

Physics of Semiconductors in Nonequilibrium

Thermal equilibrium is characterized by the condition $np = n_i^2$. When this relation holds, the equations in the previous handouts can be used to calculate E_f , n , p , etc.

Injection: extra carriers provided (electrically or by light) $np > n_i^2$.

Extraction: carriers removed (electrically) $np < n_i^2$.

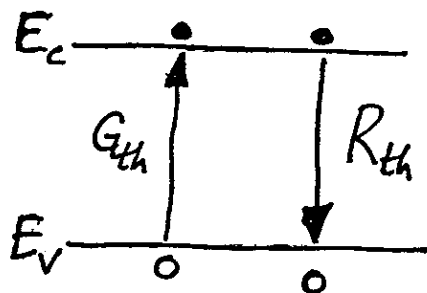
- Many important semiconductor devices depend for their operation on nonequilibrium conditions.
- Excess carriers will, given time, recombine to re-establish thermal equilibrium.

$\tau \equiv$ carrier lifetime (1ns - 1ms)

$R \equiv$ recombination rate (carriers/cm³s)

$G \equiv$ generation rate (carriers/cm³s)

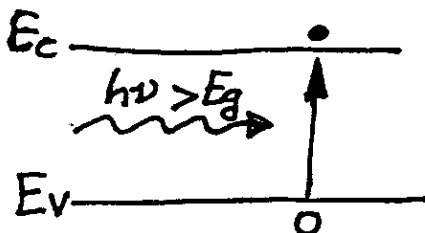
- The manner in which thermal equilibrium is re-established is of fundamental importance to the operation of semiconductor devices.



At any $T > 0^\circ\text{K}$, some electrons gain enough energy to jump into the conduction band. This leaves behind a hole so that

$$n = p = n_i. \quad (\text{intrinsic})$$

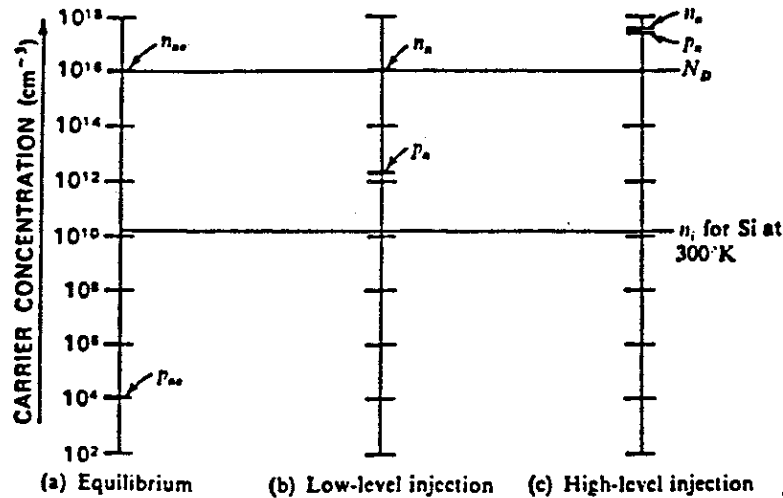
Suppose we shine light on the semiconductor with $E = h\nu > E_g$ so that by absorbing a photon an electron can be excited into the conduction band.



$$\left. \begin{aligned} n &= n_i + \Delta n \\ p &= n_i + \Delta p \\ n, p &> n_i \\ \Delta n &= \Delta p \end{aligned} \right\} \text{Injection}$$

$\nu = \text{freq}$
 $h = \text{Planck's}$
 constant

If we do the same thing for an extrinsic semiconductor (ie. n-type, $N_d = 10^{16} \text{cm}^{-3}$).



$$n \cong N_d + \Delta n$$

$$p \cong \frac{n_i^2}{N_d} + \Delta p$$

$$\Delta n = \Delta p$$

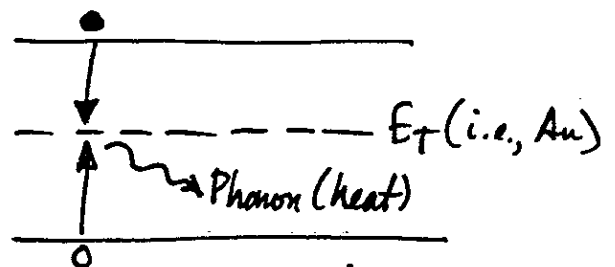
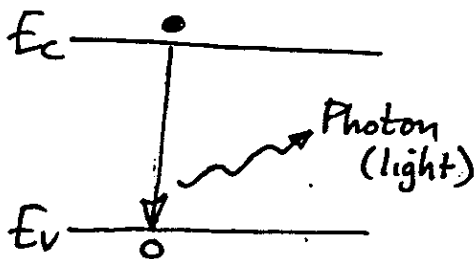
$$\text{Low level: } \Delta n \ll N_d$$

$$\text{High level: } \Delta n \geq N_d$$

Recombination

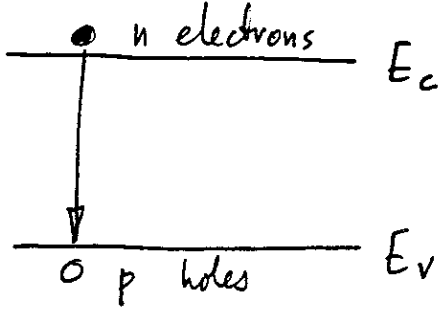
Mechanisms:

- Conduction band electrons and valence band holes mutually annihilate each other (band to band or direct recombination, $e^- + h^+ \rightarrow \emptyset$).
 - photons produced (radiative recombination)
 - only phonons produced (non-radiative recombination)
- Conduction band electrons and valence band holes recombine by sequentially being trapped in an intermediate deep level (indirect recombination, $e^- + \text{Au}^x \rightarrow \text{Au}^-$, $\text{Au}^- + h^+ \rightarrow \text{Au}^x$).



The material responds to the generation of excess carriers by increasing the recombination rate, attempting to return the material to equilibrium ($pn = n_i^2$). In the case of carrier extraction ($pn < n_i^2$), the opposite phenomena takes place. Carriers are thermally generated faster than they recombine moving the material towards equilibrium ($pn = n_i^2$).

Direct (Band to Band) Recombination



R = rate of recombination

$R \propto n$ (number of electrons in conduction band)

$R \propto p$ (number of empty sites (holes) in valence band)

$$R = Kpn \quad (K = \text{constant}) \quad (1)$$

In thermal equilibrium,

$$G_{th} = R_{th} = Kp_0n_0 \quad (n_0, p_0 \text{ are equilibrium values}) \quad (2)$$

Suppose we generate excess carriers at a rate G_L (due to incident light for example), then

$$R = Kpn = K(p_0 + \Delta p)(n_0 + \Delta n) \quad (3)$$

where $\Delta p \equiv p - p_0$ and $\Delta n \equiv n - n_0$ are the excess carrier concentrations and

$$G = G_{th} = G_L \quad (4)$$

In steady-state (not equilibrium) the time derivatives go to zero,

$$G = G_L + G_{th} = R \quad (5)$$

$$U \equiv R - G_{th} = \text{net recombination rate} = G_L. \quad (6)$$

Substituting (2) and (3) into (5) gives

$$G_L = K(n_0 + \Delta n)(p_0 + \Delta p) - Kn_0p_0 = K(n_0\Delta p + \Delta np_0 + \Delta n\Delta p). \quad (7)$$

But $\Delta n = \Delta p$ (carriers are created in pairs) so

$$G_L = K\Delta p(n_0 + p_0 + \Delta p). \quad (8)$$

For low level injection, the excess carrier concentration is much less than the majority carrier concentration ($\Delta p \ll n_0 + p_0$) so

$$G_L = K\Delta p(n_0 + p_0). \quad (9)$$

The excess carrier lifetime (τ) which is the average time which a generated carrier remains free is defined as

$$\tau \equiv \frac{\Delta p}{U} = \frac{\Delta n}{U} = \frac{1}{K(n_0 + p_0)} \quad \text{for low-level injection} \quad (10)$$

so

$$\Delta p = \tau G_L = \tau U = \frac{G_L}{K(n_0 + p_0)} \quad \text{in steady-state, l.l.i.} \quad (11)$$

Note that $\tau = \tau_n = \tau_p$ in direct recombination since a single phenomena eliminates an electron and a hole simultaneously.

In summary, if we have a semiconductor undergoing low-level injection with equilibrium carrier concentrations n_0 and p_0 , then the steady-state carrier concentrations are:

$$n = n_0 + \tau G_L \quad (12)$$

$$p = p_0 + \tau G_L \quad (13)$$

with $n_0p_0 = n_i^2$ and $np > n_i^2$.

High Level Injection

Assume direct recombination with $\Delta n = \Delta p \sim n_0, p_0$

$$U = G_L = K\Delta p(n_0 + p_0 + \Delta p)$$

$$\tau_{hl} = \frac{\Delta p}{U} = \frac{1}{K(n_0 + p_0 + \Delta p)} \cong \frac{1}{Kn} (\text{n-type}) \ll \tau_n$$

The effective lifetime depends on the amount of injection. If the system with high level injection is not in steady-state ($\Delta p \neq \text{constant}$) the effective lifetime will not be constant.

Example: Consider a sample of n-type GaAs with a doping of $N_d = 10^{15} \text{cm}^{-3}$. $n_0 \cong N_d$ and $p_0 = (n_i^2/n_0) = 8 \times 10^{-3} \text{cm}^{-3}$. Light is incident on the sample creating $G_L = 10^{25}$ hole-electron pairs per cm^3 per second.

The dark conductivity of this sample would be $\sigma = q(n\mu_n + p\mu_p) \cong q(n\mu_n) \cong 0.96 (\Omega\text{-cm})^{-1}$. Assume that the low-level injection lifetime is known:

$$\tau_0 = 1 \text{ns} = 1/(Kn_0)$$

$$K = 1/n_0\tau_0 = 10^{-6} \text{cm}^3/\text{s}$$

$$U = G_L = 10^{25} \text{cm}^{-3}\text{s}^{-1} = K(\Delta p^2 + (n_0 + p_0)\Delta p)$$

$$\Delta p^2 + n_0\Delta p - \frac{G_L}{K} = 0$$

Can solve this quadratic equation for Δp .

$$\Delta p = 2.7 \times 10^{15} \text{cm}^{-3}.$$

$$n = 3.7 \times 10^{15} \text{cm}^{-3}, p = 2.7 \times 10^{15} \text{cm}^{-3}.$$

$$\sigma = q(n\mu_n + p\mu_p) \cong 3.7 (\Omega\text{-cm})^{-1},$$

$$\tau_{\text{eff}} = \frac{1}{K(n_0 + p_0 + \Delta p)} = 0.11 \text{ns}.$$

As expected the lifetime of the excess carriers is reduced under high level injection conditions since the number of majority carriers is increased resulting in more electrons for the holes to recombine with.

This is an example of a photoconductor. The conductance of the material depends on the intensity of the light shining on it.

Transient Recombination Response

Suppose light is shining on a semiconductor and is shut off at time $t = 0$. What is the time response of the excess carrier density?

In general,

$$\frac{d\Delta p}{dt} = G_L - U$$

$$U = \begin{cases} (8) & \text{direct recombination} \\ (28) & \text{indirect recombination} \end{cases}$$

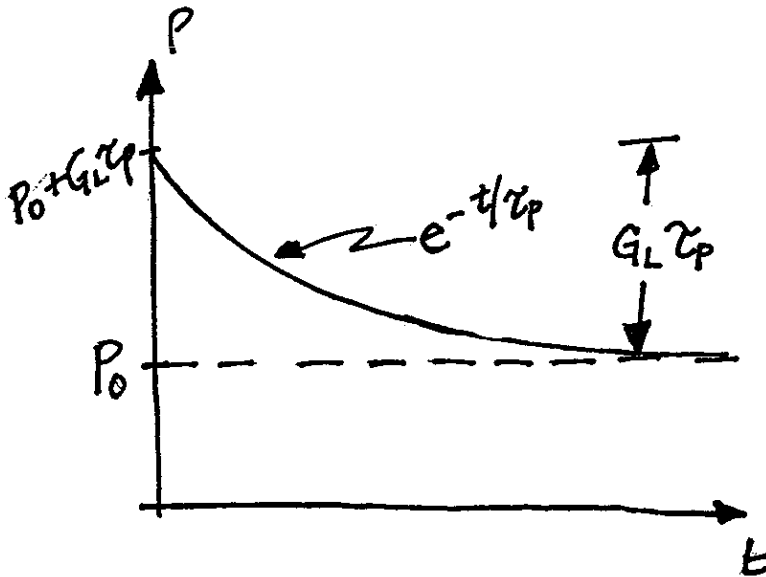
To simplify the math we will assume low level injection. If $G_L = 0$,

$$\frac{d\Delta p}{dt} = -\frac{\Delta p}{\tau_p}$$

$$\Delta p = \Delta p(0) \exp\left(-\frac{t}{\tau_p}\right)$$

We know that $\Delta p(0^-) = G_L \tau_p$ just before the light was turned off so

$$\Delta p = G_L \tau_p \exp\left(-\frac{t}{\tau_p}\right) \quad (14)$$

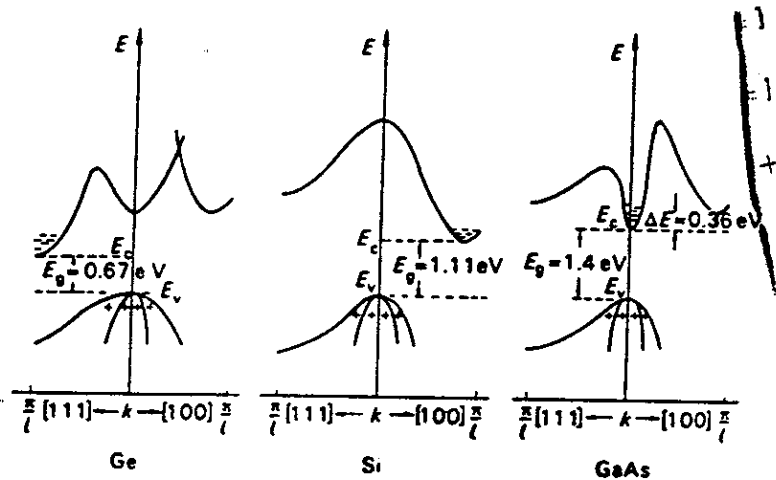


Note that both the electron and hole concentrations decay at the same rate. The excess minority carrier density is usually the important one in devices.

The excess carrier concentration decays to its equilibrium value of 0 ($p = p_0$) with a time constant given by the lifetime. The indirect recombination case is more complex but Equation (14) is often a good approximation.

Indirect Recombination (through trapping levels)

In silicon and germanium, this is usually the dominant mechanism since they are "indirect band semiconductors". Physically this means that the minima of the conduction band does not occur at the same point in momentum space (k -space) as the maxima of the valence band.

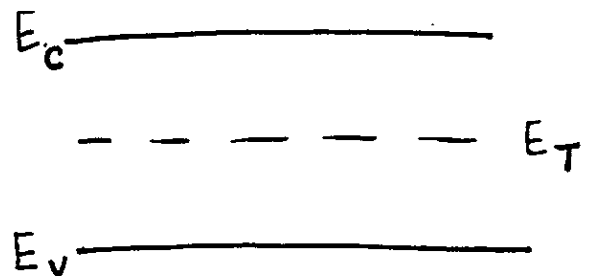


"Indirect Bandgap"

"Direct Bandgap"

Momentum as well as energy must be conserved in any energy level transition. Therefore, GaAs can easily make direct (band to band) transitions between the full states at the bottom of the conduction band (electrons) and the empty states at the top of the valence band (holes), which makes it useful for LEDs since these direct transitions are usually radiative with $E_p = h\nu = E_g$. This is not true for Si and Ge since, due to the momentum difference between electrons and holes, a direct transition must involve both a photon (large energy, small momentum) and a phonon (small energy, large momentum). Since the probability of three body reactions are small, recombination in Si and Ge occurs predominately through intermediate trapping levels.

Consider the following example. An impurity (Au, for example) is introduced which provides a "trapping level" or a set of allowed states at energy E_t . This level is assumed to be capable of trapping both holes and electrons, ie. to have both donor and acceptor characteristics. (In fact the donor and acceptor levels will differ, but assuming a single trap level simplifies the analysis.)



"1" = electron capture

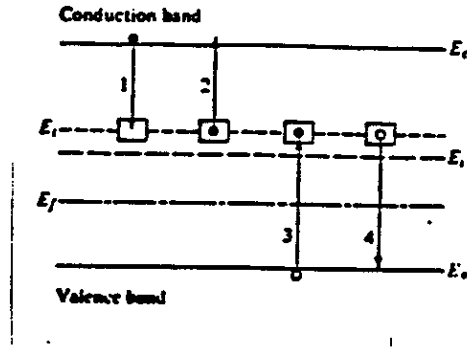
"2" = electron emission

"3" = hole capture

"4" = hole emission

1 + 3 or 3 + 1 \equiv recombination

2 + 4 or 4 + 2 \equiv generation



The probability of each of these events occurring depends on the numbers of occupied and unoccupied states in the different levels and therefore depends on Fermi-Dirac statistics (ie. the position of the Fermi level). Note however that the Fermi level is only a valid concept under equilibrium conditions ($np = n_i^2$). We will assume to begin then that system is in equilibrium.

$$f_t = \text{prob. that } e^- \text{ occupies trap} = \frac{1}{1 + \exp \frac{E_t - E_f}{kT}}$$

$1 - f_t$ = prob. that trap level is unoccupied (or occupied by hole)

$N_t f_t$ = number of occupied centers

$N_t(1 - f_t)$ = number of empty centers

We will begin by assuming that the trapping processes are all first-order. For example, we assume that the rate of electron capture is proportional to the number of electrons in the conduction band and to the number of empty trapping centers (just as in direct recombination).

$$R_1 = v_{thn} \sigma_n n N_t (1 - f_t) \quad (15)$$

where

v_{thn} = thermal velocity of electrons ($\sim 10^7$ cm/s),

σ_n = capture cross-section for electrons ($\sim 10^{-15}$ cm²).

The electron emission rate is proportional to the number of occupied trapping centers (the conduction band is nearly empty).

$$R_2 = e_n N_t f_t \quad (16)$$

where e_n = emission probability.

For holes we can write similar equations,

$$R_3 = v_{thp}\sigma_p p N_t f_t \quad (17)$$

$$R_4 = e_p N_t (1 - f_t) \quad (18)$$

Under thermal equilibrium, forward and reverse reaction rates are equal so $R_1 = R_2$ and $R_3 = R_4$.

$$v_{thn}\sigma_n n N_t (1 - f_t) = e_n N_t f_t \quad (19)$$

$$v_{thp}\sigma_p p N_t f_t = e_p N_t (1 - f_t) \quad (20)$$

Recall that

$$n = n_i \exp \frac{E_f - E_i}{kT},$$

$$p = n_i \exp \frac{E_i - E_f}{kT}.$$

Therefore,

$$v_{thn}\sigma_n n_i \exp \frac{E_f - E_i}{kT} N_t (1 - f_t) = e_n N_t f_t \quad (21)$$

$$v_{thp}\sigma_p n_i \exp \frac{E_i - E_f}{kT} N_t f_t = e_p N_t (1 - f_t) \quad (22)$$

Also,

$$\frac{1 - f_t}{f_t} = \exp \frac{E_i - E_f}{kT}.$$

The emission rates can then be expressed as:

$$e_n = v_{thn}\sigma_n n_i \exp \frac{E_i - E_f}{kT} \text{ carriers/s} \quad (23)$$

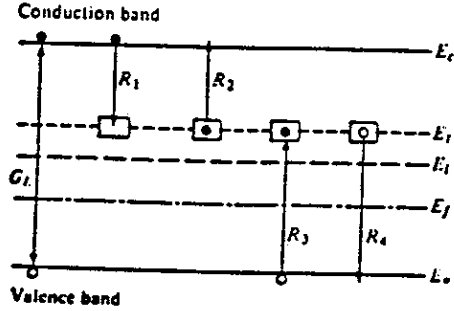
$$e_p = v_{thp}\sigma_p n_i \exp \frac{E_i - E_f}{kT} \text{ carriers/s} \quad (24)$$

- As $E_i \rightarrow E_c$, e_n increases and e_p decreases as we would have predicted.
- As $E_i \rightarrow E_v$, e_p increases and e_n decreases.

The relationship between the capture and emission rates as expressed in Eqs. (23) and (24) do not depend on the Fermi level and are valid in nonequilibrium although they were derived using equilibrium assumptions. However, equations which include the probability of occupation depend on E_f and are only valid in equilibrium.

Now consider the situation in which we shine light on the semiconductor to generate excess electrons and holes.

In steady-state (not equilibrium),



$$G_L = R_1 - R_2,$$

$$G_L = R_3 - R_4.$$

The numbers of electrons and holes in the conduction and valence bands are constant.

$$R_1 - R_2 = R_3 - R_4. \quad (25)$$

Using our previous expressions for the capture and emission rates (Eqs. (15-18)),

$$v_{thn}\sigma_n n N_t (1 - f_t) - e_n N_t f_t = v_{thp}\sigma_p p N_t f_t - e_p N_t (1 - f_t) \quad (26)$$

Using Eqs. (23) and (24) for the emission probabilities,

$$v_{thn}\sigma_n N_t \left[n(1 - f_t) - n_i \exp \frac{E_t - E_i}{kT} f_t \right] = v_{thp}\sigma_p N_t \left[p f_t - n_i \exp \frac{E_i - E_t}{kT} (1 - f_t) \right].$$

For reasons which will become apparent, define

$$\tau_n \equiv \frac{1}{v_{thn}\sigma_n N_t},$$

$$\tau_p \equiv \frac{1}{v_{thp}\sigma_p N_t}.$$

Then,

$$\frac{1}{\tau_n} \left[n(1 - f_t) - n_i \exp \frac{E_t - E_i}{kT} f_t \right] = \frac{1}{\tau_p} \left[p f_t - n_i \exp \frac{E_i - E_t}{kT} (1 - f_t) \right].$$

This equation can be solved for f_t , the occupation probability of the trap level.

$$f_t = \frac{\tau_n \tau_p \left[n + n_i \exp \frac{E_i - E_t}{kT} \right]}{\tau_p \left[n + n_i \exp \frac{E_t - E_i}{kT} \right] + \tau_n \left[p + n_i \exp \frac{E_i - E_t}{kT} \right]}. \quad (27)$$

Using this result and $U = R_1 - R_2 = \text{net recombination rate}$,

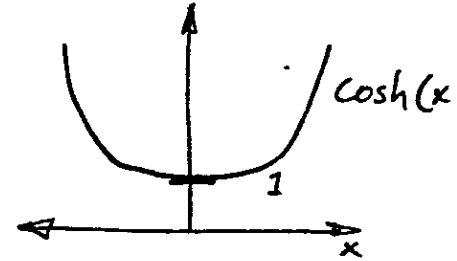
$$U = \frac{pn - n_i^2}{\tau_p \left[n + n_i \exp \frac{E_t - E_i}{kT} \right] + \tau_n \left[p + n_i \exp \frac{E_i - E_t}{kT} \right]} \quad (28)$$

If we assume that $\tau = \tau_n = \tau_p = (v_{th}\sigma N_t)^{-1}$ then the net recombination rate can be simplified to:

$$U = \frac{pn - n_i^2}{\tau \left[n + p + 2n_i \cosh \frac{E_t - E_i}{kT} \right]} \quad (29)$$

This is an extremely important result, based on the work of Shockley, Read and Hall, and is known as SRH recombination.

- The numerator is proportional to the restoring force. The net recombination rate is always of the proper sign to move the system towards equilibrium ($pn = n_i^2$).



- The net recombination rate U is maximum when the denominator is minimized. That occurs when $E_t = E_i$. In other words, the trapping levels near midgap are the most efficient recombination centers.

- $E_t \rightarrow E_c$ makes electron capture more probable, but hole capture less probable. Electron emission also increases as $E_t \rightarrow E_c$ reducing net recombination rate.
- $E_t \rightarrow E_v$ makes hole capture more probable, but electron capture less probable. Hole emission also increases as $E_t \rightarrow E_v$ reducing net recombination rate.

For trap levels at midgap ($E_t = E_i$),

$$U = \frac{pn - n_i^2}{\tau_p n + \tau_n p + (\tau_n + \tau_p)n_i} \quad (30)$$

Examples: Low-level injection in doped semiconductors.

In n-type semiconductor, with trap level at midgap, $n \gg p, n_i$.

$$U = \frac{pn - n_i^2}{\tau_p n}$$

But $n_i^2 = n_{n0}p_{n0}$ (equilibrium values) and $n \cong n_{n0}$ (assume low-level injection) so

$$U = \frac{pn_{n0} - p_{n0}n_{n0}}{\tau_p n_{n0}} = \frac{p - p_{n0}}{\tau_p} = \frac{\Delta p}{\tau_p}, \quad (31)$$

just as for direct recombination with the lifetime equal to that for holes (the minority carrier). In n-type material, there are lots of electrons available for capture, so the rate-limiting step in recombination is hole (minority) carrier capture and the lifetime for excess carriers (both holes and electrons) is the minority carrier lifetime τ_p .

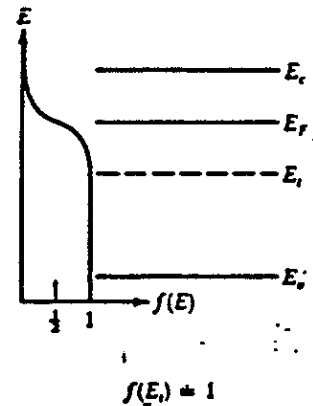
In p-type material, using the same analysis with $p \gg n, n_i$ and $p \cong p_{p0}$,

$$U = \frac{\Delta n}{\tau_n} = \frac{\Delta p}{\tau_n}. \quad (32)$$

Again the limiting factor is the minority carrier lifetime.

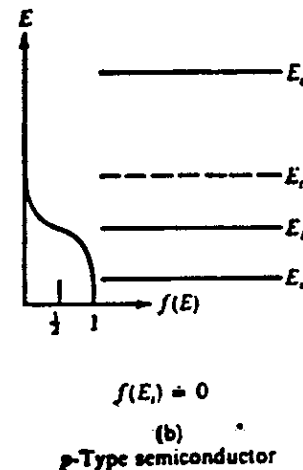
- N-type

- E_f near E_c
- E_t filled with electrons
- hole capture limits process
- τ_p dominates



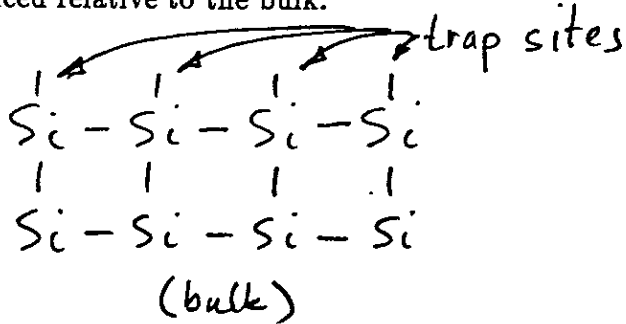
- P-type material

- E_f near E_v
- E_t empty of electrons
- electron capture limits process
- τ_n dominates



Surface Recombination

So far, we have considered only bulk recombination. Recombination at surfaces is often enhanced relative to the bulk.



Incompletely bonded silicon atoms at the surface can provide efficient traps for recombination.

By analogy with bulk recombination we can write the net recombination rate as

$$U = v_{th}\sigma N_{ts} [p(0) - p_0] \quad (33)$$

where

N_{ts} = surface density of recombination centers per area

$p(0) - p_0$ = excess carrier density at surface

We can define

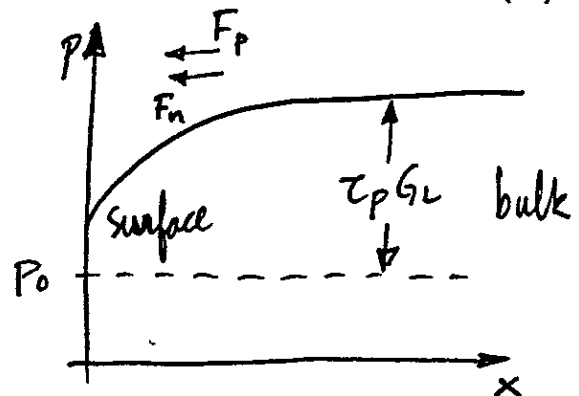
$$s \equiv v_{th}\sigma N_{ts} \quad (34)$$

to be the surface recombination velocity (cm/sec).

Thus,

$$U = s [p(0) - p_0] \quad (35)$$

If the interface has these excess recombination sites, then the excess carrier concentration will be lower at the surface than in the bulk.



In that case, there is a net flow of carriers towards the surface (both h^+ and e^- for charge neutrality and no net current).

The resulting differential equation is

$$\underbrace{\frac{dp}{dt}}_{0 \text{ in s-s}} = \underbrace{D_p \frac{d^2 p}{dx^2}}_{\text{diffusion}} + \underbrace{G_L}_{\text{generation}} - \underbrace{\frac{p - p_0}{\tau_p}}_{\text{recombination}} \quad (36)$$

Two boundary conditions apply:

Deep in the bulk the carrier concentration becomes independent of the surface.

$$p(\infty) = p_0 + \tau_p G_L$$

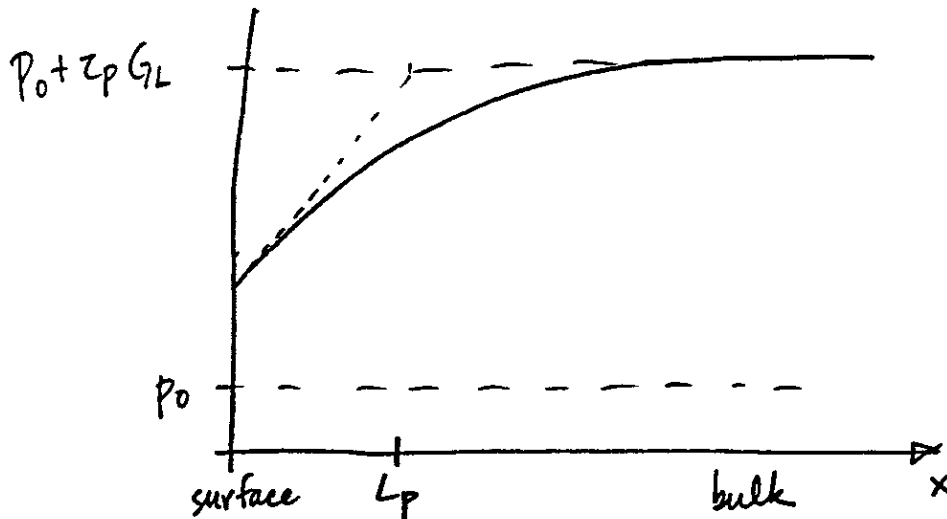
At the surface, the diffusion flux into the surface must equal the surface recombination rate.

$$D_p \left. \frac{dp}{dx} \right|_{x=0} = s [p(0) - p_0] \quad (37)$$

The solution to this differential equation is

$$\Delta p(x) = p(x) - p_0 = \tau_p G_L \left(1 - \frac{(s\tau_p)/L_p}{1 + (s\tau_p)/L_p} e^{-x/L_p} \right) \quad (38)$$

where $L_p \equiv \sqrt{D_p \tau_p}$ is the minority carrier diffusion length.



- The distance over which the surface disturbs the bulk carrier concentration depends on L_p (or D_p and τ_p).
- How strongly the surface affects the excess carrier concentration depends on s , D_p and τ_p .

In the limit of $s \rightarrow \infty$ (infinite recombination velocity at the surface), $p(0) \rightarrow p_0$ and

$$\Delta p(x) = p(x) - p_0 = \tau_p G_L (1 - e^{-x/L_p})$$

Quasi Fermi Levels

Under thermal equilibrium, we know that

$$n = n_i \exp\left(\frac{E_f - E_i}{kT}\right)$$
$$p = n_i \exp\left(\frac{E_i - E_f}{kT}\right)$$

If we have injection ($np > n_i^2$) or extraction ($np < n_i^2$) then we are not in equilibrium and therefore cannot use these relationships because E_f is meaningless.

To replace E_f , we will define two new quantities called quasi Fermi levels so that relationships similar to the above equations hold.

$$n = n_i \exp\left(\frac{E_{fn} - E_i}{kT}\right) = N_c \exp\left(-\frac{E_c - E_{fn}}{kT}\right) \quad (39)$$

where E_{fn} is defined as the quasi Fermi level for electrons.

$$p = n_i \exp\left(\frac{E_i - E_{fp}}{kT}\right) = N_v \exp\left(-\frac{E_{fp} - E_v}{kT}\right) \quad (40)$$

where E_{fp} is defined as the quasi Fermi level for holes.

The quasi Fermi levels are mathematical tools and their values are chosen so that we can extend our familiar equilibrium equations to nonequilibrium situations.

Out of equilibrium, $E_{fn} \neq E_{fp}$. $|E_{fn} - E_{fp}|$ is a measure how far removed the semiconductor is from equilibrium.

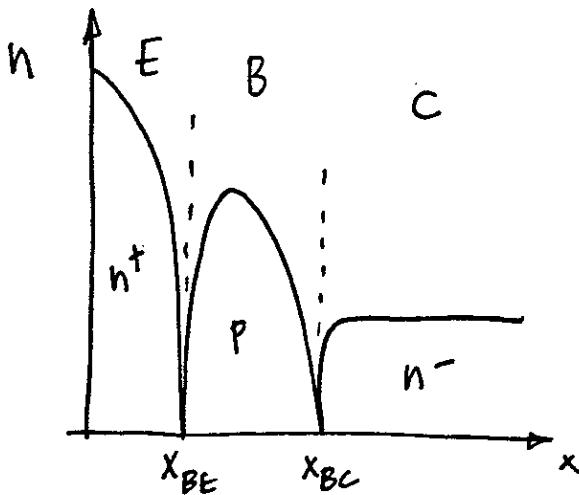
$$pn = n_i^2 \exp\left(\frac{E_{fn} - E_{fp}}{kT}\right)$$

We will find the concept of quasi Fermi levels useful when we consider PN junctions.

Non-Uniform Doping

Up to this point we have assumed that doping levels are constant with position. This is usually not the case.

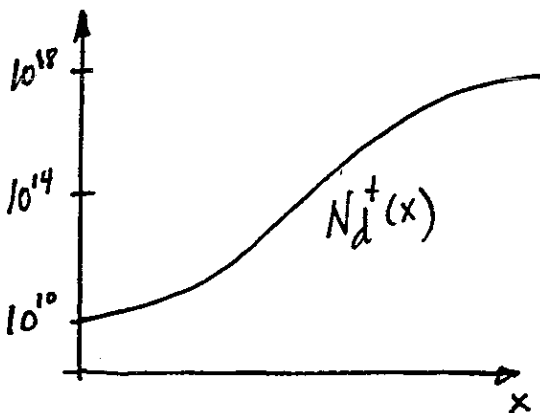
Example: Bipolar Transistor



The impurity profile at left represents the positions of the fixed ionized dopant atoms.

The carriers (electrons and holes) are mobile and will tend to diffuse from areas of high concentration to low concentration.

We will consider first the case of just one dopant type (donors in this case) so there are no junctions but N_d varies with position.



The diffusion flux is $F_n = -D_n(dn/dx)$.

As the electrons move away from the areas of heavy doping, they leave behind positively charged ionized donors which attract the electrons back.

In equilibrium, a balance is established and the drift and diffusion fluxes cancel. $n(x)$ is not exactly the same as $N_d(x)$ so an electric field remains.

The total electron flux is given by

$$F_n = - \underbrace{D_n \frac{dn}{dx}}_{\text{diffusion}} - \underbrace{\mu_n n \mathcal{E}}_{\text{drift}} \equiv 0 \quad \text{in equilibrium} \quad (41)$$

Since $n = n_i \exp[(E_f - E_i)/kT]$,

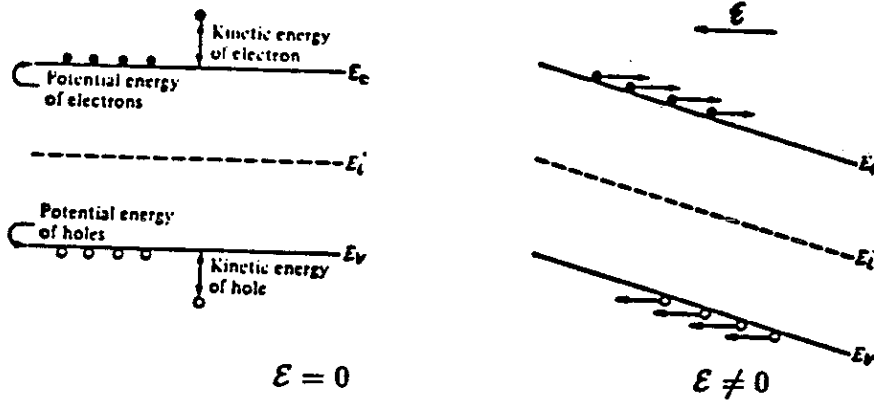
$$\frac{dn}{dx} = \frac{n}{kT} \left(\frac{dE_f}{dx} - \frac{dE_i}{dx} \right) \quad (42)$$

Therefore,

$$F_n = - \frac{n D_n}{kT} \left(\frac{dE_f}{dx} - \frac{dE_i}{dx} \right) - \mu_n n \mathcal{E} \quad (43)$$

The electric field is defined as the negative gradient of the electrical potential V .

$$\mathcal{E} \equiv - \frac{dV}{dx} \quad (44)$$



Therefore,

$$\mathcal{E} = \frac{1}{q} \frac{dE_i}{dx} = \frac{1}{q} \frac{dE_c}{dx} = \frac{1}{q} \frac{dE_v}{dx} \quad (45)$$

and

$$F_n = - \frac{n D_n}{kT} \left(\frac{dE_f}{dx} - \frac{dE_i}{dx} \right) - \frac{\mu_n n}{q} \frac{dE_i}{dx} \quad (46)$$

But we may recall the Einstein relationship,

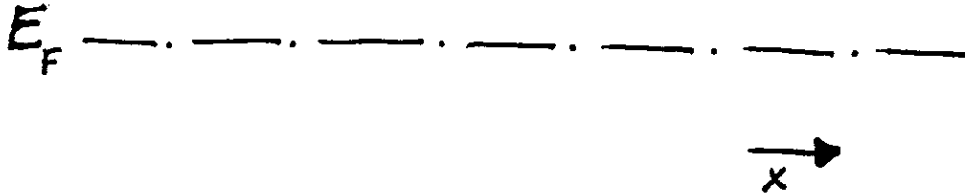
$$\frac{\mu_n}{q} = \frac{D_n}{kT}$$

Thus,

$$F_n = - \frac{n D_n}{kT} \frac{dE_f}{dx} \quad (47)$$

In equilibrium $F_n = 0$, therefore, $E_f = \text{constant}$. This is a very important result. As we have said earlier, E_f can be considered to be the equilibrium electrochemical potential for electrons. For a system in equilibrium, this potential must be constant throughout the system.

We can use this result to analyze the example of varying donor concentration. In equilibrium we can draw the Fermi level immediately since $E_f = \text{constant}$.



The Fermi level can also be expressed as

$$E_f = E_i + kT \ln \frac{n}{n_i}$$

We can use this expression to draw the rest of the band diagram.

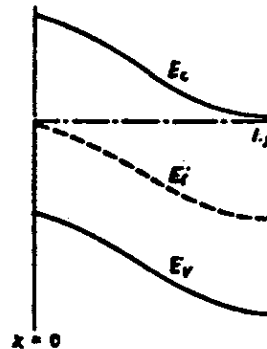
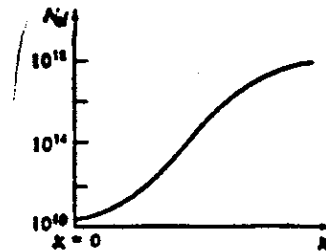
We must also assume "quasi neutrality". This says that $n \cong N_d^+$. This is a reasonable assumption since for normal doping concentrations, small deviations from neutrality cause a large enough electric field to keep the carriers near the dopant ions.

Thus, assuming complete ionization,

$$E_f - E_i \cong kT \ln \frac{N_d(x)}{n_i} \quad (48)$$

We can now draw the band diagram for doping varying with position.

1. $E_f = \text{constant}$.
2. When N_d is large, E_f is close to E_c .
3. When N_d is small (on the order of n_i), E_f is near E_i .
4. E_i and E_v track E_c since the bandgap is constant.



Also recall that in equilibrium

$$F_n = -D_n \frac{dn}{dx} - \mu_n n \mathcal{E}_x \equiv 0$$

Therefore,

$$\mathcal{E}_x = -\frac{D_n}{\mu_n n} \frac{dn}{dx}$$

which, using the Einstein relationship, can be written as

$$\mathcal{E}_x = -\frac{kT}{q} \frac{1}{n} \frac{dn}{dx} \quad (49)$$

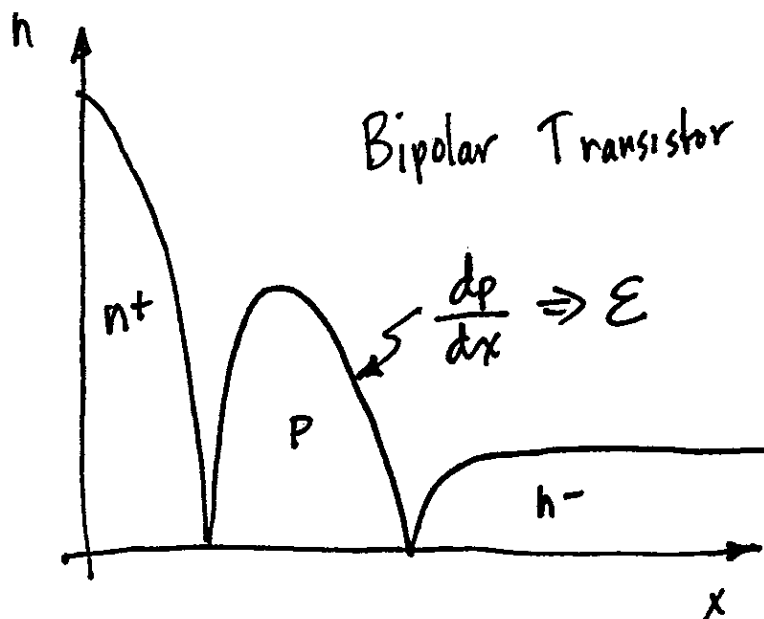
We may conclude, therefore, that an impurity gradient results in a built-in electric field in the semiconductor.

We may also write a similar expression for holes.

$$\mathcal{E}_x = \frac{kT}{q} \frac{1}{p} \frac{dp}{dx} \quad (50)$$

The concept of built-in field will be an important concept to use in analyzing devices.

For example, in a bipolar transistor the doping in the base is graded such that minority carriers injected into the base will be accelerated by the built-in electric field across the base, speeding up the response time.



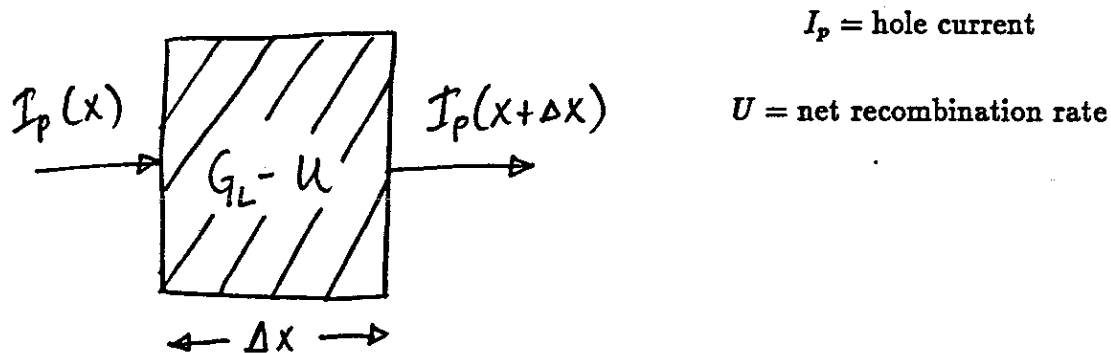
Fundamental Semiconductor Equations

So far, we have considered

1. Carrier transport
 - (a) drift
 - (b) diffusion
2. Generation of carriers
3. Recombination of carriers

All of these phenomena can be considered together in terms of current continuity.

Considering one-dimensional hole current into and out of a small region.



Since charge can neither be created nor destroyed,

$$(\text{rate of change of number of holes in region}) = -(\text{net recombination}) - (\text{net flow out of region})$$

From our discussion of recombination and generation, the net recombination rate (including light generation) is

$$U - G_L \approx \frac{p - p_0}{\tau_p} - G_L \quad (l.l.i.)$$

Therefore, in one dimension, the net rate of hole recombination is given by

$$(U - G_L) \Delta x = \left(\frac{p - p_0}{\tau_p} - G_L \right) \Delta x$$

The net rate at which holes flow out of the region is

$$\frac{1}{q}I_p(x + \Delta x) - \frac{1}{q}I_p(x) = \frac{1}{q} \frac{dI_p}{dx} \Delta x$$

using the definition of the derivative. The rate of loss of holes from the region is given by

$$-\frac{dp}{dt} \Delta x$$

Therefore,

$$-\frac{dp}{dt} \Delta x = \frac{1}{q} \frac{dI_p}{dx} \Delta x + \frac{p - p_0}{\tau_p} \Delta x - G_L \Delta x$$

so

$$\frac{dp}{dt} = -\frac{1}{q} \frac{dI_p}{dx} - \frac{p - p_0}{\tau_p} + G_L \quad (51)$$

Similarly, for electrons,

$$\frac{dn}{dt} = \frac{1}{q} \frac{dI_n}{dx} - \frac{n - n_0}{\tau_n} + G_L \quad (52)$$

These equations are called the continuity equations and are essential to understanding semiconductor device operation.

The transport equations which we already derived are also fundamental.

$$I_p = q \left(\mu_p p \mathcal{E} - D_p \frac{dp}{dx} \right) \quad (53)$$

$$I_n = q \left(\mu_n n \mathcal{E} + D_n \frac{dn}{dx} \right) \quad (54)$$

The continuity equations can also be written in terms of the carrier concentrations rather than the currents.

$$\begin{aligned} \frac{dp}{dt} &= D_p \frac{d^2 p}{dx^2} - \mu_p \left(\mathcal{E} \frac{dp}{dx} + p \frac{d\mathcal{E}}{dx} \right) - \frac{p - p_0}{\tau_p} + G_L \\ \frac{dn}{dt} &= D_n \frac{d^2 n}{dx^2} + \mu_n \left(\mathcal{E} \frac{dn}{dx} + n \frac{d\mathcal{E}}{dx} \right) - \frac{n - n_0}{\tau_n} + G_L \end{aligned}$$

The final basic equation is some form of Poisson's equation.

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{K\epsilon_0}$$

where

\mathcal{E} = electric field

ρ = space charge density (C/cm³)

K = dielectric constant (also ϵ_r or ϵ_s/ϵ_0)

ϵ_0 = permittivity of free space = 8.86×10^{-14} F/cm

Alternatively,

$$\mathcal{E} = \int \frac{\rho}{K\epsilon_0} dx$$

Thus if we know the space charge density, we determine the electric field. We know that the net charge density in a semiconductor is given by

$$\rho = q(p + N_d^+ - n - N_a^-)$$

and

$$\mathcal{E} = -\frac{dV}{dx} \quad (55)$$

Therefore,

$$\frac{d^2V}{dx^2} = \frac{q}{K\epsilon_0} [(n - p) + (N_a^- - N_d^+)] \quad (56)$$

Equations (51) through (56) constitute a system of six equations in six unknowns (n , p , I_n , I_p , \mathcal{E} and V) and, given appropriate boundary conditions, can be used to analyze the carrier concentrations, currents and fields in an arbitrary device structure. We will often be able to considerably simplify these equations by appropriate use of approximations.