Name $\underline{\text{Solutions}}$

Final Exam — EE 482

Fall 2002

The test is open book/open notes. Show all work. Be sure to state all assumptions made and **check** them when possible. The number of points per problem are indicated in parentheses. Total of 160 points in 8 problems on 8 pages. If not otherwise specified, assume $T = 300\text{K}$ and the material is silicon.

1. A bulk silicon sample is doped with 10^{15}cm^{-3} of arsenic. Recombination occurs primarily via a density of 10^{12}cm^{-3} traps 0.1 eV above the intrinsic Fermi level. Assume traps have a capture cross-section of 10^{-14}cm^2 and the thermal velocities are $v_{thn} = 10^7\text{cm/sec}$ and $v_{thp} = 6 \times 10^6\text{cm/sec}$.

   If light generates $10^{20}\text{cm}^{-3}\text{sec}^{-1}$ carriers throughout the sample, what would be the steady-state electron concentration? (15)

$\tau_n = (v_{thn} N_t \sigma_t)^{-1} = 10^{-5}\,s \qquad \tau_p = (v_{th} N_t \sigma_t)^{-1} = 1.67\times10^{-5}\,s$

$p_1 < n_1 = n_i \exp\left(\frac{0.1}{kT}\right) = 7.1\times10^{11}\,cm^{-3} << n_0 = N_{As} = 10^{15}\,cm^{-3}$

Assume low level injection, $\Delta p = \tau_p G_L$ (n-type)

$= 1.67\times10^{15}\,cm^{-3} \sim n_0 = 10^{15}$

$\Rightarrow$ <u>not</u> low level injection

In steady-state

$$G_L = \frac{pn - n_i^2}{\tau_p(n+n_1) + \tau_n(p+p_1)}$$

$$\cong \frac{(p_0+\Delta p)(n_0+\Delta p) - n_0 p_0}{\tau_p(n_0+\Delta p+n_1) + \tau_n(p_0+p_1+\Delta p)} \cong \frac{\Delta p(n_0+\Delta p)}{\tau_p(n_0+\Delta p) + \tau_n \Delta p}$$

$$\Delta p^2 + \Delta p(n_0 - G_L(\tau_p+\tau_n)) - G_L \tau_p n_0 = 0$$

$$\Delta p^2 + \Delta p(10^{15} - 2.67\times10^{15}) - 1.67\times10^{30} = 0 \qquad \Delta p > 0$$

$$\Delta p = \frac{1.67\times10^{15} \pm \sqrt{(1.67\times10^{15})^2 + 4(1.67\times10^{30})}}{2} = 2.4\times10^{15}\,cm^{-3}$$

$$\underline{\underline{n = n_0 + \Delta p = 3.4\times10^{15}\,cm^{-3}}}$$

2. The structure of the conduction band in silicon is changed by strain such that the minima elongated in the z-direction are raised in energy by 30 meV relative to those elongated in the x and y directions. Assuming that the shape of the minima are otherwise unchanged ($m_t^* = 0.19m_0$, $m_l^* = 0.98m_0$), what would be the electron effective mass for an electric field in the x-direction? Hint: Think about the relative number of carriers in each minima. (12)

States in higher minima are $\exp\left(-\frac{30\text{ meV}}{kT}\right) = 0.31$ times as likely to be occupied

Average acceleration is $\langle\vec{a}\rangle = \langle M^{-1}\vec{F}\rangle =$

x-minima, y-minima, z-minima

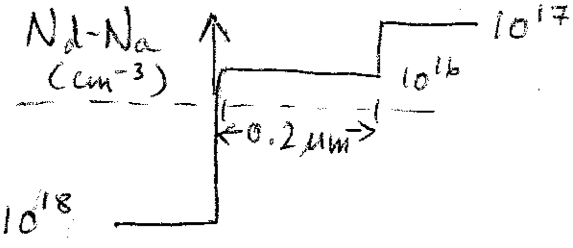
$$\langle m_x^{-1}\rangle = \frac{m_l^{-1} + m_t^{-1} + 0.31\, m_t^{-1}}{1 + 1 + 0.31} = \frac{(0.98)^{-1} + 1.31(0.19)^{-1}}{2.31} m_0^{-1}$$

$$= 3.43\, m_0^{-1}$$

$$\langle m_x\rangle = \frac{1}{3.43} = 0.29\, m_0$$ (compared to $0.26 m_0$ for unstrained Si)

3. A silicon p-n junction has the doping profile shown to the right.

   If the width of the depletion region on the n-side is 0.25μm, sketch the charge density, electric field and potential versus distance. What is the applied bias? (20)



$$Q_n' = q\left[(2\times10^{-5}\ \mathrm{cm})(10^{16}\ \mathrm{cm}^{-3}) + (5\times10^{-6}\ \mathrm{cm})(10^{17}\ \mathrm{cm}^{-3})\right]$$

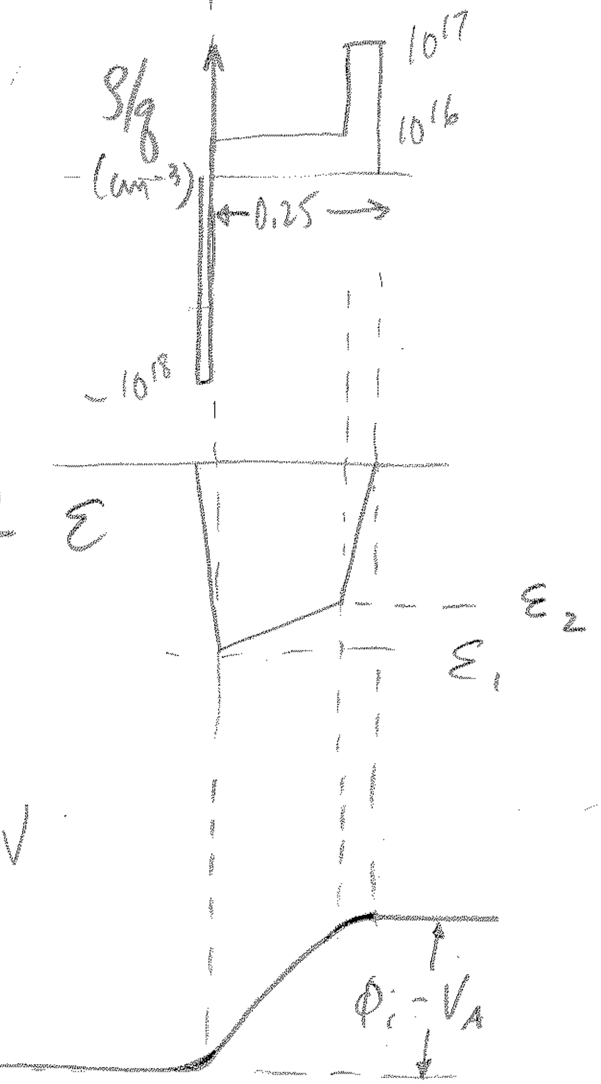
$$= 1.1\times10^{-7}\ \mathrm{C/cm^2}$$

$$\mathcal{E}_1 = \frac{Q_p'}{K_S\epsilon_0} = -\frac{Q_n'}{K_S\epsilon_0} = -1.08\ 10^5\ \frac{\mathrm{V}}{\mathrm{cm}}$$

$$\mathcal{E}_2 = -\frac{(5\times10^{-6}\ \mathrm{cm})(10^{17}\ \mathrm{cm}^{-3})(1.6\times10^{-19}\ \mathrm{C})}{K_S\epsilon_0}$$

$$= \frac{5}{7}\mathcal{E}_1 = -7.7\times10^{4}\ \frac{\mathrm{V}}{\mathrm{cm}}$$

$$x_p = \frac{Q_n'}{q N_a} = 7\times10^{-7}\ \mathrm{cm}$$



$$V_A - \phi_i = \frac{\mathcal{E}_1 x_p}{2} + \frac{\mathcal{E}_1+\mathcal{E}_2}{2}\,2\times10^{-5}\ \mathrm{cm} + \frac{\mathcal{E}_2}{2}\,5\times10^{-6}\ \mathrm{cm} = \frac{\mathcal{E}_1}{2}\left[7\times10^{-7}\ \mathrm{cm} + \frac{12}{7}(2\times10^{-5}\ \mathrm{cm}) + \frac{5}{7}(5\times10^{-6}\ \mathrm{cm})\right]$$

$$= -2.08\ \mathrm{V}$$

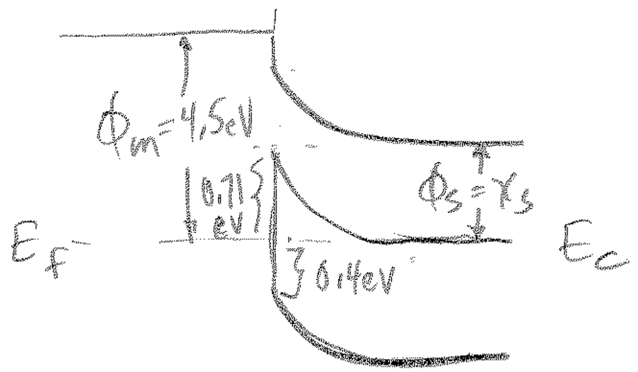
$$\phi_i = \frac{kT}{q}\ln\left(\frac{10^{17}\,10^{18}\ \mathrm{cm}^{-6}}{2\times10^{20}\ \mathrm{cm}^{-6}}\right) = 0.88\ \mathrm{V}$$

$$V_A = \Phi_i - 2.08\ \mathrm{V}$$

$$= \underline{\underline{-1.20\ \mathrm{V}}}$$

4. A contact is made between tungsten ($\phi_m = 4.5\text{eV}$) and silicon ($\chi_s = 4.05\text{eV}$). A high density of surface states pins the Fermi level at 0.4eV above the valence band maxima.

   How high would the donor doping have to be to reduce the equilibrium depletion region width to 30 Å so that tunneling will occur readily? Assume that for the heavy doping required, $E_f \cong E_c$ in the bulk (far from contact). (15)



$$x_d = \sqrt{\frac{2K_s\epsilon_0 \phi_i}{q N_d}}$$

$$N_d = \frac{2K_s\epsilon_0 \phi_i}{q x_d^2}$$

$$= \frac{2(11.7)(8.854\times10^{-14}\ \frac{F}{cm})(0.71V)}{q(30\times10^{-8}\,cm)^2}$$

$$\phi_i = E_g - 0.4\text{eV} = 0.71\text{eV}$$

$$\underline{\underline{N_d = 1.0\times10^{20}\ cm^{-3}}}$$

5. A silicon n-channel MOS transistor with a polysilicon gate heavily doped with phosphorus such that $E_f = E_c + 0.1\text{eV}$ has a substrate doping of $N_a = 5 \times 10^{17}\text{cm}^{-3}$ and an oxide thickness of $x_{ox} = 30\,\text{Å}$. $W = L = 0.25\mu\text{m}$. Oxide charges are negligible. The device is biased with $V_{SB} = 1\,V$ and $V_{DS} = 0$, and with V_{GS} such that the voltage dropped across the oxide is $1.0\,\text{V}$ (more positive near gate).

Calculate the **charge on the semiconductor** and the **applied voltage between the gate and source**. Sketch the **charge density** and **band diagram** for this transistor near the center of the channel (half-way between source and drain). (25)

$$C_{ox}' = \frac{K_{ox}\epsilon_0}{x_{ox}} = 1.15\times10^{-6}\ F/cm^2$$

$$Q_s = -Q_m = -C_{ox}' V_{ox} = -1.15\times10^{-6}\ C/cm^2$$

$Q_s < 0$, n-type, so depleted or inverted

$$\phi_F = -\frac{kT}{q}\ln\left(\frac{N_a}{n_i}\right) = -0.45V$$

$$Q_{d,max}' = -\sqrt{2K_s\epsilon_0 q N_a |V_{SB}-2\phi_F|}$$

$$V_{SB}-2\phi_F = 1.9V$$

$$= -5.6\times10^{-7}\ C/cm^2$$

$|Q_{d,max}'| < Q_s'$, so inverted

$$V_s = V_{SB} - 2\phi_F = 1.9V$$

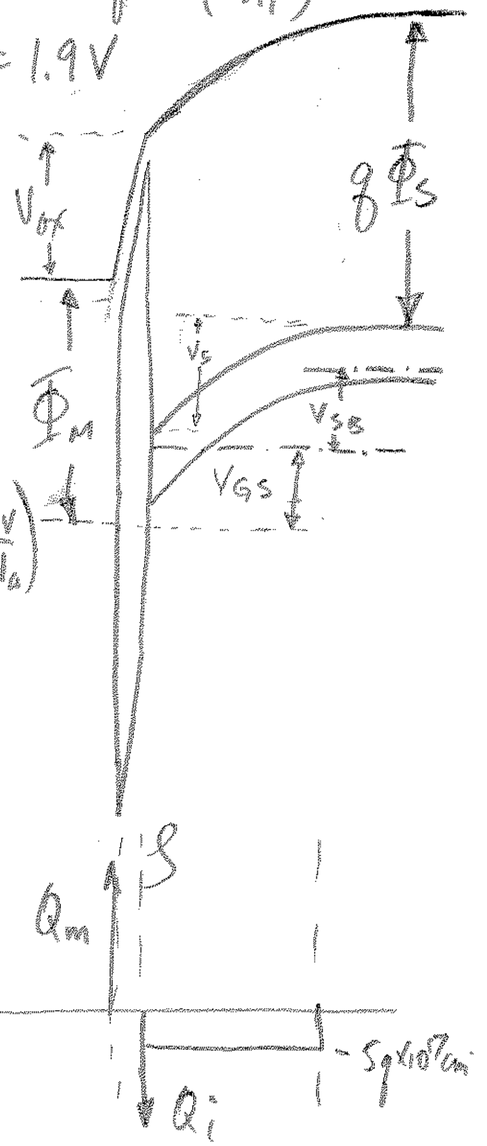
$$\Phi_M = \chi_s - 0.1V = 3.95eV$$

$$V_{GB} = V_{ox} + V_s + \phi_{MS}$$

$$\Phi_s = \chi_s + E_g - \frac{kT}{q}\ln\left(\frac{N_v}{N_a}\right) = 5.08\,eV$$

$$= 1 + 1.9 - 1.13 = 1.77V$$

$$V_{GS} = V_{GB} - V_{SB} = \underline{0.77\,V}$$



6. The same transistor is biased with $V_{SB} = 1\,V$ and $V_{GS} = 2$V.

(a) Calculate V_T and δ for $V_{SB} = 1\,V$. (12)

$$V_T = \phi_{ms} - 2\phi_F - \frac{Q'_{d,max}}{C'_{ox}} = -1.13\,V + 0.90\,V + \frac{5.6\times10^{-7}\,C/cm^2}{1.1\times10^{-6}\,C/cm^2} = \underline{0.28\,V}$$

$$\delta = \frac{1}{C'_{ox}}\left|\frac{dQ'_{d,max}}{dV_{SB}}\right|_{V_{SB}=1V} = \frac{1}{2C'_{ox}}\frac{|Q'_{d,max}|}{|V_{SB}-2\phi_F|} = \frac{5.6\times10^{-7}\,C/cm^2}{2\left(1.1\times10^{-6}\,\frac{F}{cm^2}\right)(1.9V)}$$

$$= 0.13$$

(b) Calculate the saturation current using the linearized model). (10)

$$I_{DS}^{sat} = \frac{W}{L}C'_{ox}\mu'_n\left(V_{GS}-V_T\right)^2/(1+\delta) \qquad \mu'_n \cong \frac{\mu_n(5\times10^{17}\,cm^{-3})}{2}$$

$$= 1.1\times10^{-6}\,\frac{C}{cm^2}\left(190\,\frac{cm^2}{V\cdot s}\right)(2-0.28)^2/(1.13) \qquad \cong 190\,\frac{cm^2}{V\cdot s}$$

$$= 5.5\times10^{-4}\,A = 0.55\,mA$$

(c) Determine the output resistance if $V_{DS} = V_{DS}^{sat} + 0.5\,$V. (11)

$$\Delta L = \sqrt{2K_s\epsilon_0(0.5V)/qN_a} = 3.6\times10^{-6}\,cm$$

$$\frac{1}{r_o} = \frac{\Delta I_{ds}}{\Delta V_{ds}} = \frac{I_{ds}^{sat}\left(\frac{L}{L-\Delta L}-1\right)}{0.5\,V} = \frac{5.5\times10^{-4}A\left(\frac{2.5\times10^{-5}}{2.14\times10^{-5}}-1\right)}{0.5\,V}$$

$$= 1.85\times10^{-4}\,\mho \qquad r_o = 5.4\times10^{3}\,\Omega = 5.4\,k\Omega$$

7. A silicon *npn* transistor with constant doping in each region has $A_E = A_C = 10^{-8}\text{cm}^2$, $N_{dE} = 2 \times 10^{19}\text{cm}^{-3}$, $N_{aB} = 2 \times 10^{18}\text{cm}^{-3}$, and $N_{dC} = 4 \times 10^{17}\text{cm}^{-3}$. The width of the undepleted (quasi-neutral) emitter is $x_E = 0.4\mu\text{m}$, the width of the undepleted base region is $0.2\mu\text{m}$ and the width of the undepleted collector region is $1\mu\text{m}$. Assume these widths are unchanged throughout this problem. In all regions, the minority carrier lifetimes are $2 \times 10^{-6}\text{sec}$. The minority carrier diffusion coefficients are $2\text{cm}^2/\text{sec}$ in the base and emitter regions and $8\text{cm}^2/\text{sec}$ in the collector.

(a) Calculate β_F and β_R at moderate current levels. (10)

$$L_{nB} = \sqrt{D_{nB}\tau_{nB}} = \left[\left(2\,\frac{cm^2}{s}\right)\left(2\times10^{-6}\,s\right)\right]^{1/2} = 2\times10^{-3}\,cm = 20\mu$$

$$\alpha_T = 1 - \frac{x_B^2}{2L_{nB}^2} = 1 - \frac{(0.2)^2}{2(20)^2} = 0.99995$$

$$\alpha_F = \frac{1}{1 + \frac{x_B}{x_E}\frac{D_{pE}}{D_{nB}}\frac{N_{aB}}{N_{dE}}} = \frac{1}{1 + \frac{0.2}{0.4}\,\frac{2}{2}\,\frac{2\times10^{18}}{2\times10^{19}}} = 0.952$$

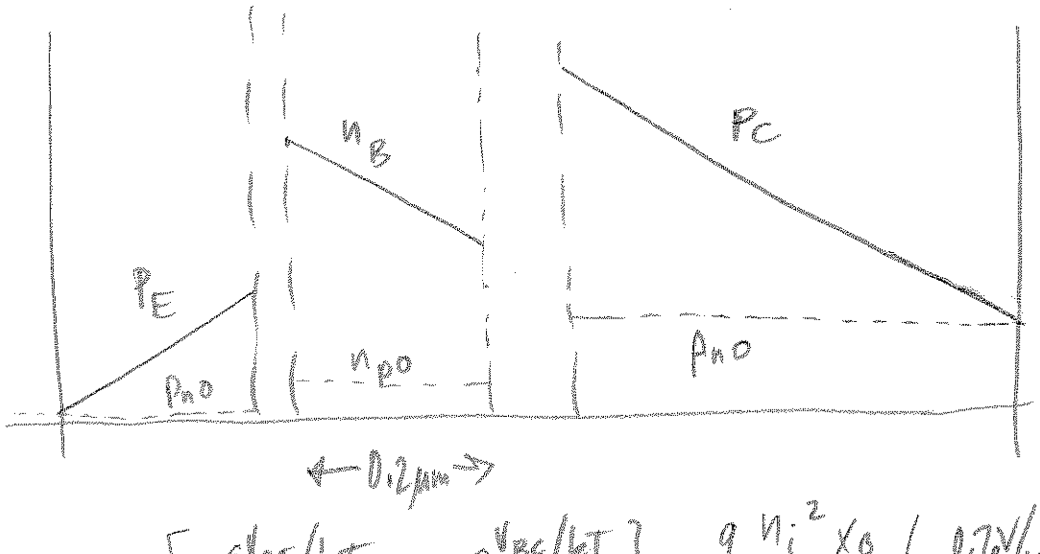
$$\beta_F = \frac{\alpha_F}{1-\alpha_F} = 20$$

$$\alpha_R = \frac{1}{1 + \frac{x_B}{x_C}\frac{D_{pC}}{D_{nB}}\frac{N_{aB}}{N_{dC}}} = \frac{1}{1 + \frac{0.2}{1}\,\frac{8}{2}\,\frac{2\times10^{18}}{4\times10^{17}}} = 0.2$$

$$\beta_R = \frac{\alpha_R}{1-\alpha_R} = \frac{1}{4}$$

(b) If $V_{BE} = 0.7V$ and $V_{CE} = 0.1V$, what mode is device in? Sketch the minority carrier concentrations through the active region of BJT as function of distance from emitter contact to collector contact. What is the excess minority charge in the base? (15)

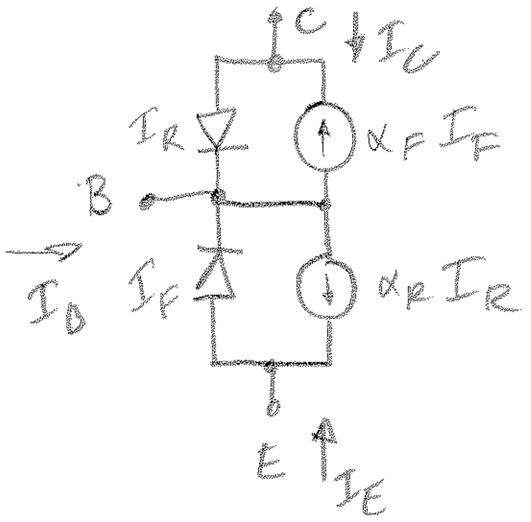
$V_{BE} > 0$, $V_{BC} = V_{BE} - V_{CE} = 0.6V > 0$ $\Rightarrow$ Device is saturated



$$q_B = \frac{n_{po}x_B}{2}\left[e^{qV_{BE}/kT} + e^{qV_{BC}/kT}\right] = \frac{q\,n_i^2\,x_B}{2N_{AB}}\left(e^{0.7eV/kT} + e^{0.6eV/kT}\right)$$

$$= 8.9\times10^{-11}\ C/cm^2$$

8. A *pnp* BJT has $\alpha_F = 0.99$, $\alpha_R = 0.1$, and $I_{ES} = 10^{-16}$ A. If $V_{BC} = -0.68$ V and $I_C = -0.1$ mA, determine the mode of the device and the value of I_B using the Ebers-Moll model. (15)



$$I_R = I_{CS}\left(e^{-qV_{BC}/kT} - 1\right)$$

$$= \frac{\alpha_F I_{ES}}{\alpha_R}\left(e^{-qV_{BC}/kT} - 1\right)$$

$$= 10^{-15}\,A\left(e^{0.68eV/kT} - 1\right) = 2.53\times10^{-4}\,A$$

$$I_C = I_R - \alpha_F I_F = 2.53\times10^{-4} - (0.99)\,I_F = -10^{-4}\,A$$

$$I_F = \frac{3.53\times10^{-4}\,A}{0.99} = 3.56\times10^{-4}\,A = I_{ES}\left(e^{-qV_{BE}/kT} - 1\right)$$

$$V_{BE} = -\frac{kT}{q}\ln\left(\frac{3.56\times10^{-4}\,A}{I_{ES}}\right) = -0.75\,V$$

V_{BE} & V_{BC} both negative for pnp, so <u>saturation</u>

$$I_B = -I_R(1-\alpha_R) - I_F(1-\alpha_F)$$

$$= -2.53\times10^{-4}\,A\,(0.9) - 3.56\times10^{-4}\,A\,(0.01) = -2.28\times10^{-4}\,A$$

$$= \underline{\underline{-0.23\,mA}}$$

End Of Exam