(\sqrt{V_L + 2\varphi_F} - \sqrt{2\varphi_F}\right) = -1.94\,V$$

The load of M_L is in saturation, as $V_{DSL} = 3.1\,V > V_{GSL} - V_{TNL} = 1.94\,V$. Then,

$$I_{DL} = \frac{K_n'}{2}\left(\frac{W}{L}\right)_L {V_{TNL}}^2$$

$$\left(\frac{W}{L}\right)_L = 0.805$$

When $v_I = V_H = 3.3\ V, v_O = 0.2\ V, M_S$ is in triode region.

$$I_D = K_n'\left(\frac{W}{L}\right)_S\left(V_I - V_{TO} - \frac{1}{2}V_L\right)V_L$$

$$\left(\frac{W}{L}\right)_S = 2.914$$

(b) SPICE model

SPICE code is:

```
* Depletion load inverter *

.model Mload nmos(level=1 vt0=-2 gamma=0.5 PHI=0.6  KP=100e-6 )

.model Ms nmos(level=1 vt0=0.6 KP=100e-6 )

* Build circuit

Vdd 3 0 3.3

Vin 1 0 3.3

Cload 2 0 5p

M1 3 2 2 0 Mload L=1u W=0.805u

M2 2 1 0 0 Ms L=1u W=2.912u


.OP

.END
```

Operation point of every node can be found in a file with suffix .ic0.

Input voltage is 3.3V, simulated output voltage on node 2 is V(2)=0.2V=V_L. Solution for a) is correct.

7. J&B P6.95

$$I_D = \frac{P_D}{V_{DD}} = 55.6\ \mu A$$

When $v_I = V_H = 1.8\,V$, $v_O = V_L = 0.2\,V$, M_L is in saturation region, M_L is in triode region.

For M_L, $V_{GSL} = -V_{DD}$, $V_{TPL} = -0.5\,V$

$$I_D = \frac{K_p'}{2}\left(\frac{W}{L}\right)_L (V_{GSL} - V_{TPL})^2$$

$$\left(\frac{W}{L}\right)_L = 1.64$$

For M_S,

$$I_D = K_n'\left(\frac{W}{L}\right)_S \left(V_H - V_{TN} - \frac{1}{2}V_L\right)V_L$$

$$\left(\frac{W}{L}\right)_S = 2.31$$

(b)

$V_{TN} = -0.5\,V$ and $V_{Tp} = 0.5\,V$.

$$K_R = \frac{K_S}{K_L} = 3.52$$

$$V_{IL} = V_{TN} + \frac{V_{DD} + V_{TP}}{\sqrt{{K_R}^2 + K_R}} = 0.826\,V$$

$$V_{OH} = V_{DD} - (V_{DD} + V_{TP})\left(1 - \sqrt{\frac{K_R}{K_R + 1}}\right) = 1.647\,V$$

$$V_{IH} = V_{TN} + \frac{2(V_{DD} + V_{TP})}{\sqrt{3K_R}} = 1.3\,V$$

$$V_{OL} = \frac{V_{DD} + V_{TP}}{\sqrt{3K_R}} = 0.4\,V$$

$$NM_H = V_{OH} - V_{IH} = 0.347\,V$$

$$NM_L = V_{IL} - V_{OL} = 0.426\,V$$

8. J&B P6.101

The load device stays the same with $W/L = 1.81/1$. Since there are 4 driver devices in series, their W/L ratios must be increased by a factor of 4 so that the same value of V_{OL} is achieved when all 4 are turned on: $W/L = 4 \times 2.22/1 = 8.88/1$. This does not account for any body bias effects, but the design is still adequate for most purposes with these values as shown.