Homework #1 - EE 482

due 10/9/02

1. Consider an electron in an infinite potential well. Assume that the well has energy $V = 0$ for $0 < x < a$.

   (a) Determine the expected value of x, p_x (momentum) and E (energy) for the lowest three energy levels.

   (b) The standard deviation of the position is $\langle \Delta x \rangle = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$. A similar expression holds for $\langle \Delta p_x \rangle$. If the operator for ${p_x}^2$ is $-\hbar^2 \partial^2 / \partial x^2$, check the Heisenberg uncertainty principle ($\Delta x \Delta p_x \geq \hbar$) for the lowest energy state.

2. A band in an imaginary two dimensional crystal is given by

$$E = A(2{k_x}^2 + {k_y}^2) - B({k_x}^4 + 3{k_y}^4),$$

   with A and B positive constants.

   (a) Find the wavevectors and corresponding energies at which the electron velocity is zero. Which of these states represent maxima or minima of the energy band. Draw a rough sketch of the E versus k diagram.

   (b) Find the effective mass tensor for electrons in each of the states found in (a). Which of these states will result in purely "electron-like" behavior and which in purely "hole-like" behavior? How can you see this from the sketch of part(a).

   (c) If a force in the (11) direction is applied to an electron with a wavevector $(k_x, k_y) = (0, 0)$, what direction will the electron move?

   (d) Repeat (c) for one of the band maxima.

3. The outer occupied energy band in a crystal with N atoms per unit volume is described by

$$E = E_0 + \frac{\hbar^2 (k - k_0)^2}{2m^*}$$

   and the band contains one electron per atom.

   (a) Is this material an insulator, a metal or a semiconductor? Explain.

   (b) Give an expression for $E_f - E_0$ at 0K in terms of m^*?

4. Sketch the Fermi-Dirac distribution and appropriate forms of the Maxwell-Boltzmann approximation versus energy on a common set of axes. How far must the energy be above the Fermi level at 300K for the appropriate M-B approximation to result in an error of less than 3% in the occupation probability? How far below? Repeat for probability of being empty (1-p_{occ}).