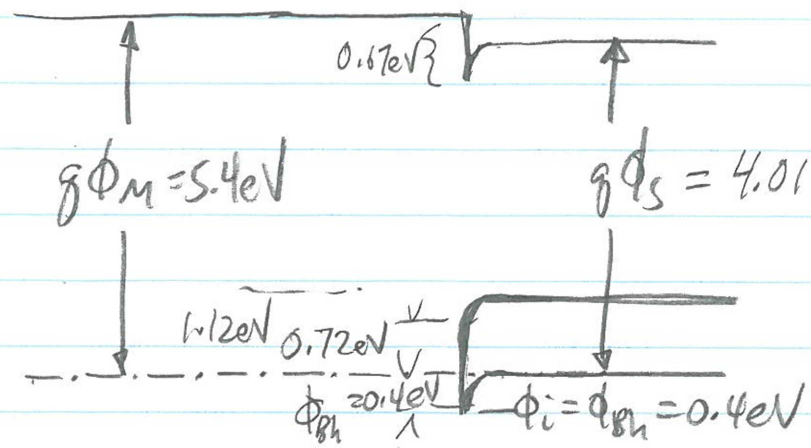
Winter 2011

Exam #2 Solutions

1\.

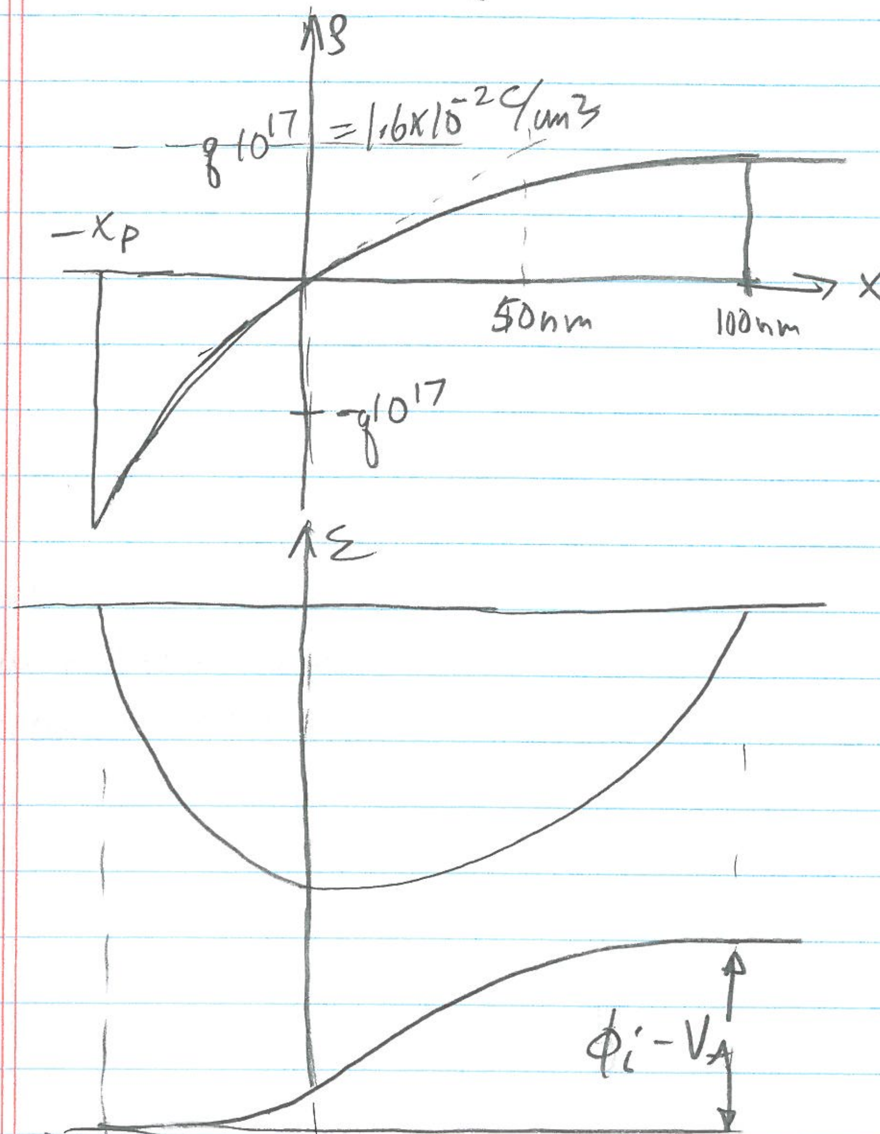
(b)



$N_a > N_V$, assume $E_F \cong E_V$

(a) Junction is ohmic. There is a barrier for majority carriers (holes), but because of high doping, the depletion region is thin enough (~2nm) to be easily tunneled through

2 (a) Metallurgical junction at $x=0$ (net doping $=0$)



$$Q_n' = \int_0^{x_n} \rho\, dx$$

$$0.016\frac{C}{cm^3}\int_0^{100nm}\left[1-e^{-\frac{x}{50nm}}\right]dx$$

$$= 0.016\left[x + 50nm\, e^{-x/50nm}\right]_0^{100nm}$$

$$= 0.016\frac{C}{cm^2}\left[10^{-5}cm + 5\times10^{-6}cm\left(e^{-2}-1\right)\right]$$

$$= 9.1\times10^{-8}\frac{C}{cm^2}$$

$$Q_p' = -Q_n' = \int_{-x_p}^{0}\rho\, dx = -9.08\times10^{-8}\frac{C}{cm^2}$$

$$= 0.016\frac{C}{cm^3}\left[-x_p + 5\times10^{-6}cm * \left(1-e^{x_p/50nm}\right)\right]$$

$$\Rightarrow x_p = 60nm$$

$$\phi_i = \frac{kT}{q}\ln\frac{N_a(-x_p)\,N_d(x_n)}{n_i^2}$$

$$= \frac{kT}{q}\ln\left[\left(\frac{10^{17^2}}{10^{20}}\right)\left(1-e^{-2}\right)\left(e^{1.2}-1\right)\right]$$

$$= 0.85\ V$$

(b) $$C_j' = \frac{K_s\epsilon_0}{x_d} = \frac{11.7\times8.854\times10^{-14}\frac{F}{cm}}{1.6\times10^{-5}cm}$$

$$= \underline{6.5\times10^{-8}F/cm^2}$$

$$\mathcal{E}_{max} = \frac{Q_p'}{K_s\epsilon_0} = \frac{-9.1\times10^{-8}C/cm^2}{11.7\times8.854\times10^{-14}F/cm}$$

$$= \underline{8.8\times10^{4}V/cm}$$

$$(\phi_i - V_A) = -\int_{-x_p}^{x_n} \mathcal{E}(x)\,dx$$

$$\mathcal{E}(x) = \int_{-x_p}^{x} \frac{\rho(x)}{K_s\epsilon_0}\,dx = \frac{0.016\,C/cm^3}{1.03\times10^{-12}F/cm}\left[x + 50nm\, e^{-\frac{x}{50nm}}\right]_{-x_p}^{x}$$

$$= 1.55\times10^{10}\frac{V}{cm^2}\left\{x + 6\times10^{-6}cm + 5\times10^{-6}cm\left[e^{-\frac{x}{50nm}} - e^{1.2}\right]\right\}$$

$$(\phi_i - V_A) = -1.55\times10^{10}\frac{V}{cm^2}\left\{\frac{x^2}{2} + 6\times10^{-6}cm\,x - 5\times10^{-6}cm\left[5\times10^{-6}cm\, e^{-\frac{x}{50nm}} + x e^{1.2}\right]\right\}_{-x_p}^{x_n}$$

$$= -1.55\times10^{10}\frac{V}{cm^2}\left\{\frac{(10^2-6^2)\times10^{-12}cm^2}{2} + 6\times10^{-6}cm\left(16\times10^{-6}cm\right) - 25\times10^{-12}cm^2\left(e^{-2} - e^{1.2}\right) - 5(16)\times10^{-12}cm^2 e^{1.2}\right\}$$

$$= -0.0155\,V\left[32 + 96 + 25\left(e^{1.2} - e^{-2}\right) - 80e^{1.2}\right]$$

$$= 0.90\,V \qquad V_A = \phi_i - 0.9V = -0.05V$$

(very small reverse bias)

3 (a) $D_{n_p} = 32.5 \frac{cm^2}{s}$, $\tau_{n_p} = 5\times10^{-6}s \Rightarrow L_{n_p} = [(32.5)(5\times10^{-6})]^{1/2}$

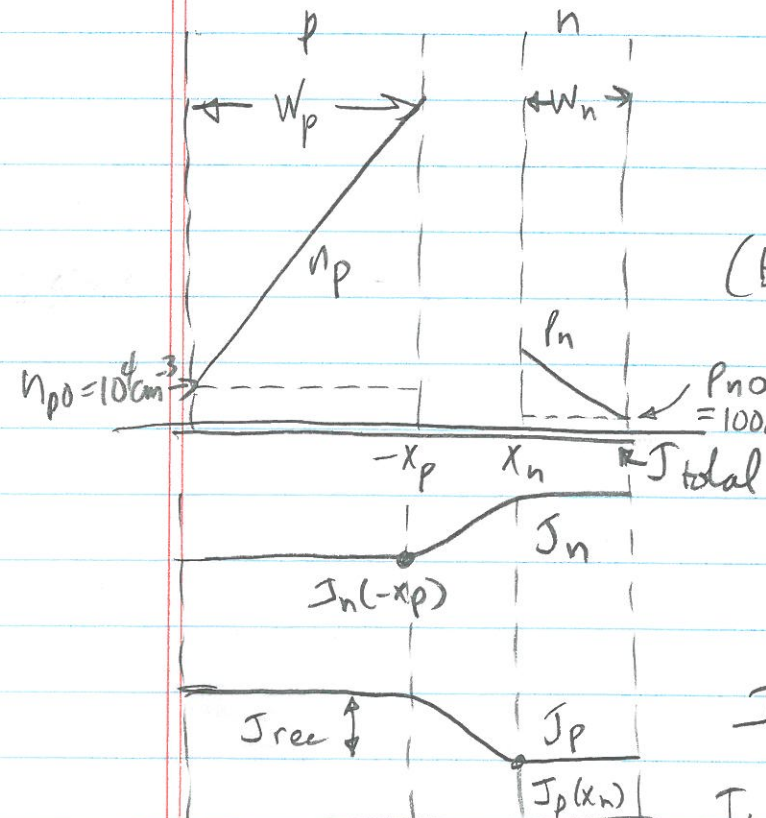
$= 12.7\times10^{-3} cm = 127 \mu m$

$D_{p_n} = 2.6 \frac{cm^2}{s}$, $\tau_{p_n} = 10^{-6}s \Rightarrow L_{p_n} = [(2.6\times10^{-6})]^{1/2} = 1.61\times10^{-3} cm$

$= 16.1 \mu m$

$L_{n_p} = 127 \mu m >> W_p = 20 \mu m \Rightarrow$ short base

$L_{p_n} = 16.1 \mu m >> W_n = 0.5 \mu m \Rightarrow$ short base also



$$\frac{J_p(x_n)}{J_n(-x_p)} = \frac{D_{p_n}}{D_{n_p}} \frac{W_p N_a}{W_n N_d} = \frac{2.6}{32.5} \frac{20}{0.5} \frac{1}{100}$$

$= 0.032$

(b) $$J_p(x_n) = \frac{q n_i^2 D_{pn}}{N_d W_n}\left(e^{qV_A/kT} - 1\right)$$

$$= \frac{(1.6\times10^{-19}C)(10^{20}cm^{-6})(2.6\frac{cm^2}{s})}{(10^{18}cm^{-3})(5\times10^{-5}cm)} e^{0.3/0.0259}$$

$= 8.9\times10^{-8} A/cm^2 = 8 \frac{nA}{cm^2}$

$= \underline{\underline{8.9 \; A/\mu m^2}}$

$J_n(x_n+W_n) = J_n(-x_p) + J_{rec}$

$$J_n(-x_p) = \frac{q n_i^2 D_{np}}{N_a W_p}\left(e^{qV_A/kT} - 1\right) = \frac{J_p(x_n)}{0.032}$$

$= 2.8\times10^{-6} \frac{A}{cm^2} = 280 \, A/\mu m^2$

$\phi_i = \frac{kT}{q} \ln\left(\frac{10^{18} 10^{16}}{10^{20}}\right) = 0.835$

$$J_{rec} \cong \frac{q n_i x_d'}{\tau_n + \tau_p} \exp\left(\frac{qV_A}{2kT}\right)$$

$n \cong p$ on lightly-doped side

$$x_d' \cong \frac{x_d}{10} = \frac{1}{10}\left[\frac{2K_s\epsilon(\phi_i - V_A)}{qN_a}\right]^{1/2} = 2.6\times10^{-6} cm$$

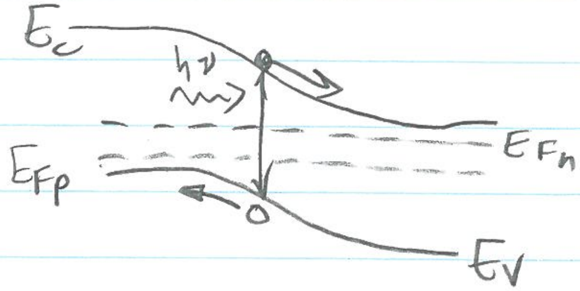
$$= \frac{(1.6\times10^{-19}C)(10^{10}cm^{-3})(2.6\times10^{-6}cm)}{1.5\times10^{-5}s} e^{\frac{0.15}{0.0259}} = 9.1\times10^{-8} A/cm^2$$

$J_n(x_n+W_n) = 2.8\times10^{-6} + 9.1\times10^{-8} = \underline{\underline{2.9\times10^{-6} A/cm^2}}$

(c) $\dfrac{Q_n'}{Q_p'} = \dfrac{\cancel{q}\cancel{n_i^2} W_p / N_a}{\cancel{q}\cancel{n_i^2} W_n / N_d} = \dfrac{W_p N_d}{W_n N_a} = \dfrac{20\mu m \; 10^{18} cm^{-3}}{0.5\mu m \; 10^{16} cm^{-3}}$

$= (40)(100) = \underline{\underline{4000}}$

(d)



Carriers generated in or near the depletion region are swept across by built-in (plus applied) field.

Holes go toward p-region, electron toward n-region. The resulting net current is negative (opposite to forward bias current in which holes are injected into n-region (and electrons into p-region). Thus the current will become smaller (less positive) or could even change sign

4 (a) $V_{ox} = -0.6V = Q_g'/C_{ox}' = -Q_s'/C_{ox}'$ (no oxide charges)

so $Q_s' = -C_{ox}' V_{ox}$ $\quad C_{ox}' = \frac{K_{ox}\epsilon_0}{x_{ox}} = 1.15\times10^{-6}\frac{F}{cm^2}$

$= -\left(1.15\times10^{-6}\frac{F}{cm^2}\right)(-0.6V)$

$= 6.9\times10^{-7}\,C/cm^2 > 0$, so N_d^+ or h^+ (depleted or inverted)

$$Q_{d,max}' = \sqrt{2K_s\epsilon_0 q N_d \underbrace{|V_{CB} - 2\phi_F|}_{|-1V-0.99V|}} \qquad \phi_F = \frac{kT}{q}\ln\left(\frac{N_d}{n_i}\right) = 0.495V$$

$Q_{d,max}' = 1.14\times10^{-6}\,C/cm^2 > Q_s'$, so just <u>depleted</u>

(b) $V_{gb} = \phi_{MS} + V_{ox} + V_s$ $\quad \phi_{MS} = \left[\left(\cancel{\chi_{si}} + \frac{E_g}{q}\right) - \left(\cancel{\chi_{si}} + \frac{E_g}{2q} - \phi_F\right)\right]$

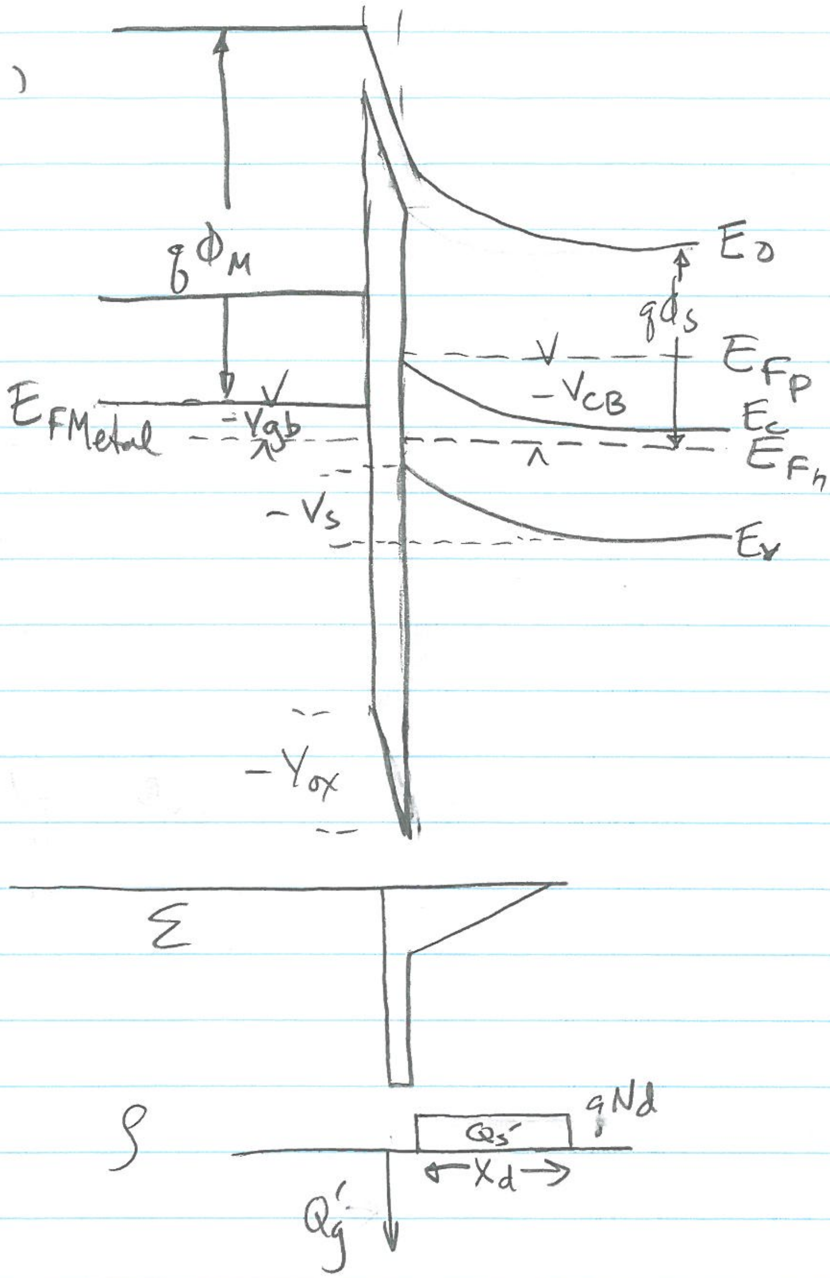
$= 1.055V$

$Q_s' = \sqrt{2K_s\epsilon_0 q N_a V_s}$

$$V_s = \left(\frac{Q_s'}{Q_{d,max}'}\right)^2 V_s^{EOSI} = \left(\frac{0.69}{1.14}\right)^2(-1.99) = -0.73V$$

$V_{gb} = 1.06 - 0.6 - 0.73 = \underline{\underline{-0.27V}}$

(c)



(d) $X_d = Q_s'/qN_d = \dfrac{6.9\times10^{-7}\,C/cm^2}{(1.6\times10^{-19}C)(2\times10^{18}cm^{-3})} = 2.15\times10^{-6}\,cm$

$C_d' = \dfrac{K_s\varepsilon_o}{X_d} = 4.8\times10^{-7}\dfrac{F}{cm^2}$

$C_g = \dfrac{C_{ox}'C_d'}{C_{ox}'+C_d'} = \dfrac{(4.8)(11.5)}{4.8+11.5}\times10^{-7}\dfrac{F}{cm^2}$

$= \underline{\underline{3.4\times10^{-7}F/cm^2}}$ at both high & low frequency as depletion region width can be modulated rapidly.