A.7 DERIVATION OF THE v–i CHARACTERISTIC OF THE PN JUNCTION DIODE

In this section, the solutions of the preceding section are used to derive the v–i characteristic of the *pn* junction diode. A qualitative discussion of the diode was given in Sections 3.3.2 and 3.3.3. The reader is urged to review these sections before proceeding with the present discussion. The representative geometry of a simple *pn* junction is depicted in Fig. A.12. The geometry shown in Fig. A.12 is a one-dimensional representation of the more realistic junction structure shown in Fig. A.13. The diagram of Fig. A.12 represents the core region (shaded in Fig. A.13), where most of the junction current flows.

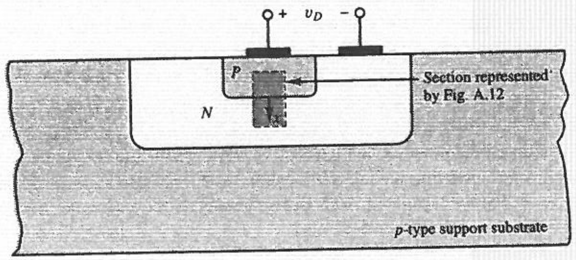


Figure A.13 Portion of an actual *pn* junction modeled by the structure of Fig. A.12.

A.7.1 Boltzmann Relations

The first step in the derivation of the diode v–i equation is the identification of the injection mechanism that operates across the diode's depletion region. As we shall soon see, this injection mechanism is related to the applied diode voltage. With no external voltage applied, the drift and diffusion currents of the holes and electrons must balance so that the net diode current is zero. Under such conditions, a built-in electric field develops in the depletion region, as discussed in Chapter 3. The magnitude of this field can be found from either the hole or the electron transport equation by setting the current density to zero. The transport equation for holes with $J_h = 0$, for example, becomes

$$J_h = qp\mu_h E - qD_h\frac{dp}{dx} = 0 \tag{A.37}$$

This equation can be solved for E, yielding

$$E = \frac{D_h}{\mu_h}\frac{1}{p}\frac{dp}{dx} \equiv \frac{kT}{q}\frac{1}{p}\frac{dp}{dx} \tag{A.38}$$

where the Einstein relation $kT/q = D_h/\mu_h$ has been used in writing the right-hand side of the equation.

The electric field E, which extends across the depletion region, creates a drift current of holes that exactly balances the corresponding hole diffusion current. A similar drift current balances the diffusion of electrons across the depletion region. In a *pn* junction with no externally applied voltage, the drift and diffusion components of the hole and electron currents must sum to zero separately. This requirement is called the principle of *detailed balance.*

By definition, an electric field is equal to the negative gradient of potential. For the junction of Fig. A.12, the potential varies only with x, so that

$$E = -\frac{dV(x)}{dx} \tag{A.39}$$

Applying this definition of E to Eq. (A.38) results in

$$-\frac{kT}{q}\frac{1}{p}\frac{dp}{dx} = \frac{dV(x)}{dx} \tag{A.40}$$

Multiplying both sides of Eq. (A.40) by dx results in

$$-\frac{kT}{q}\frac{dp}{p} = dV(x) \tag{A.41}$$

Equation (A.41) can be integrated from the p side of the depletion region to the n side (i.e., from $x = 0^-$ to $x = 0^+$):

$$\int_{p(0^-)}^{p(0^+)} -\frac{kT}{q}\frac{dp}{p} = \int_{V(0^-)}^{V(0^+)} dV(x) \tag{A.42}$$

where the depletion region is assumed to have negligible thickness. In this case, $V(0^-)$ and $V(0^+)$ represent the values of $V(x)$ on the p and n sides of the depletion region, respectively. Performing the indicated integration yields

$$\frac{kT}{q}\ln\frac{p(0^-)}{p(0^+)} = V(0^+) - V(0^-) \triangleq \Psi_o \tag{A.43}$$

The voltage Ψ_o, defined as $V(0^+) - V(0^-)$, is called the *built-in voltage.* It is equal to the potential difference across the depletion region, as defined from the n side to the p side. The built-in field associated with Ψ_o extends from the positive donor ions on the n side of the depletion region to the negative acceptor ions on the p side of depletion region (see Figs. 3.16 and 3.17).

With no external voltage applied, the boundary conditions $p(0^-)$ and $p(0^+)$ become just the equilibrium hole concentrations on the p and n sides of the junction, respectively. Under these conditions, Eq. (A.43) can be expressed in the form

$$\Psi_o = \frac{kT}{q}\ln\frac{p_{po}}{p_{no}} \tag{A.44}$$

In this latter equation, a second subscript, referring to either the p side or the n side of the junction, has been added to the symbol for p_o. Specifically, p_{po} refers to the equilibrium hole concentration on the p side of the junction (where holes are the majority carrier), and p_{no} refers to the equilibrium hole concentration on the n side of the junction (where holes are the minority carrier). Equation (A.44), called the *Boltzmann relation,* can be inverted to express the carrier concentrations at the edges of the depletion region in terms of the built-in voltage:

$$p_{no} = p_{po}e^{-\Psi_o/V_T} \tag{A.45}$$

The quantity $V_T = kT/q$ again defines the *thermal voltage.*

Note that the built-in voltage Ψ_o, as given by Eq. (A.44), depends only on the carrier concentrations p_{po} and p_{no} at the edges of the depletion region. It does not depend on the general distribution $p(x)$. The Boltzmann relation is generally applicable to hole concentrations measured at any two points in the diode; in this case, it has been specifically applied to the two sides of the depletion region.

A parallel analysis yields the corresponding Boltzmann relation for electrons. Specifically, for the case $J_e = 0$, the transport equation for electrons becomes

$$J_e = qn\mu_e E + qD_e\frac{dn}{dx} = 0 \tag{A.46}$$

or

$$E = -\frac{D_e}{\mu_e}\frac{1}{n}\frac{dn}{dx} = -\frac{kT}{q}\frac{1}{n}\frac{dn}{dx} \tag{A.47}$$

This electric field is the same one described by Eq. (A.38). Applying the relation $E = -dV/dx$ to Eq. (A.47) yields

$$\frac{kT}{q}\frac{dn}{n} = dV(x) \tag{A.48}$$

Equation (A.48) can be integrated from $x = 0^+$ to $x = 0^-$ (again from the n side of the junction to the p side), to yield

$$\Psi_o \triangleq V(0^+) - V(0^-) = \frac{kT}{q}\ln\frac{n_{no}}{n_{po}} \tag{A.49}$$

This second Boltzmann relation also can be expressed in the form

$$n_{po} = n_{no}e^{-\Psi_o/V_T} \tag{A.50}$$

The beauty of the Boltzmann relations (A.45) and (A.50) is that they also apply to the diode even when an external voltage is applied. The Boltzmann relations thus can be used to relate the carrier concentrations at either edge of the depletion region to an externally applied voltage. This concept is explored in the next section.

A.7.2 Carrier Injection Mechanism

The depletion region of a pn junction is void of carriers, hence it behaves like a minimally conductive region compared to the rest of the semiconductor material in the diode. Consequently, when an external voltage v_D is applied to the diode, it appears mainly across the depletion region, where it becomes superimposed on the built-in voltage. When the applied voltage is of such a polarity that the field it produces counteracts the built-in field, the diode becomes forward biased. The reduction in the net field upsets the zero-current balance of the junction and allows carriers to diffuse across the depletion region. The flow of holes under this mechanism, which constitutes an injection of holes into the n side of the diode, can be quantified by adding an external voltage term to the Boltzmann relation for holes. The substitution of $(\Psi_o - v_D)$ for Ψ_o in Eq. (A.45) yields the total hole concentration p_n at the $x = 0^+$ side of the depletion region:

$$p_n(0^+) = p_p(0^-)e^{(-\Psi_o+v_D)/V_T} \tag{A.51}$$

A similar consideration involving the Boltzmann relation for electrons yields the total electron concentration n_p measured at the $x = 0^-$ side of the depletion region:

$$n_p(0^-) = n_n(0^+)e^{(-\Psi_o+v_D)/V_T} \tag{A.52}$$

The holes are the majority carriers on the p side of the diode. If the low-level injection condition is met, $p_p(0^-)$ will be approximately equal to p_{po}. Equation (A.51) therefore may be expressed as

$$p_n(0^+) = p_{po}e^{-\Psi_o/V_T}e^{v_D/V_T} \tag{A.53}$$

The Boltzmann relation (A.35) can be used to further transform Eq. (A.53) into

$$p_n(0^+) = p_{no}e^{v_D/V_T} \tag{A.54}$$

Similarly, Eq. (A.52) for electrons becomes

$$n_p(0^-) = n_{no}e^{-\Psi_o/V_T}e^{v_D/V_T} \tag{A.55}$$

which can be transformed using the Boltzmann relation (A.50) into

$$n_p(0^-) = n_{po}e^{v_D/V_T} \tag{A.56}$$

Equations (A.54) and (A.56) relate the injected carrier concentration boundary conditions $p_n(0^+)$ and $n_p(0^-)$ to the applied diode voltage v_D.

A.7.3 Diode Current Components

In a forward-biased diode, holes and electrons are injected in opposite directions across the depletion region by the externally applied voltage. This injection results in exponential carrier gradients on either side of the diode, subject to the carrier boundary conditions $n(0^-)$ and $p(0^+)$ given by Eqs. (A.56) and (A.54). As these equations show, the boundary conditions are dependent on the applied diode voltage v_D. Equations (A.54) and (A.56) express the values of the *total* minority concentrations $p_n(0^+)$ and $n_p(0^-)$ at the edges of the depletion region. These quantities can be transformed into boundary conditions for the *excess* concentrations by subtracting the equilibrium concentrations p_{no} and n_{po}:

$$p'_n(0^+) = p_n(0^+) - p_{no} = p_{no}(e^{v_D/V_T} - 1) \tag{A.57}$$

and

$$n'_p(0^-) = n_p(0^-) - n_{po} = n_{po}(e^{v_D/V_T} - 1) \tag{A.58}$$

Equations (A.57) and (A.58) can be combined with the expressions (A.33) and (A.35) derived in Section A.6 to find the diode current flow under forward-biased conditions. Specifically, the distribution of excess injected holes on the n-side of the diode can be expressed as a function of position by

$$p'_n(x) = p'_n(0^+)e^{-x/L_h} \tag{A.59}$$

where $p'_n(0^+)$ is given by Eq. (A.57), and $L_h = \sqrt{D_h\tau_h}$. As was done in Section A.6, the diffusion current density associated with this distribution can be found by substituting it into the transport equation for holes:

$$J_{h\text{-diff}} = -qD_h\frac{dp_n(x)}{dx} = \frac{qD_h}{L_h}p_{no}(e^{v_D/V_T} - 1)e^{-x/L_h} \tag{A.60}$$

Equation (A.60) is approximately equal to the total hole current on the n side of the diode, because the drift-current component of the minority holes is negligible in comparison.

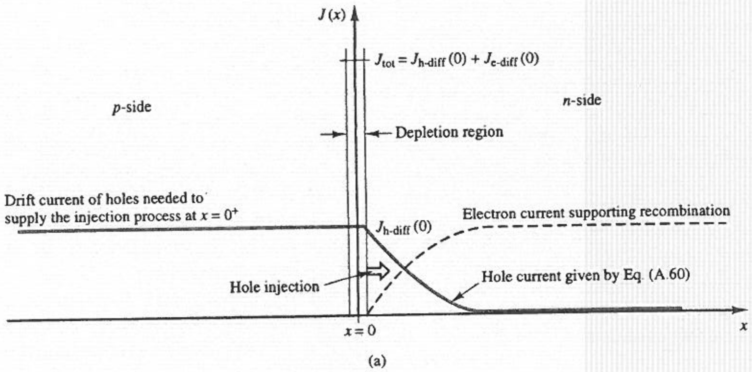
The electron current density injected from the n side of the diode to the p side can be found in a similar manner. The excess electron distribution becomes

$$n'_p(x) = n'_p(0^-)e^{x/L_e} \tag{A.61}$$

where $n'_p(0^-)$ is given by Eq. (A.58), and $L_e = \sqrt{D_e\tau_e}$. The exponent in Eq. (A.61) has a positive coefficient because the distribution decays in the negative x-direction defined in Fig. A.12. Applying Eq. (A.61) to the transport equation for electrons yields the diffusion component of the electron current density on the p side of the diode:

$$J_{e\text{-diff}} = qD_e\frac{dn_p}{dx}(x) = \frac{qD_e}{L_e}n_{po}(e^{v_D/V_T} - 1)e^{x/L_e} \tag{A.62}$$

Equations (A.60) and (A.62) describe the minority-carrier diffusion-current densities on either side of the diode. These expressions are plotted in Figs. A.14(a) and A.14(b). The majority-carrier current densities on each side, also plotted in Fig. A.14, are composed of two components. The first component consists of the majority carriers that flow from the outer ends of the diode to recombine with the injected minority carriers flowing from the depletion region. The second component consists of the majority carriers destined to be injected across the depletion region. At any point along the length of the diode, which has a constant cross-sectional area, KCL must be satisfied; hence the sum of all the current-density components must be independent of x.



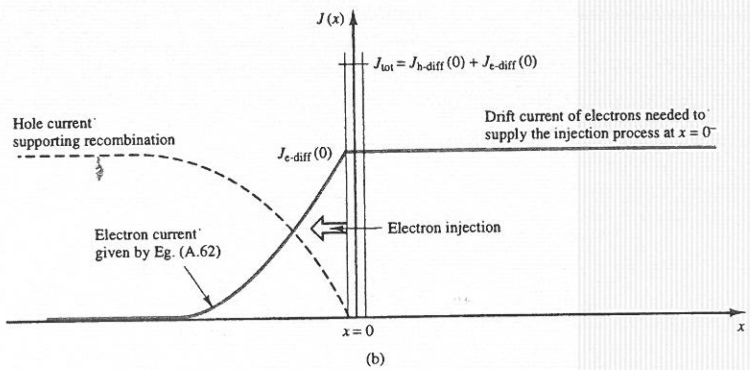


Figure A.14 Current components in a forward-biased *pn* junction diode: (a) current flow associated with hole injection; (b) current flow associated with electron injection.

The total current density in the diode is equal to the sum of the hole and electron currents at any position x. It is easiest to evaluate the total current at the depletion region, where the majority-carrier components associated with recombination are equal to zero. Only those carriers associated with injection are nonzero at $x = 0$. The total current density flowing in the diode will thus be equal to the sum of $J_{h\text{-diff}}$ and $J_{e\text{-diff}}$ evaluated at $x = 0$. Performing the summation of Eqs. (A.60) and (A.62) with $x = 0$ results in

$$J_{tot} = \Big(\underbrace{\frac{qD_e}{L_e}n_{po}}_{\text{electron component}} + \underbrace{\frac{qD_h}{L_h}p_{no}}_{\text{hole component}}\Big)(e^{v_D/V_T} - 1) \tag{A.63}$$

Multiplying this current-density equation by the cross-sectional area A to obtain the total current through the diode yields the familiar diode equation:

$$i_D = I_s(e^{v_D/V_T} - 1) \tag{A.64}$$

where

$$I_s = qA\left(\frac{D_e}{L_e}n_{po} + \frac{D_h}{L_h}p_{no}\right) \tag{A.65}$$

Note that $p_{no} = n_i^2/n_{no} \approx n_i^2/N_D$ on the n side of the diode. Similarly, $n_{po} = n_i^2/p_{po} \approx n_i^2/N_A$ on the p side. The parameter I_s thus can be expressed in the alternative form

$$I_s = qAn_i^2\left(\frac{D_e}{N_AL_e} + \frac{D_h}{N_DL_h}\right) \tag{A.66}$$

As Eq. (A.66) shows, I_s depends on the donor and acceptor concentrations N_D and N_A, where N_D applies to the n side of the diode and N_A to the p side. The parameter I_s is also a strong function of temperature due to the temperature dependency of n_i^2. Even though the thermal voltage V_T appears in the exponent of the diode equation (A.64), the exponential rise of n_i with temperature dominates the temperature dependence of the diode.

A.7.4 Correction for Depletion-Region Recombination

In the derivation of Section A.7.3, the depletion region was assumed to have negligible width, and recombination was neglected within the depletion region. In reality, the depletion region has finite width; hence some recombination does occur within it. A detailed analysis that includes depletion-region combination results in a minor modification to the diode equation that can be accommodated by adding an empirical constant η to the exponential term:

$$i_D = I_s(e^{v_D/\eta V_T} - 1) \tag{A.67}$$

The parameter η, called the *emission coefficient*, has a value between 1 and 2 for silicon diodes and is approximately equal to 1 for germanium and gallium arsenide diodes. The value of η also changes slightly with the width of the depletion region, and hence is a mild function of the diode voltage. In some of the literature, the emission coefficient is represented by the letter n.

A.7.5 The *pn* Junction at Extreme Operating Points

The physical model developed in Sections A.7.1 to A.7.3 must be expanded to accommodate diode operation at extremes of voltage and current. The v–i equation for the diode derived in Section A.7.3 uses the diffusion-limited model for hole and electron flow in the *pn* junction under forward-biased conditions. The effect of ohmic resistance in the semiconductor materials and diode contacts is ignored. This omission is valid if the diode is operated with low forward-bias current. When the diode current is small, the voltage drop across its internal ohmic resistance is minimal compared to the principal voltage drop across the depletion region. At larger values of current, the voltage drop across the diode's internal ohmic resistance becomes comparable to, or greater than, the depletion-region voltage drop. This ohmic resistance is a real, physical resistance and should not be confused with r_d, the equivalent incremental resistance that represents the slope of the diode's v–i characteristic in the piecewise linear model. At very high currents, the diode's ohmic resistance limits the exponential rise of current with voltage. In essence, a diode at very high currents begins to behave more like the series combination of a diode and a small-valued resistor.

In the reverse-biased region of operation, other processes limit the validity of the simple diode equation. As the reverse-bias diode voltage is increased, resistive leakage paths around the physical extremities of the diode act in parallel to pass current through the diode terminals. At even moderately large voltages, this reverse leakage current can exceed the reverse saturation current $-I_s$ by several orders of magnitude. In the reverse-biased mode, a diode often behaves more like the parallel combination of a diode and a large-valued resistor.

If the reverse bias voltage is very large, the diode may succumb to reverse breakdown via the avalanche or zener effects discussed in Section 3.3.5. The avalanche and zener mechanisms thus act to further limit the general validity of the diode equation (A.67).