Name Solutions

Example Exam #2 Problems — EE 482

Fall 2000

The test is open book/open notes. Show all work. Be sure to state all assumptions made and **check** them when possible. The number of points per problem are indicated in parentheses.

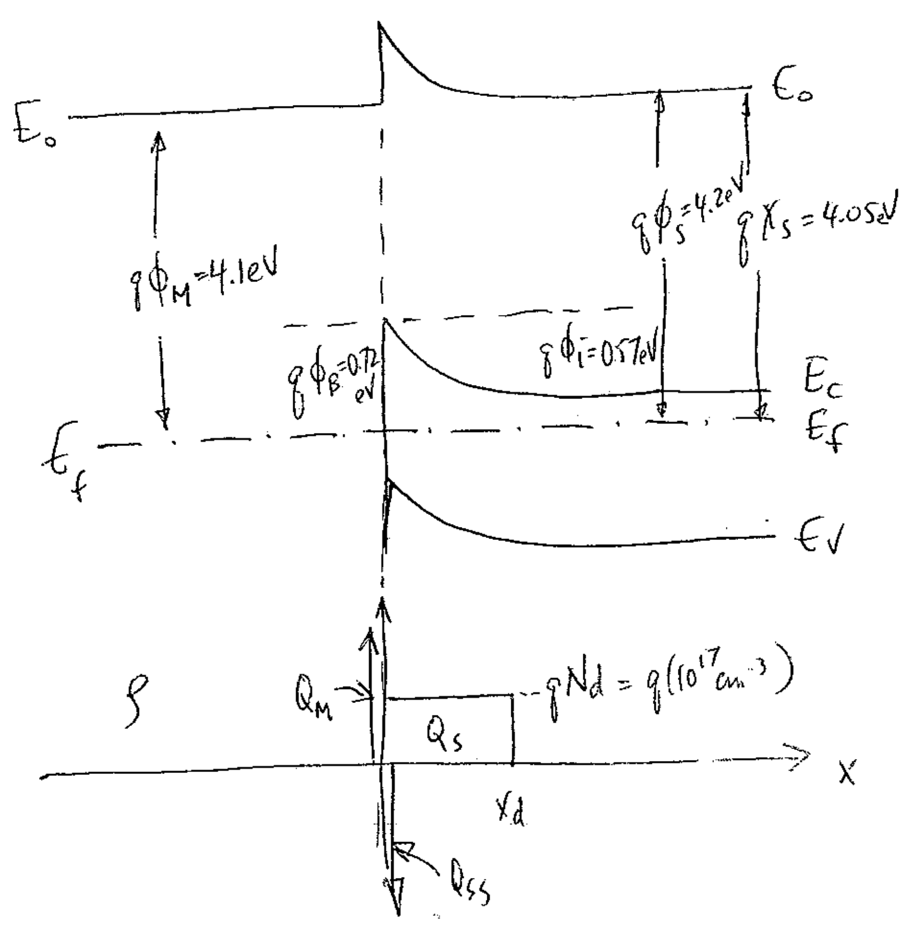
1. A contact is made between silicon ($\chi_s = 4.05\text{eV}$) doped with 10^{17}cm^{-3} of arsenic and aluminum ($\phi_m = 4.1\text{eV}$).

   (a) If a high density of surface states pins the Fermi level at 0.4eV above the valence band maxima, calculate ϕ_s, ϕ_B and ϕ_i and sketch the band diagram (including vacuum level and with barriers indicated) and the charge density versus position for the contact in equilibrium. (15)

$$\phi_s = \chi_s + \frac{kT}{q}\ln\left(\frac{N_c}{N_d}\right) = 4.05 + (0.026\text{V})\ln\left(\frac{2.8\times10^{19}\text{cm}^{-3}}{10^{17}\text{cm}^{-3}}\right) = \underline{4.2\text{V}}$$

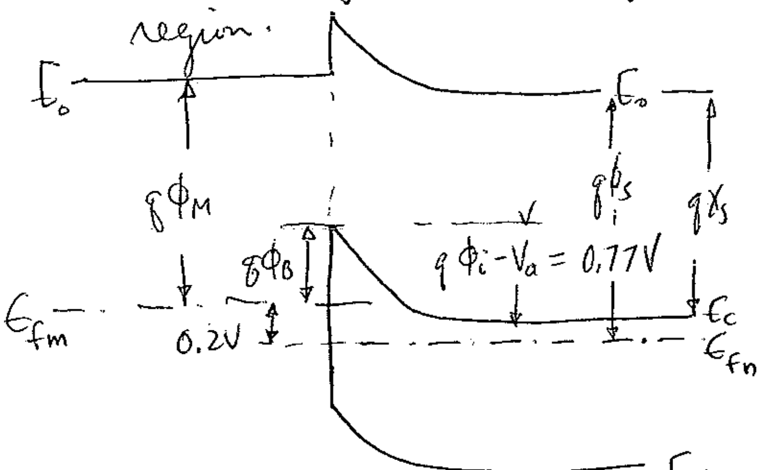
$$\phi_B = \frac{1}{q}(E_c - E_f)_{surf} = \frac{1}{q}(E_g - 0.4\text{eV}) = 1.12 - 0.4 = \underline{0.72\text{V}}$$

$$\phi_i = \frac{1}{q}\left[(E_c - E_f)_{surf} - (E_c - E_f)_{bulk}\right] = 0.72\text{ V} - 0.15\text{V} = \underline{0.57\text{V}}$$



(b) If a bias of 0.2V is applied to the semiconductor relative to the metal, calculate the junction capacitance per unit area and sketch the band diagram. (10)

+0.2 V applied to n-type semiconductor is reverse bias
For blocking contact, voltage is dropped across depletion region.



Reverse bias adds to built-in voltage

$\phi_i \rightarrow \phi_i - V_A$
$= 0.57 - (-.2V)$
$= 0.77V$

$$C' = \sqrt{\frac{q\epsilon_s N_d}{2(\phi_i + 0.2V)}} = \left(\frac{(1.6\times10^{-19}c)(11.8)(8.854\times10^{-14}\frac{F}{cm})(10^{17}cm^{-3})}{2(0.77V)}\right)^{1/2}$$

$$= 1.04\times10^{-7}\ F/cm^2$$

(c) If it was possible to lower the surface state density, how would the built-in potential (ϕ_i) change (larger, smaller, no change)? Explain briefly. (5)

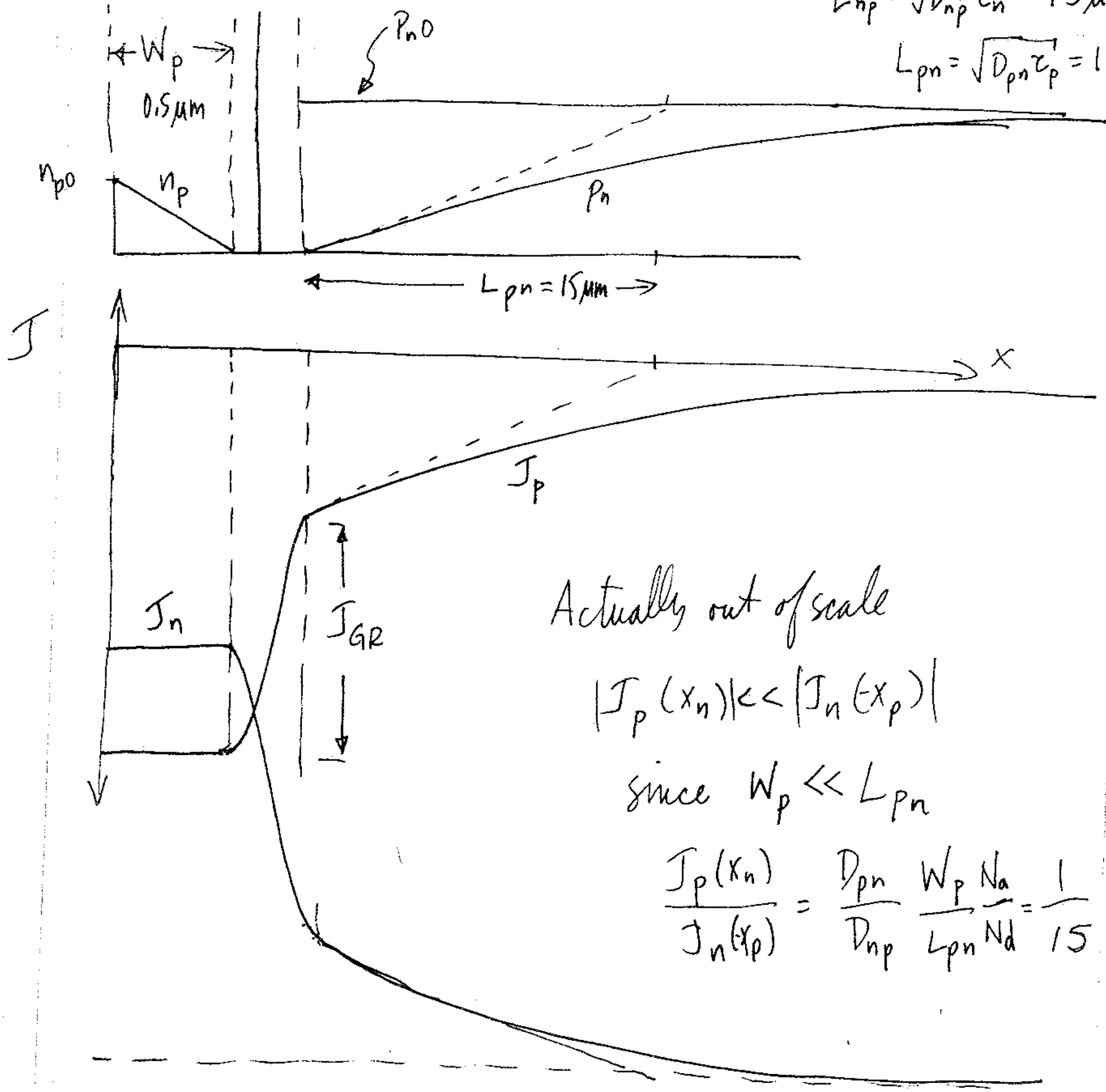
With fewer surface states, Φ_i would decrease.
The behavior of the contact would move closer to ideal behavior as lower density of surface states weakens pinning. $\Phi_i^{ideal} = \Phi_M - \Phi_S = 4.1 - 4.2 = -0.1V$
which is lower than pinned value of 0.57V.

2. In an abrupt silicon *p-n* junction, $N_a = 10^{18}\text{cm}^{-3}$, $N_d = 5 \times 10^{17}\text{cm}^{-3}$, $\tau_n = \tau_p = 0.25\mu\text{s}$, $D_n = 9\,\text{cm}^2/\text{s}$ and $D_p = 4\,\text{cm}^2/\text{s}$ in the *p*-region and $D_n = 25\,\text{cm}^2/\text{s}$ and $D_p = 9\,\text{cm}^2/\text{s}$ in the *n*-region, $W_p = 0.5\mu\text{m}$ and $W_n = 500\mu\text{m}$. $T = 300\text{K}$.

   (a) Sketch the minority carrier densities and the hole and electron current densities as functions of position under reverse bias. Do not ignore generation/recombination in the depletion region. (8)

$L_{np} = \sqrt{D_{np}\tau_n} = 15\mu m \gg W_p$

$L_{pn} = \sqrt{D_{pn}\tau_p} = 15\mu m \ll W_n$



Actually out of scale

$|J_p(x_n)| \ll |J_n(-x_p)|$

since $W_p \ll L_{pn}$

$$\frac{J_p(x_n)}{J_n(-x_p)} = \frac{D_{pn}}{D_{np}} \frac{W_p}{L_{pn}} \frac{N_a}{N_d} = \frac{1}{15}$$

(b) Calculate the current density in this junction with −2V applied Assume that $x_d' = x_d/4$ (8)

$$J = J_p(x_n) + J_n(-x_p) + J_{GR}$$

$$\phi_i = \frac{kT}{q}\ln\frac{N_d N_a}{n_i^2} = 0.92V$$

$$J_p(x_n) = \frac{q n_i^2 D_{pn}}{N_d L_{pn}}\left(\exp\frac{qV_A}{kT} - 1\right) = -4.\times10^{-13}\ A/cm^2$$

$$J_n(-x_p) = \frac{q n_i^2 D_{np}}{N_a W_p}\left(\exp\frac{qV_A}{kT} - 1\right) = -6.0\times10^{-12}\ A/cm^2$$

$$J_{GR} = -\frac{q x_d' n_i}{2\tau_0} \quad \text{(under reverse bias)}$$

$$= -1.25\times10^{-8}\ A/cm^2$$

$$x_d' = \frac{x_d}{4} = \frac{1}{4}\sqrt{\frac{2K_s\epsilon_0(\phi_i - V_A)(N_a+N_d)}{qN_dN_a}}$$

$$= 2.7\times10^{-6}\ cm = 0.027\ \mu m$$

$$J = J_p(x_n) + J_n(-x_p) + J_{GR} \cong J_{GR} = -1.25\times10^{-8}\ \frac{A}{cm^2}$$

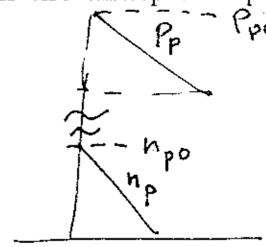
(c) If light of energy $h\nu > E_g$ was incident on this diode, causing the generation of $10^{12} cm^{-2} s^{-1}$ hole-electron pairs within a diffusion length of the depletion region, what would the new current be? (4)

$$J_{ph} = -qG = -(1.6\times10^{-19}C)(10^{12}\ cm^{-2}s^{-1}) = -1.6\times10^{-7}\ A/cm^2$$

$$J = J_{ph} + J_{dark} = -1.6\times10^{-7}\ \frac{A}{cm^2} - 1.25\times10^{-8}\ \frac{A}{cm^2}$$

$$= -1.7\times10^{-7}\ \frac{A}{cm^2}$$

(d) Using the diffusion approximation and assuming low-level injection and quasi-neutrality in the undepleted regions, calculate the electric field in the undepleted p region ($x < -x_p$) Justify the diffusion approximation there (10)



$$\Delta p \cong \Delta n \quad (\text{quasi-neutrality})$$

$$= -n_{po}\frac{x}{W_p}$$

$$J_{p_p} = q\mu_{p_p} p \mathcal{E} - q D_{p_p}\frac{dp}{dx} = J_i(x_n) + J_{GR} = -1.25\times10^{-8}\frac{A}{cm^2} \ (\text{from (a) \& (b)})$$

$$\mathcal{E} = \left(\frac{q D_{p_p}\frac{dp}{dx}}{q\mu_{p_p} p} - \frac{1.25\times10^{-8}\frac{A}{cm^2}}{q\mu_{p_p} p}\right) = \frac{kT}{q}\frac{1}{p}\frac{dp}{dx} - \frac{1.25\times10^{-8}\,A/cm^2}{q D_{p_p} p/(kT/q)}$$

$$p = 10^{18},\ D_{p_p} = 4\,cm^2/s,\ \frac{kT}{q} = 0.026\,V,\ \frac{dp}{dx} = -\frac{n_{po}}{W_p} = -\frac{n_i^2}{(10^{18}\,cm^{-3})(5\times10^{-5}\,cm)} = -4.2\times10^{6}\,cm^{-4}$$

$$\mathcal{E} = (0.026\,V)\left(\frac{-4.2\times10^{6}\,cm^{-4}}{10^{18}\,cm^{-3}}\right) - \frac{(1.25\times10^{-8}\,A/cm^2)(0.026\,V)}{(1.6\times10^{-19}\,C)(4\,cm^2/s)(10^{18}\,cm^{-3})}$$

$$= -1.1\times10^{-13}\frac{V}{cm} - 5.1\times10^{-10}\,V/cm = \underline{-5.1\times10^{-10}\frac{V}{cm}}$$

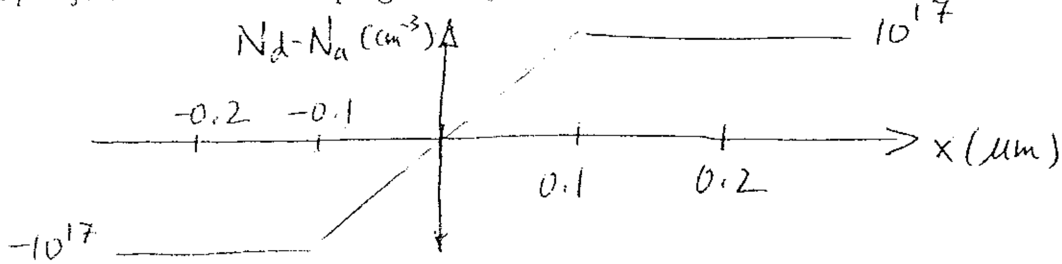
$$\left|J_{n_p}^{drift}\right| = q\mu_n \mathcal{E} n = q\mu_n \mathcal{E} n_{po}\left(1 - \frac{x}{W_p}\right)$$

$$= \left|(1.6\times10^{-19}\,C)\left(\frac{4\,cm^2/s}{0.026\,eV}\right)\left(-5.1\times10^{-10}\frac{V}{cm}\right)(210\,cm^{-3})\left(1 - \frac{x}{W_p}\right)\right|$$

$$= 2.6\times10^{-24}\frac{A}{cm^2}\left(1 - \frac{x}{W_p}\right) \lll \left|J_{n_p}^{diff}\right| = 6\times10^{-12}\frac{A}{cm^2}$$

Diffusion Approximation is Excellent!

3 A silicon p-n junction has the doping density shown below.



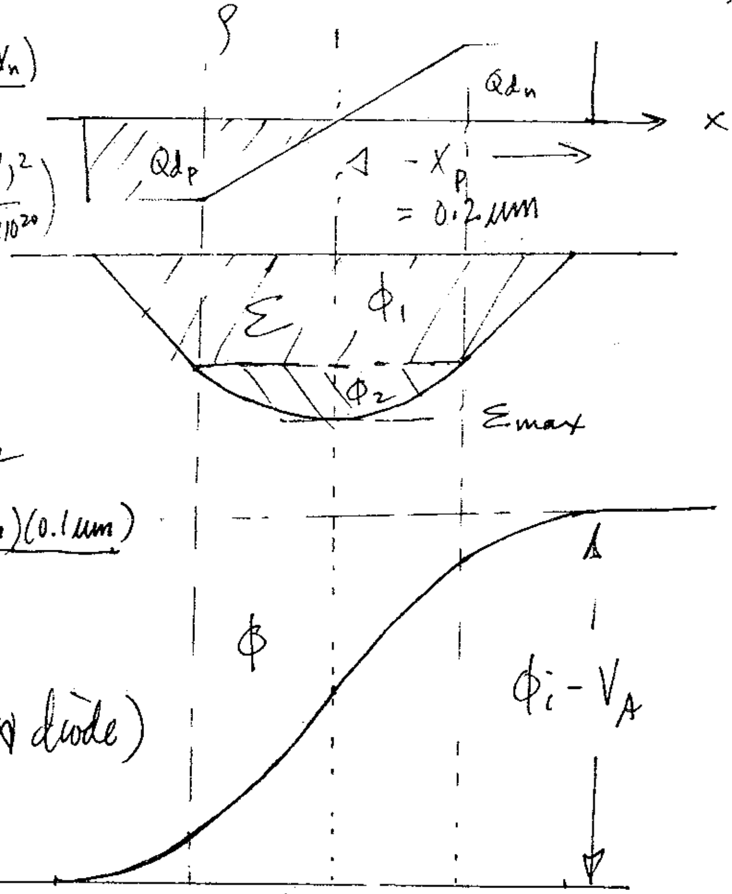
(a) Sketch the charge density, electric field and voltage as functions of position assuming $x_p = 0.2\mu m$. Determine the maximum electric field (in magnitude) and the applied bias. ($T = 300°K$) (16)

$X_n = X_p = 0.2\ \mu m$

$\phi_i = \frac{kT}{q} \ln \frac{N_a(-x_p) N_d(x_n)}{n_i^2}$

$= 0.\ \ V \ln\left(\frac{(10^{17})^2}{2.1 \times 10^{20}}\right)$

$= 0.82V$



$ℰ_{max} = \int_{-x_n}^{0} \frac{\rho(x)}{K_s \epsilon_0} dx$

$= -\frac{Q_{dp}}{K_s \epsilon_0} = \frac{q N_a (0.1 + \frac{0.1}{2})\mu}{K_s \epsilon_0}$

$= \frac{(1.6 \times 10^{-19} C)(10^{17} cm^{-3})(0.15\ \mu m)}{(11.8)(8.854 \times 10^{-14} F/cm)}$

$= 2.3 \times 10^5\ V/cm$

$\phi_i - V_A = \phi_1 + \phi_2$

$\phi_1 = \frac{q N_a (0.3\ \mu m)(0.1\ \mu m)}{K_s \epsilon_0}$

$= 4.6\ V$

(just as for PiN diode)

$V_A = \phi_i - (\phi_1 + \phi_2)$

$= 0.82V - (5.6V)$

$V_A = -4.8V$

$\phi_2 = \frac{2qa(0.1\ \mu m)^3}{3K_s \epsilon_0} = \frac{2(1.6 \times 10^{-19} C)(10^{17} cm^{-3})(0.1\ \mu m)^2}{3(11.8)(8.854 \times 10^{-14} F/cm)} = 1.0\ V$

Just as in graded junction with $x_p = 0.1\ \mu m$

(b) At large reverse biases, by what factor will the capacitance change if the bias is increased by a factor of 2. (4)

For large reverse biases, doping is uniform ($x_d >> 0.1\ \mu m$)

so as for step junction $x_d \propto \sqrt{V_R}$ and $C \propto \frac{1}{x_d} \propto V_R^{-1/2}$

$V_R * 2 \Rightarrow C * \frac{1}{\sqrt{2}} = C * 0.71$

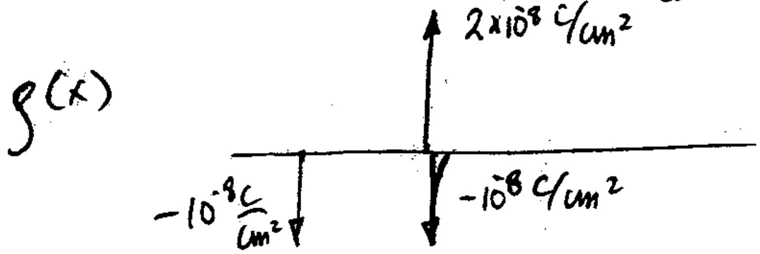
4. In a silicon ($\chi_s = 4.15\,\text{V}$) MOS ~~transistor~~ capacitor with an n-type substrate and an aluminum gate ($\phi_m = 4.1\,\text{V}$), $V_B = 0$. The substrate doping is $N_d = 10^{16}\text{cm}^{-3}$ and the oxide thickness is 400 Å. There is a positive oxide charge of $Q_{ss} = 2 \times 10^{-8}\,\text{C/cm}^2$ located at the oxide/semiconductor interface. There is a charge on the gate of $Q_g = -10^{-8}\,\text{C/cm}^2$.

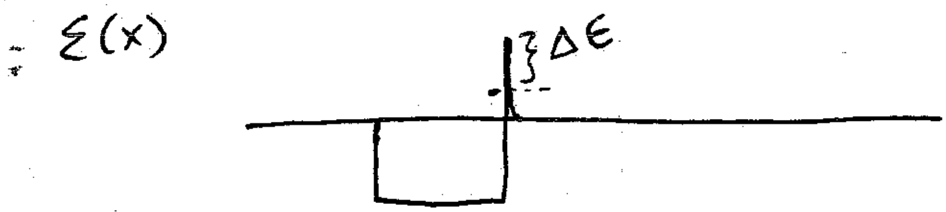
(a) Determine the state of the channel region (accumulation, flat-band, depletion, strong inversion, etc.). (5)

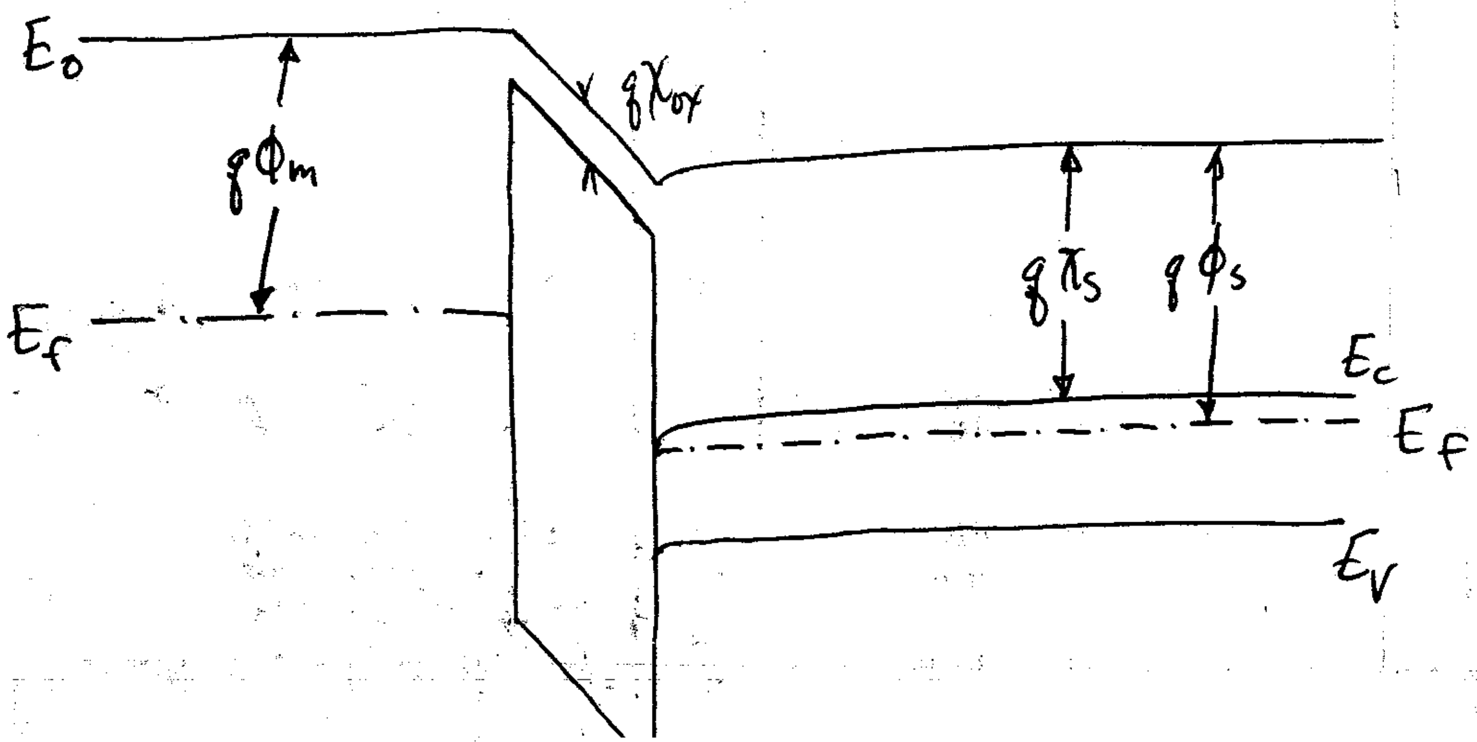
$$Q_s' = -(Q_g' + Q_{ss}') = -\left(-10^{-8}\frac{\text{C}}{\text{cm}^2} + 2\times 10^{-8}\frac{\text{C}}{\text{cm}^2}\right) = -10^{-8}\frac{\text{C}}{\text{cm}^2}$$

$Q_s' < 0$, n-type substrate $\Rightarrow$ <u>accumulation</u>

(b) Sketch the charge density, electric field and energy band diagram for the system. (10)







(c) Determine the applied gate voltage. (7)

$$V_{gb} - \phi_{ms} = V_{ox} + V_s \qquad V_s << V_{ox}$$

$$\cong V_{ox} \cong \frac{Q_g'}{C_{ox}'}$$

$$C_{ox}' = \frac{\epsilon_{ox}}{X_{ox}} = \frac{3.9 \times 8.854 \times 10^{-14}\ F/cm}{400 \times 10^{-8}\ cm} = 8.63 \times 10^{-8}\ \frac{F}{cm^2}$$

$$\phi_{ms} = \phi_m - \left(\chi_s + \frac{(E_c - E_f)_{bulk}}{q}\right)$$

$$= 4.1V - (4.15V + 0.21V)$$

$$= -0.26V$$

$$\frac{1}{q}(E_c - E_f)_{bulk} = \frac{kT}{q} \ln\left(\frac{N_c}{N_d}\right)$$

$$= .026V \ln\left(\frac{3 \times 10^{19}\ cm^{-3}}{10^{16}\ cm^{-3}}\right)$$

$$= 0.21V$$

$$V_{gb} \cong \phi_{ms} + V_{ox}$$

$$\cong -0.26V + \frac{-10^{-8}\ C/cm^2}{8.63 \times 10^{-8}\ F/cm^2} = \underline{\underline{-0.38\ V}}$$

End Of Exam